

## Sliding Luttinger liquid phases

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We study systems of coupled spin-gapped and gapless Luttinger liquids. First, we establish the existence of a sliding Luttinger liquid phase for a system of weakly coupled parallel quantum wires, with and without disorder. It is shown that the coupling can *stabilize* a Luttinger liquid phase in the presence of disorder. We then extend our analysis to a system of crossed Luttinger liquids and establish the stability of a non-Fermi-liquid state: the crossed sliding Luttinger liquid phase. In this phase the system exhibits a finite-temperature, long-wavelength, isotropic electric conductivity that diverges as a power law in temperature  $T$  as  $T \rightarrow 0$ . This two-dimensional system has many properties of a true isotropic Luttinger liquid, though at zero temperature it becomes anisotropic. An extension of this model to a three-dimensional stack exhibits a much higher in-plane conductivity than the conductivity in a perpendicular direction.

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### I. INTRODUCTION

For over two decades a central theme in the study of correlated electronic systems has been the drive to understand and classify electronic states that do not conform to Landau's Fermi-liquid theory. A clear example of such "non-Fermi liquid" physics occurs in one dimension (1D),<sup>1</sup> where arbitrarily weak interactions destroy the Fermi surface and invalidate the notion of independent quasiparticles at low energy. Away from charge- and spin-density-wave instabilities, the 1D interacting electron gas forms a Luttinger liquid. The discontinuity in occupation at the Fermi energy is replaced by a power-law singularity, and the low-lying excitations are bosonic collective modes in which spin and charge decouple.

In this paper, we investigate how such non-Fermi liquid behavior could arise in two- and three-dimensional systems consisting of arrays of quantum wires or chains. Our analysis is largely motivated by the unusual normal-state properties of high  $T_c$  materials. Prominent among these are

- (1) Resistivity perpendicular to the  $\text{CuO}_2$  plane much larger than the in-plane resistivity.<sup>2</sup>
- (2) Linear temperature ( $T$ ) dependence of resistivity along the conducting planes, and  $1/T^2$  divergence of the Hall angle.<sup>3</sup>
- (3) Angle-resolved photoemission data showing a pseudogap and absence of dispersion in the  $c$  direction.<sup>4</sup>
- (4) Linear temperature dependence of the nuclear magnetic resonance relaxation  $1/T_1$ .<sup>5</sup>

Anderson has suggested that these unusual normal-state properties of the cuprates are the result of non-Fermi-liquid physics in two dimensions.<sup>6</sup> The study of non-Fermi liquids in dimensions greater than one has, however, proven to be quite difficult. Since the Fermi liquid is stable for weak interactions, perturbative methods about this state fail to locate non-Fermi liquid states.<sup>7</sup> Moreover, generalizations of the bosonization technique to isotropic systems in higher dimensions have indicated that Fermi-liquid theory survives provided the interactions are not pathologically long ranged.<sup>8</sup>

An alternative approach has been to study anisotropic systems consisting of arrays of parallel weakly coupled 1D

wires.<sup>9</sup> Coupled Luttinger liquids have been studied extensively for the past thirty years, mostly in the context of quasi one-dimensional conductors. Single-particle hoppings as well as pair-hopping correlations were shown to destabilize the Luttinger liquid phase. Following Anderson's suggestions regarding high-temperature superconductors, there have been a series of studies on coupled quantum chains, which confirm this view<sup>10</sup> though the issue remains controversial.<sup>11</sup> It has recently been proposed<sup>12,13</sup> that for a range of interwire charge and current interactions, there is a *smectic-metal* (SM) phase in which Josephson, charge- and spin-density-wave, and single-particle couplings are *irrelevant*. This phase is a two or three-dimensional anisotropic sliding Luttinger liquid whose transport properties exhibit power-law singularities like those of a 1D Luttinger liquid. It is the quantum analog of the sliding phases of coupled classical XY models first identified in the context of phases of DNA-lipid complexes.<sup>14,15</sup> The study of coupled Luttinger liquids becomes particularly relevant in the context of striped phases, which have recently been found in quantum Hall systems<sup>16</sup> and in the cuprates.<sup>17</sup> These phases are characterized by inhomogeneous distributions of charge and spin, where, for example, the charge carriers might be confined to separate linear regions, thus resembling stripes. Sliding Luttinger liquid phases might also be relevant for a variety of other systems such as quasi-one-dimensional organic conductors and ropes of nanotubes.

The work on sliding Luttinger liquid phases has also been extended to a square network of 1D wires formed by coupling two perpendicular smectic metals,<sup>18</sup> which has been shown to exhibit a *crossed sliding Luttinger liquid* (CSLL) phase. At finite temperature  $T$ , the CSLL phase is a 2D Luttinger liquid with an isotropic long-wavelength conductivity that diverges as a power law in  $T$  as  $T \rightarrow 0$ . At  $T=0$ , it is essentially two independent smectic metals. This model could be realized in man-made structures constructed from quantum wires such as carbon nanotubes. The extension of the model to a three-dimensional stack may be relevant to the stripe phases of the cuprates. Based on neutron and x-ray scattering measurements, it has been suggested that spin-

charge stripes in the adjacent  $\text{CuO}_2$  plane are orthogonal to each other.<sup>17</sup>

In order to understand the nature of the sliding phase, it is easiest to think in terms of the classical analog. Imagine a stack of 2D  $XY$  models coupled by the Josephson coupling  $\cos(\theta_n - \theta_{n+1})$  where  $\theta_n$  is the  $XY$ -angle variable in layer  $n$ . Such a system always goes directly from a 3D ordered phase to a completely disordered phase as a function of the temperature  $T$ . However in the presence of interlayer gradient coupling terms of the form  $\nabla \theta_m \cdot \nabla \theta_n$ , the system may have an intermediate sliding phase,<sup>15</sup> that exhibits an in-plane 2D order with power-law decay of correlations along the planes. In the absence of Josephson couplings, in the sliding phase,  $\theta$  in neighboring layers freely slide over each other with no energy cost, and two-point correlation functions are identical in form to those of a stack of decoupled 2D layers. When interlayer Josephson couplings are present, though irrelevant, two-point correlation functions decay exponentially perpendicular to the layers. Thus the sliding phase has in-plane 2D order but is disordered in the third direction. For sliding Luttinger liquid models, the 2D layers are replaced by 1D quantum wires, and interwire, current-current, and density-density couplings play the role of the gradient couplings. Thus, in the sliding Luttinger liquid phases, the correlation functions along a given wire exhibit the same power-law functional form as in a 1D Luttinger liquid.

In this paper, we review and discuss in detail, results on the sliding Luttinger liquid phase. Our discussion of the parallel Luttinger liquids follows Ref. 13, but it establishes the stability of the sliding phase to a more complete set of interwire operators and also to disorder. For the CSLL phase, we provide detailed calculations that were presented only briefly in Ref. 18. The stability and transport properties of the CSLL phase form the central results of the paper. We also extend the analysis to a three-dimensional stack of crossed Luttinger liquids, and show that we can still obtain a sliding Luttinger liquid phase, with hopping between planes being irrelevant. In this phase, the in-plane and perpendicular conductivity can be quite different and exhibit different temperature dependencies. This could be of relevance to the normal state of the cuprates.

This paper is organized as follows. In Sec. II we explore the stability of the sliding phase for an array of parallel coupled Luttinger liquids with respect to a large (but not complete) set of interwire operators. For a one-dimensional system of interacting spin-1/2 fermions, the spin excitations could either be gapped or gapless. In the spin-gapped Luther-Emery regime, the system can be described by a single Luttinger liquid for charge. In the gapless case, both spin and charge are dynamical degrees of freedom, and there are two Luttinger parameters ( $\kappa_\sigma, \kappa_\rho$ ), and two velocities ( $v_\sigma, v_\rho$ ). We study coupled Luttinger liquids, both for the spin-gapped and the gapless case, and in each case demonstrate the stability of the sliding phase. In addition, we study the effect of disorder. We find that density-density and current-current interactions that stabilize the sliding phase, also make disorder more strongly irrelevant. Note, however, that in this paper, we do not establish stability with respect to all multiwire operators. Since the overall strength of these higher-order

interactions that we neglect are expected to be much smaller than those we consider here, their relevance becomes important only at very small temperatures. We delay a more complete study of their effects to a future publication. Throughout the paper we will use the term ‘‘stable’’ somewhat loosely to refer to stability with respect to a restricted set of interactions that include all two-wire interactions. In Sec. III we extend the above analysis to two identical coupled Luttinger liquid arrays, arranged such that the wires of one array run perpendicular to those of the other. In this case we establish the stability of a CSLL phase to a large class of operators. The interarray density-density couplings effectively renormalize the intraarray couplings; however, the stability of the CSLL phase turns out to be identical to the stability of the smectic-metal phase for each array, but with the intraarray couplings replaced by the renormalized couplings. We then generalize our analysis to a three-dimensional stack of arrays, with wires on each array running perpendicular to the wires of the consecutive array, and obtain once more a sliding phase that appears stable. In Sec IV we explore the transport properties of sliding phases. In particular, we focus on the power-law singularities of the conductivity as a function of  $T$  as  $T \rightarrow 0$ . Finally Sec V sums up our principal results. Appendix A sketches out the steps used to obtain intrawire correlation functions, Appendices B and C present details of some integrals used in Secs. II and III, and Appendix D presents details of conductivity calculations using the Kubo formula.

## II. COUPLED PARALLEL LUTTINGER LIQUIDS

Our goal is to construct non-Fermi-liquid electron systems in two and three dimensions. Our basic building blocks are one-dimensional quantum wires that exhibit Luttinger-liquid phases with non-Fermi-liquid power-law decay of correlation functions. We couple these wires together in arrays in such a way that they retain their one-dimensional non-Fermi-liquid character yet allow nonvanishing interwire electron transport at nonzero frequency or temperature. We will consider several types of arrays, and it is useful to introduce a nomenclature for them. The long axis of parallel wires defines one direction in space. The wires can be arranged in either a one- or a two-dimensional Bravais lattice of points in the plane perpendicular to that direction. We, therefore introduce the notation  $d_\perp : 1$  to denote an array of parallel wires centered on lattice points in a  $d_\perp$ -dimensional space. The resulting structure occupies a  $d_\perp + 1$  dimensional space. Thus a 1:1 array is a planar array of equally spaced parallel wires, and a 2:1 array is a three-dimensional columnar array of wires. As stated, we will restrict our attention to arrays in which the wires lie on a  $d_\perp$ -dimensional periodic lattice. For 2:1 arrays, we will only consider, in detail, those based on a two-dimensional rectangular lattice. Both 1:1 and 2:1 arrays can exhibit anisotropic sliding phases for an appropriate choice of interwire potentials.

In this section we will consider anisotropic sliding metal phases in 1:1 and 2:1 arrays. In the succeeding sections, we will discuss how crossed 1:1 and 2:1 arrays can produce two and three-dimensional sliding phases with  $C_{4v}$  symmetry and

isotropic long-wavelength finite-temperature conductivity. We begin our discussion with a review of 1D Luttinger liquids.

### A. Review of one-dimensional Luttinger liquids

One-dimensional Luttinger liquids are most easily described in terms of collective bosonic charge and spin-density modes. The bosonized form of the fermion operators near the left and right Fermi points are<sup>19</sup>

$$R_{s,j}(x) = \frac{1}{\sqrt{4\pi\epsilon}} \eta_{R,s,j} e^{ik_F x} e^{i\Phi_{R,s,j}(x)},$$

$$L_{s,j}(x) = \frac{1}{\sqrt{4\pi\epsilon}} \eta_{L,s,j} e^{-ik_F x} e^{i\Phi_{L,s,j}(x)}, \quad (2.1)$$

where  $R$  and  $L$  stand for the right and left moving electrons, respectively,  $s$  is the spin index,  $j$  denotes the wire number,  $\epsilon$  is some intrachain cutoff, and  $\eta_{R/L,s,j}$  are the Klein factors. We can write down an effective theory for the low-energy excitations in terms of the boson operators  $\Phi$ . It is convenient to define

$$\theta_{s,j} = (\Phi_{R,s,j} + \Phi_{L,s,j}) / \sqrt{4\pi},$$

$$\phi_{s,j} = (\Phi_{R,s,j} - \Phi_{L,s,j}) / \sqrt{4\pi}, \quad (2.2)$$

where  $\theta_{s,j}$  is a phase variable, and  $\phi_{s,j}$  is the conjugate density variable.<sup>20</sup> Since the bosonic excitations can be characterized in terms of charge and spin excitations, we further define

$$\theta_{\rho,j} = (\theta_{\uparrow,j} + \theta_{\downarrow,j}) / \sqrt{2}, \quad \theta_{\sigma,j} = (\theta_{\uparrow,j} - \theta_{\downarrow,j}) / \sqrt{2}, \quad (2.3)$$

and similarly for the  $\phi$  variables. Here  $\rho$  characterizes the charge excitations, and  $\sigma$  the spin excitations. For a single Luttinger liquid, we can write down an effective Hamiltonian for the charge sector

$$H_\rho = \int dx \frac{v_\rho}{2} \left[ \frac{(\partial_x \theta_\rho)^2}{\kappa_\rho} + \kappa_\rho (\partial_x \phi_\rho)^2 \right], \quad (2.4)$$

where  $v_\rho$  is the velocity, and  $\kappa_\rho$  is a Luttinger liquid parameter (this is the inverse of the usual Luttinger liquid parameter  $g$ ). Alternately, we could write down the action, in a path-integral formulation, as

$$S_\rho = \int dx d\tau \left\{ \frac{v_\rho}{2} \left[ \frac{(\partial_x \theta_\rho)^2}{\kappa_\rho} + \kappa_\rho (\partial_x \phi_\rho)^2 \right] - 2i(\partial_x \theta_\rho) \times (\partial_\tau \phi_\rho) \right\}. \quad (2.5)$$

Integrating out either the  $\theta_\rho$  or the  $\phi_\rho$  variables, we obtain

$$S_{\rho,\phi} = \frac{\kappa_\rho}{2} \int dx d\tau \left[ v_\rho (\partial_x \phi_\rho)^2 + \frac{1}{v_\rho} (\partial_\tau \phi_\rho)^2 \right],$$

$$S_{\rho,\theta} = \frac{1}{2\kappa_\rho} \int dx d\tau \left[ v_\rho (\partial_x \theta_\rho)^2 + \frac{1}{v_\rho} (\partial_\tau \theta_\rho)^2 \right]. \quad (2.6)$$

In the spin sector, we might either have a spin gap corresponding to the Luther-Emery regime, or have a gapless phase where the spin excitations are described by a Hamiltonian of the same form as Eq. (2.4) with the parameters  $\kappa_\sigma$  and  $v_\sigma$ . In the gapless phase,  $SU(2)$  symmetry imposes the constraint  $\kappa_\sigma = 1$ .

### B. The spin-gapped 1:1 array

In this subsection we consider the simplest array of quantum wires, the two-dimensional 1:1 array of Luttinger liquids in the spin-gapped phase. It has been suggested that this case might describe the striped phases of high-temperature superconductors.<sup>21</sup> In this subsection, all bosonic variables refer to the charge sector, and we do not write explicitly the subscript  $\rho$ . In general, we expect density-density and current-current interactions between the wires, which can be represented by an action of the form

$$S_{\text{int}} = \frac{1}{2} \sum_{n,n',\mu} \int dx d\tau j_{\mu,n}(x,\tau) \tilde{W}_\mu(n-n') j_{\mu,n'}(x,\tau), \quad (2.7)$$

where  $j_{\mu,n} = [\rho_n(x,\tau), J_n(x,\tau)]$  with  $\rho_n = \partial_x \phi_{\mu,n}(x,\tau)$  the density and  $J_n = \partial_x \theta_n(x,\tau)$  the current on the  $n$ th wire. The density-density interaction is an effective interaction generated by both the screened Coulomb and the electron-phonon interaction. In the striped phases, stripe fluctuations lead to current-current interactions.<sup>21</sup> These current-current and density-density interactions are marginal and should be included in the fixed-point action. They are invariant under the ‘‘sliding’’ transformations  $\phi_n \rightarrow \phi_n + \alpha_n$  and  $\theta_n \rightarrow \theta_n + \alpha'_n$ . Equations (2.4) and (2.7) define the fixed-point action of the smectic-metal phase,<sup>12</sup> which can be written as

$$S = \sum_n \int dx d\tau \left[ \sum_j V_j^\theta(\partial_x \theta_n)(\partial_x \theta_{n+j}) + \sum_j V_j^\phi(\partial_x \phi_n) \times (\partial_x \phi_{n+j}) + 2i(\partial_x \theta_n)(\partial_\tau \phi_n) \right]. \quad (2.8)$$

Upon integration over  $\phi_n$  or  $\theta_n$ , respectively, the effective action for  $\theta_n$  and  $\phi_n$  become

$$S_\theta = \int \frac{d^3 Q}{(2\pi)^3} \frac{1}{2} \kappa(q_\perp) \left\{ \frac{1}{v(q_\perp)} \omega^2 + v(\vec{q}_\perp) q_\parallel^2 \right\} |\phi(\mathbf{Q})|^2,$$

$$S_\phi = \int \frac{d^3 Q}{(2\pi)^3} \frac{1}{2\kappa(q_\perp)} \left\{ \frac{1}{v(q_\perp)} \omega^2 + v(\vec{q}_\perp) q_\parallel^2 \right\} |\theta(\mathbf{Q})|^2, \quad (2.9)$$

where  $\mathbf{Q} = (\omega, q_\parallel, q_\perp)$ , with  $q_\parallel$  the momentum along the chain and  $q_\perp$  perpendicular to the chains. Here

$$\begin{aligned}\kappa(q_{\perp}) &= \sqrt{V^{\phi}(q_{\perp})/V^{\theta}(q_{\perp})}, \\ v(q_{\perp}) &= \sqrt{V^{\phi}(q_{\perp})V^{\theta}(q_{\perp})},\end{aligned}\quad (2.10)$$

where  $V^{\theta}(q_{\perp})$  and  $V^{\phi}(q_{\perp})$  are the Fourier transforms of  $V_j^{\theta}$  and  $V_j^{\phi}$  with respect to the wire index  $j$ . Equations (2.8) and (2.9) define the action of the ideal 2D sliding Luttinger liquid or SM phase. Even though they include interwire interactions described by Eq. (2.7), they yield power-law correlations characteristic of a 1D Luttinger liquid.

We could consider, for example, correlation functions involving the density variable  $\rho$ . We note that

$$\begin{aligned}\rho \equiv \psi^{\dagger} \psi &= R^{\dagger} R + L^{\dagger} L + [L^{\dagger} R + \text{c.c.}] \approx \partial_x \phi \\ &+ \{\exp[i2k_F x + i\sqrt{2\pi}\phi(x, \tau)] + \text{c.c.}\}.\end{aligned}\quad (2.11)$$

Thus the density has two pieces: the first piece is the coarse-grained density and the second is modulated at  $2k_F$ , where  $k_F$  is the Fermi wave vector. Correspondingly, the density-density correlation has two pieces. We consider  $G_{\phi}(x, \tau)$ ; the component of the density-density correlation function with  $2k_F$  modulation. Thus

$$G_{\phi}(x, \tau) \equiv \langle \exp\{i\sqrt{2\pi}[\phi_j(x, \tau) - \phi_j(0, 0)] + i2k_F x\} \rangle + \text{c.c.}$$

It is easy to show that for large  $x$  (see Appendix A)

$$G_{\phi}(x, 0) \approx \frac{A_1 \cos(2k_F x)}{x^{\Delta_{\text{CDW}, \infty}}}, \quad (2.12)$$

where

$$\Delta_{\text{CDW}, \infty} = \int_{-\pi}^{\pi} \frac{dq_{\perp}}{2\pi} \frac{1}{\kappa(q_{\perp})}, \quad (2.13)$$

and  $A_1$  is a constant. Alternately, for large  $\tau$ ,

$$G_{\phi}(0, \tau) \approx A_2 \tau^{-\Delta_{\text{CDW}, \infty}}, \quad (2.14)$$

where  $A_2$  is a different constant. In general,  $G_{\phi}(x, \tau)$  can be written in the scaling form

$$G_{\phi}(x, \tau) \approx x^{-\Delta_{\text{CDW}, \infty}} F\left(\frac{x}{\tau}\right), \quad (2.15)$$

where

$$\begin{aligned}F(y) &\rightarrow A_1 \quad \text{as } y \rightarrow 0 \\ &\rightarrow A_2 y^{\Delta_{\text{CDW}, \infty}} \quad \text{as } y \rightarrow \infty.\end{aligned}\quad (2.16)$$

A variety of interactions, other than those of Eq. (2.7), couple neighboring wires (see Fig. 1). The sliding phase is stable only if the interactions are irrelevant, that is, only if they scale to zero with system size. We will now investigate perturbatively the relevance of these interchain interactions to determine under what conditions the sliding phase is stable. Due to the spin gap, single-particle hopping between chains is irrelevant, and the interchain interactions that could become relevant are the Josephson (SC) and charge-density wave (CDW) couplings, which are represented by operators of the form

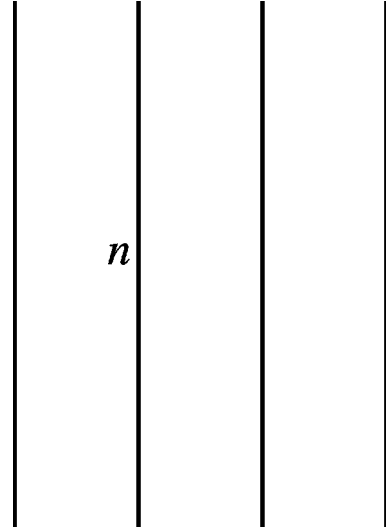


FIG. 1. A schematic depiction of a two-dimensional array of quantum wires.

$$\begin{aligned}\mathcal{H}_{\text{SC}, n} &= \sum_j \int dx [(R_{\uparrow, j}^{\dagger} L_{\downarrow, j}^{\dagger} + R_{\downarrow, j}^{\dagger} L_{\uparrow, j}^{\dagger})(R_{\uparrow, j+n} L_{\downarrow, j+n} \\ &+ R_{\downarrow, j+n} L_{\uparrow, j+n})] + \text{c.c.},\end{aligned}\quad (2.17)$$

and

$$\begin{aligned}\mathcal{H}_{\text{CDW}, n} &= \sum_j \int dx [(R_{\uparrow, j}^{\dagger} L_{\uparrow, j} + R_{\downarrow, j}^{\dagger} L_{\downarrow, j})(L_{\uparrow, j+n}^{\dagger} R_{\uparrow, j+n} \\ &+ L_{\downarrow, j+n}^{\dagger} R_{\downarrow, j+n})] + \text{c.c.}\end{aligned}\quad (2.18)$$

At low temperatures, the spin variable  $\phi_{\sigma}$  is effectively frozen, and these interactions depend only on  $\theta_i$ . Their associated actions can be expressed as

$$S_{\text{SC}, n} = \mathcal{J}_n \sum_i \int dx d\tau \cos[\sqrt{2\pi}(\theta_i - \theta_{i+n})],$$

$$S_{\text{CDW}, n} = \mathcal{V}_n \sum_i \int dx d\tau \cos[\sqrt{2\pi}(\phi_i - \phi_{i+n})]. \quad (2.19)$$

The relevance of these terms are determined by the scaling dimensions of the corresponding operators,  $\cos[\sqrt{2\pi}(\theta_i - \theta_{i+n})]$  and  $\cos[\sqrt{2\pi}(\phi_i - \phi_{i+n})]$ , which are, respectively,

$$\begin{aligned}\Delta_{\text{SC}, n} &= \int_{-\pi}^{\pi} \frac{dq_{\perp}}{2\pi} (1 - \cos nq_{\perp}) \kappa(q_{\perp}), \\ \Delta_{\text{CDW}, n} &= \int_{-\pi}^{\pi} \frac{dq_{\perp}}{2\pi} \frac{(1 - \cos nq_{\perp})}{\kappa(q_{\perp})}.\end{aligned}\quad (2.20)$$

The exponent  $\Delta_{\text{SC}, n}$  follows from

$$\langle \cos \sqrt{2\pi}(\theta_i - \theta_{i+n}) \rangle = \exp[-\pi \langle (\theta_i - \theta_{i+n})^2 \rangle], \quad (2.21)$$

and

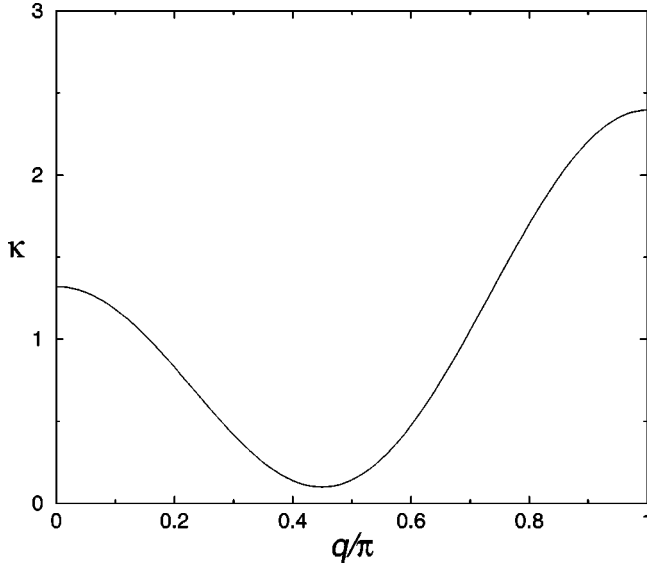


FIG. 2.  $\kappa(q)$  as a function of  $q/\pi$ , with a minimum,  $\kappa_{\min}$  at  $q = q_0$ .

$$\begin{aligned} \frac{1}{2} \langle (\theta_i - \theta_{i+n})^2 \rangle &= \int \frac{d^3 Q}{(2\pi)^3} \langle |\tilde{\theta}(\mathbf{q}, \omega)|^2 \rangle [1 - \cos(q_\perp n)] \\ &= \int_{-\pi}^{\pi} \frac{dq_\perp}{2\pi} \left\{ \kappa(q_\perp) (1 - \cos q_\perp n) \right. \\ &\quad \left. \times \left[ \int \frac{dq_\parallel d\omega}{(2\pi)^2} \frac{1}{v q_\parallel^2 + \omega^2/v} \right] \right\}, \quad (2.22) \end{aligned}$$

where  $\langle \rangle$  denotes averaging with respect to  $S$  in Eq. (2.8). The integral in the square brackets diverges logarithmically with system size  $L$  ( $\sim C \ln L$ ).<sup>22</sup> Using this, we find

$$\langle \cos \sqrt{2\pi}(\theta_j - \theta_{j+n}) \rangle \sim L^{-\Delta_{SC,n}}, \quad (2.23)$$

where  $\Delta_{SC,n}$  is given by Eq. (2.20). From the above equation, it follows that

$$\langle S_{SC,n} \rangle \sim L^{2-\Delta_{SC,n}}. \quad (2.24)$$

Similar calculations produce  $\Delta_{CDW,\infty}$ . For a stable smectic-metal phase, these terms have to be irrelevant, implying

$$\Delta_{CDW,n} > 2, \quad \Delta_{SC,n'} > 2 \quad (2.25)$$

for all  $n$  and  $n'$ .

If any  $\Delta_{SC,n} < 2$ , the SM phase is unstable to the formation of an anisotropic 2D superconductor. If any  $\Delta_{CDW,n} < 2$ , the SM phase will flow to a 2D longitudinal CDW-crystalline phase with  $2k_F$  density modulations along the wires and the phase locked from wire to wire. Notice that if  $\kappa(q_\perp)$  is uniformly small, the  $\Delta_{SC,n}$ 's are small and the  $\Delta_{CDW,n}$ 's are large, and for large  $\kappa$ , the  $\Delta_{SC,n}$ 's are large and the  $\Delta_{CDW,n}$ 's small. For a stable sliding phase we need all the  $\Delta_{SC,n}$ 's and  $\Delta_{CDW,n}$ 's to be greater than two. Our strategy to create a stable sliding phase is to choose  $\kappa(q_\perp)$  of the form shown in Fig. 2, with  $\kappa$  having a minimum,  $\kappa_{\min}$ , at  $q_\perp = q_0$ . When  $\kappa_{\min}$  becomes zero, the system undergoes a transition to a transverse CDW modulation with wave vector  $q_0$ ,

where the charge-density varies from wire to wire. For small but positive  $\kappa_{\min}$ , the system is close to the transverse CDW instability. As pointed out in (Ref. 13), a transverse CDW would frustrate the crystallization of fermions, since  $k_F$  is now a function of the chain index  $j$ . Strong fluctuations of this kind prevent the locking in of density fluctuations along the wires. Thus, in order to stabilize the sliding Luttinger liquid phase, we need to tune  $\kappa_{\min}$  to be very small compared to average  $\kappa(q_\perp)$ , so that  $\Delta_{SC,n}$  and  $\Delta_{CDW,n}$  can both be made large. Note that in addition, we should also consider interchain operators of the form  $R_{\uparrow,j}^\dagger R_{\downarrow,j}^\dagger R_{\uparrow,j+n} R_{\downarrow,j+n}$ . These interactions, however, turn out to be automatically irrelevant if the superconducting and CDW interactions are irrelevant, and hence merit no further consideration.

In addition to the lowest-order interactions between pairs of chains described by  $S_{SC,n}$  and  $S_{CDW,n}$ , there can, in principle, be higher-order multichain interactions with actions of the form

$$\begin{aligned} S_{SC,s_p} &= \sum_i \int dx d\tau \mathcal{J}_{s_p} \cos \left[ \sqrt{2\pi} \left( \sum_p s_p \theta_{i+p} \right) \right], \\ S_{CDW,s_p} &= \sum_i \int dx d\tau \mathcal{V}_{s_p} \cos \left[ \sqrt{2\pi} \left( \sum_p s_p \phi_{i+p} \right) \right], \quad (2.26) \end{aligned}$$

where  $\mathcal{J}_{s_p}$  are the interchain Josephson couplings,  $\mathcal{V}_{s_p}$  the interchain particle-hole (CDW) interactions, and  $s_p$  is an integer-valued function of the chain number  $p$  satisfying  $\sum_p s_p = 0$ . The scaling dimensions of  $\cos[\sqrt{2\pi}(\sum_p s_p \theta_{i+p})]$  and  $\cos[\sqrt{2\pi}(\sum_p s_p \phi_{i+p})]$  are, respectively,

$$\begin{aligned} \Delta_{SC,s_p} &= \int_{-\pi}^{\pi} \frac{dq_\perp}{2\pi} \kappa(q_\perp) \left[ \sum_{p,p'} s_p s_{p'} \cos\{(p-p')q_\perp\} \right], \\ \Delta_{CDW,s_p} &= \int_{-\pi}^{\pi} \frac{dq_\perp}{2\pi} \frac{1}{\kappa(q_\perp)} \left[ \sum_{p,p'} s_p s_{p'} \cos\{(p-p')q_\perp\} \right]. \quad (2.27) \end{aligned}$$

These perturbations are irrelevant if

$$\Delta_{CDW,s_p} > 2, \quad \Delta_{SC,s'_p} > 2 \quad (2.28)$$

for all sets of  $s_p$  and  $s'_p$ . The relevance of higher-order terms of this form is a subtle issue, and we will elaborate on this in a future publication. However the strength of these terms, as measured, for example, by  $\mathcal{V}_{s_p}$ , is small and they become important only at very small temperatures even if they are relevant. In this paper we will restrict ourselves to pairwise CDW couplings of the form given by Eq. (2.17); however, we comment on the higher-order superconducting terms below.

To explore the regions of stability of the SM phase, we follow Refs. 13 and 15 and take

$$\kappa(q_\perp) = K[1 + \lambda_1 \cos(q_\perp) + \lambda_2 \cos(2q_\perp)]. \quad (2.29)$$

The parameters  $\lambda_1$  and  $\lambda_2$  can be tuned to set the value  $q_0$  of  $q_\perp$ , at which  $\kappa(q_\perp)$  reaches a minimum and the minimum value of  $\kappa(q_\perp)$  and  $\kappa(q_0) = K\Delta$ . For a specific value of  $\Delta$  and  $k$ ,

$$\begin{aligned}\lambda_1 &= -\frac{4(1-\Delta)\cos q_0}{[1+2\cos^2(q_0)]}, \\ \lambda_2 &= \frac{(1-\Delta)}{(1+2\cos^2 q_0)},\end{aligned}\quad (2.30)$$

unless  $q_0 = 0$  or  $\pi$ , in which case only  $\lambda_1 + \lambda_2$  is fixed by  $\Delta$ . We note that, in the above equation (i.e., for  $q_0 \neq 0, \pi$ ), while  $\lambda_2$  is always positive for  $\Delta < 1$ ,  $\lambda_1$  can be either positive or negative. Typically positive  $\lambda$ 's correspond to repulsive interactions.

In this paper we treat  $K$  as the control parameter. For small  $K$  the system becomes superconducting, while for large  $K$  the system goes into a CDW crystalline phase. We will show that through judicious tuning of  $\lambda_1$  and  $\lambda_2$  it is possible to have an intermediate window of  $K$  where the smectic-metal phase is stable. For this purpose, we define  $a_{s_n} = \Delta_{SC, s_n}/K$  and  $b_n = \Delta_{CDW, n}K$ , where  $a_{s_n}$  and  $b_n$  depend only on  $\lambda_1$  and  $\lambda_2$ , and

$$\begin{aligned}a_{s_p} &= \sum_p \frac{1}{2} [s_p^2 + s_p(s_{p+1} + s_{p-1})\lambda_1/2 \\ &+ s_p(s_{p+2} + s_{p-2})\lambda_2/2].\end{aligned}\quad (2.31)$$

The SM phase becomes unstable to interchain Josephson couplings for  $K$  less than  $K_{SC} = \max_p (2/a_{s_p})$  and unstable to interchain CDW interactions for  $K$  greater than  $K_{CDW} = \min_n (b_n/2)$ . Thus the smectic-metal phase is stable over a window of  $K$  and  $K_{SC} < K < K_{CDW}$ , provided

$$\beta \equiv \frac{K_{CDW}}{K_{SC}} = \frac{a_{s_p} b_m}{4} \Big|_{\min. \text{ wrt. } m \& s_p} > 1. \quad (2.32)$$

We note, once more, that in this paper we consider stability with respect to all superconducting terms, but only pairwise CDW terms of the form given by Eq. (2.17). If  $\beta < 1$ , the system goes directly from a 2D SC phase to a CDW crystal as  $K$  is increased, without passing through the SM phase. For a stable sliding phase, we need to make  $\Delta$  very small. The value of  $\Delta_{CDW, n}$  for  $\Delta$  small is determined by values of  $q_\perp$  near  $q_0$ . We can therefore set  $\kappa(q_\perp)$

$$\kappa(q_\perp) \approx K[\Delta + C(q_\perp - q_0)^2], \quad (2.33)$$

where

$$C \equiv \frac{\kappa''(q_0)}{2K} = 2\lambda_2 \sin^2 q_0. \quad (2.34)$$

This gives us

$$\Delta_{CDW, n} \approx \frac{K[1 - \cos(nq_0)e^{-n\sqrt{\Delta/C}}]}{\sqrt{C\Delta}}, \quad (2.35)$$

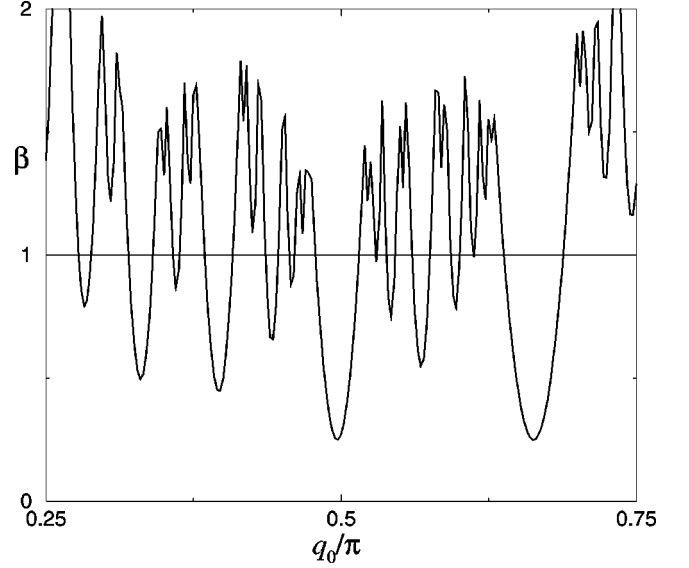


FIG. 3. Plot of  $\beta \equiv K_{CDW}/K_{SC}$  as a function of  $q_0/\pi$ . For  $\beta > 1$ , there exists a region of  $K$  over which the non-Fermi-liquid phase is stable.

where  $C$  has been defined in Eq. (2.34). We set  $\Delta = 10^{-5}$ . We consider the range of  $q_0/\pi$  lying between 0.25 and 0.75. This range can be broken into three sections:

(1)  $0.25 < q_0/\pi < 0.41957$ : In this range  $\lambda_1 < 0$  and  $|\lambda_1| > \lambda_2$ . Here the most relevant superconducting term corresponds to the multichain operator  $\cos[\sqrt{2}\pi(\theta_i + \theta_{i+1} - \theta_{i+3} - \theta_{i+4})]$ .<sup>23</sup> The dimension of this operator sets the minimum of  $a_{s_p}$ . Thus, in this range,  $\min a_{s_p} = (2 + \lambda_1 - \lambda_2/2)$  and  $K_{SC} = 1/(2 + \lambda_1 - \lambda_2/2)$ .

(2)  $0.41957 < q_0/\pi < 0.5804$ : In this region  $|\lambda_1| < \lambda_2$ . We find that  $a_{s_p}$  is smallest for the set  $s_n = \delta_{n,0} - \delta_{n,2}$ . Thus, in this range  $K_{SC} = 2/(1 - \lambda_2/2)$ .

(3)  $0.5804 < q_0/\pi < 0.75$ : Here  $\lambda_1 > \lambda_2$ , and  $a_{s_p}$  is the smallest for the set  $s_n = \delta_{n,0} - \delta_{n,1}$ . Thus  $K_{SC} = 2/(1 - \lambda_1/2)$ .

In Fig. 3 we plot  $\beta$  as a function of  $q_0$ . The minima of the curve corresponds to  $q_0 = 2\pi l/m$ , where  $l$  and  $m$  are integers. Also, note that since  $\lambda_1$  has the same sign as  $(-\cos q_0)$ , there are regions of stable smectic phase for positive as well as negative values of  $\lambda_1$ .

Having established a stable smectic phase for the pure system, we now study the relevance of quenched disorder in this phase. Disorder gives rise to a random electron potential  $D(x)$ , with associated action

$$S_{\text{dis}} = \sum_j \int dx d\tau D_j(x) \cos[\sqrt{2}\pi\phi_j]. \quad (2.36)$$

$D(x)$  can be treated as a Gaussian random variable, with zero mean and local fluctuations such that

$$\overline{D(x)} = 0,$$

$$\overline{D(x)D(x')} = \Delta_D \delta(x - x'), \quad (2.37)$$

where the over line signifies averages over the randomness. By a generalization of the Harris criterion,<sup>24</sup> it can be shown

quite easily that (also, see Giamarchi and Schulz<sup>25</sup>) disorder is irrelevant if  $\Delta_{\text{CDW},\infty} > 3$ , where

$$\Delta_{\text{CDW},\infty} \equiv \int_{-\pi}^{\pi} \frac{dq}{2\pi} \frac{1}{\kappa(q)} \simeq \frac{1}{K} \frac{1}{\sqrt{C\Delta}}. \quad (2.38)$$

For the range of parameters we are considering where the SM phase is stable,  $\Delta_{\text{CDW},\infty}$  is large and thus disorder is strongly irrelevant. This is an important point. For a single Luttinger liquid in the repulsive region,  $\kappa > 1$  and disorder is always relevant. However, interwire interactions can drive disorder irrelevant for  $\kappa(q_{\perp}=0) > 1$ , even in regions of phase space where all interactions are repulsive.

Thus, there is a small but finite region of phase space where the smectic-metal phase appears stable. We should note that over a larger region of phase space, the only relevant operators involve nonlocal interactions of the form  $\mathcal{V}_n \cos[\sqrt{2\pi}(\phi_i - \phi_{i+n})]$ , where  $\mathcal{V}_n$  is expected to be exponentially small, for large  $n$ , in the bare Hamiltonian. Though relevant, these operators would only play a role for  $k_B T$  smaller than some energy scale set by  $\mathcal{V}_n$ . So, for example, there will be a range of temperatures, where we will only need to consider the relevance of  $\mathcal{V}_1$  and  $\mathcal{J}_1$ . These can be made irrelevant over a reasonably large region of phase space (see Ref. 13). Thus, even though the region of phase space where the smectic phase is strictly stable is highly restricted, at finite temperature and for weak coupling, we expect a much larger region of phase space whose behavior is governed by the sliding Luttinger liquid ground state.

### C. The gapless 1:1 array

We now consider 1:1 arrays of wires in which both charge and spin excitations are gapless. In this case, there are two Luttinger liquid parameters ( $\kappa_{\rho}, \kappa_{\sigma}$ ) for the charge and spin modes, respectively, and two velocities ( $v_{\rho}, v_{\sigma}$ ) on each wire. To maintain gapless Luttinger liquids and  $SU(2)$  spin symmetry, we do not include any marginal spin-spin coupling terms in the Hamiltonian. Thus the spin degrees of freedom are represented by the fixed-point action

$$S_{\phi,\sigma} = \kappa_{\sigma} \sum_j \int dx d\tau \left[ v_{\sigma} (\partial_x \phi_{\sigma,j})^2 + \frac{(\partial_{\tau} \phi_{\sigma,j})^2}{v_{\sigma}} \right] \quad (2.39)$$

with  $\kappa_{\sigma} = 1$ . In a more general treatment, one could include spin-spin coupling terms and consider their relevance, maintaining, however, the  $SU(2)$  symmetry of the spin sector. We leave that for a future consideration. The charge modes are still represented by Eq. (2.9), with  $\kappa(q_{\perp})$  and  $v(q_{\perp})$  replaced by  $\kappa_{\rho}(q_{\perp})$  and  $v_{\rho}(q_{\perp})$ . The form of  $\kappa_{\rho}$  is still given by Eq. (2.29).

We again consider the relevance of single-particle, CDW and SC tunneling. The SC and CDW tunneling were already considered in the previous subsection. When the spin variables are included, Eqs. (2.11) and (2.12) become

$$\begin{aligned} S_{\text{SC},n} &= \mathcal{J}_n \int dx d\tau \sum_j \cos[\sqrt{2\pi}(\theta_{\rho,j} - \theta_{\rho,j+n})] \\ &\quad \times \cos(\sqrt{2\pi}\phi_{\sigma,j}) \cos(\sqrt{2\pi}\phi_{\sigma,j+n}), \\ S_{\text{CDW},n} &= \mathcal{V}_n \int dx d\tau \sum_j \cos[\sqrt{2\pi}(\phi_{\rho,j} - \phi_{\rho,j+n})] \\ &\quad \times \cos(\sqrt{2\pi}\phi_{\sigma,j}) \cos(\sqrt{2\pi}\phi_{\sigma,j+n}). \end{aligned} \quad (2.40)$$

The  $\phi_{\sigma}$  variables now contribute to the dimensions of these terms. Because  $\kappa_{\sigma}$  is constrained to be one, the contribution of the  $\sigma$  variables is trivial, and the dimensions of these terms are given by

$$\begin{aligned} \Delta_{\text{SC},n} &= \Delta_{\text{SC},n}^{(\text{gap})} + 1, \\ \Delta_{\text{CDW},n} &= \Delta_{\text{CDW},n}^{(\text{gap})} + 1, \end{aligned} \quad (2.41)$$

where  $\Delta_{\text{CDW},n}^{(\text{gap})}$  and  $\Delta_{\text{SC},n}^{(\text{gap})}$  are given by Eqs. (2.13) and (2.14) with  $\kappa$  replaced by  $\kappa_{\rho}$ .

Since the  $\sigma$  variables are no longer gapped, single-electron tunneling is no longer irrelevant. Single-particle hopping is described by operators such as  $R_{j,\uparrow} R_{j+n,\uparrow}^{\dagger}$ , which can be represented by terms of the form

$$\begin{aligned} S_{\text{el},n} &= \int dx d\tau \sum_j \mathcal{T}_n \exp \left[ -i \sqrt{\frac{\pi}{2}} (\phi_{\rho,j} - \phi_{\rho,j+n}) \right] \\ &\quad \times \exp \left[ -i \sqrt{\frac{\pi}{2}} (\theta_{\rho,j} - \theta_{\rho,j+n}) \right] \\ &\quad \times \left\{ \exp \left[ -i \sqrt{\frac{\pi}{2}} (\phi_{\sigma,j} - \phi_{\sigma,j+n} + \theta_{\sigma,j} - \theta_{\sigma,j+n}) \right] \right\}. \end{aligned} \quad (2.42)$$

The expectation value of the term in the curly bracket goes as  $L^{-1/2}$  as system size  $L$  goes to infinity. Thus  $\langle S_{\text{el}} \rangle \sim L^{2-\Delta_{\text{el},n}}$ , where

$$\Delta_{\text{el},n} = \frac{1}{4} [\Delta_{\text{CDW},n}^{(\text{gap})} + \Delta_{\text{SC},n}^{(\text{gap})}] + \frac{1}{2}. \quad (2.43)$$

Regions of phase space, where  $S_{\text{SC},n}$  is relevant, corresponds to the superconducting phase, whereas regions of relevance of  $S_{\text{CDW},n}$  correspond to the CDW crystal phase. Regions where both of these are irrelevant, but single-particle hopping is relevant, correspond to the Fermi-metal phase. For a stable smectic-metal phase, we require that all these operators be irrelevant. The superconducting and CDW coupling terms are irrelevant if

$$\Delta_{\text{SC},n}^{(\text{gap})} > 1, \quad \Delta_{\text{CDW},n'}^{(\text{gap})} > 1, \quad (2.44)$$

for all  $n$  and  $n'$ . The condition for single-particle hopping to be irrelevant is that

$$\Delta_{\text{SC},n}^{(\text{gap})} + \Delta_{\text{CDW},n}^{(\text{gap})} > 6 \quad (2.45)$$

for all  $n$ .

We now proceed exactly as for the gapped case, assuming  $\kappa_{\rho}$  to have a form as given by Eq. (2.29). As before, we may

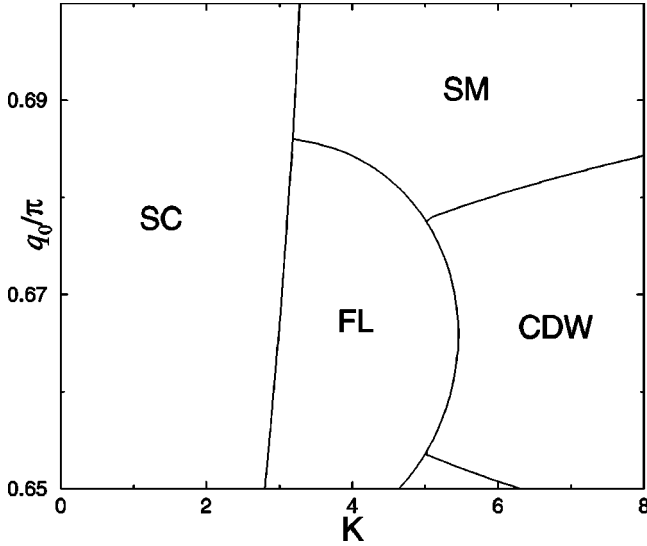


FIG. 4. A plot of the phase diagram in  $q_0$ ,  $K$  space with  $\Delta = 10^{-5}$ . SC stands for superconducting, FL for Fermi-liquid, SM for smectic metal, and CDW for charge-density wave crystal.

write  $\Delta_{\text{CDW},n}^{(\text{gap})} = a_n/K$ ,  $\Delta_{\text{SC},n}^{(\text{gap})} = b_n K$ ,  $K_{\text{SC}} = \max_n(1/a_n)$ , and  $K_{\text{CDW}} = \min_n(b_n)$ . Provided  $K_{\text{CDW}}/K_{\text{SC}} > 1$ , there is a window of  $K$ ,  $K_{\text{SC}} < K < K_{\text{CDW}}$ , where the system is stable with respect to both the CDW and superconducting couplings. For the smectic-metal phase to be stable, the single-particle hopping has to be irrelevant as well, which indicates that

$$\frac{a_n}{K} + b_n K > 6. \quad (2.46)$$

This condition is violated for  $K$  lying between  $K_-$  and  $K_+$ , where  $K_- = \min_n K_{-,n}$  and  $K_+ = \max_n K_{+,n}$ , with

$$K_{\pm,n} = \frac{3 \pm \sqrt{9 - a_n b_n}}{b_n}. \quad (2.47)$$

The single electron hopping is relevant in a large region of phase space, indicating an instability towards a Fermi-liquid FL phase. We write  $\lambda_1$  and  $\lambda_2$  as functions of  $\Delta$  and  $q_0$  [see Eq. (2.30)] and set  $\Delta = 10^{-5}$ . Higher-order terms involving  $\theta_\sigma$  and  $\phi_\sigma$ , in general, are less relevant in this case, and we do not need to consider the whole set of operators.<sup>26</sup> Depending on  $q_0$ , we have the following possibilities for phases as  $K$  is increased: (1) SC  $\rightarrow$  FL  $\rightarrow$  CDW crystal, (2) SC  $\rightarrow$  SM  $\rightarrow$  CDW crystal, (3) SC  $\rightarrow$  FL  $\rightarrow$  SM  $\rightarrow$  CDW crystal, and (4) SC  $\rightarrow$  SM  $\rightarrow$  FL  $\rightarrow$  SM  $\rightarrow$  CDW crystal.

The phase diagram is complicated, and we plot a region of  $K$ ,  $q_0$  space in Fig. 4, in the absence of disorder. Backscattering due to disorder is irrelevant for  $\Delta_{\text{CDW},\infty}^{(\text{gap})} > 2$ , which is automatically satisfied in the SM phase.

#### D. The three-dimensional anisotropic sliding phase

We now turn to three-dimensional 2:1 arrays with wires on a periodic 2D lattice with primitive translation vectors  $\vec{a}_1$  and  $\vec{a}_2$ . Each wire occupies a position  $n_1 \vec{a}_1 + n_2 \vec{a}_2$  on a 2D lattice and is labeled by the integer valued vector  $\mathbf{n}$

$= (n_1, n_2)$ . We will focus on the spin-gapped case, though extensions to the gapless case proceed exactly as in the previous section. Taking into account interwire density-density and current-current interactions, we could write down an action of the form (2.7), but where we now have sums over columns in a 2D lattice, with  $j_{\mu,\mathbf{n}}(x, \tau) \rightarrow j_{\mu,\mathbf{n}}(x, \tau)$  and  $\mathbf{n} = (n_x, n_y)$ . When transformed to Fourier space, this action becomes

$$S = \frac{1}{2} \int \frac{d^4 Q}{(2\pi)^4} [V^\theta(\vec{q}_\perp) q_\parallel^2 |\theta|^2 + V^\phi(\vec{q}_\perp) q_\parallel^2 |\phi|^2 - i\omega q_\parallel \{\theta^* \phi_x + \text{c.c.}\}], \quad (2.48)$$

where  $\mathbf{Q} = (\omega, q_\parallel, \vec{q}_\perp)$  with  $\vec{q}_\perp$  a vector in the first Brillouin zone of the 2D lattice of columns. We choose the  $x$  axis to lie along wires, so that  $\vec{q}_\perp = (q_y, q_z)$ . The  $\theta$  or the  $\phi$  variables may be integrated out, giving us the effective actions

$$S_\theta = \int \frac{d^4 Q}{(2\pi)^4} \frac{1}{2\kappa(\vec{q}_\perp)} \left\{ \frac{1}{v(\vec{q}_\perp)} \omega^2 + v(\vec{q}_\perp) q_\parallel^2 \right\} |\phi(\mathbf{Q})|^2, \\ S_\phi = \int \frac{d^4 Q}{(2\pi)^2} \frac{1}{2\kappa(\vec{q}_\perp)} \left\{ \frac{1}{v(\vec{q}_\perp)} \omega^2 + v(\vec{q}_\perp) q_\parallel^2 \right\} |\theta(\mathbf{Q})|^2, \quad (2.49)$$

where

$$\kappa(\vec{q}_\perp) = \sqrt{V^\phi(\vec{q}_\perp)/V^\theta(\vec{q}_\perp)}, \\ v(\vec{q}_\perp) = \sqrt{V^\phi(\vec{q}_\perp)V^\theta(\vec{q}_\perp)}. \quad (2.50)$$

In three dimensions it turns out that the stability of the sliding phase with respect to the complete set of operators, requires an even further fine-tuning of the generalized current-current coupling terms. In particular,  $\kappa(q_y, q_z)$  should have a minimum  $K\Delta$  at some  $q_y = q_{0,y}$ ,  $q_z = q_{0,z}$ , with both  $\Delta$  and the second derivative of  $\kappa/K$  being much smaller than unity at the minimum. Let us consider two examples of the form that  $\kappa(q_y, q_z)$  could assume in order to obtain a stable sliding phase (see Fig. 5).

The first example is one that is symmetric with respect to  $q_y$  and  $q_z$ . We assume that the wires are arranged in a square or rectangular pattern, and align the  $y$ - and  $z$ - axes along the edges of the rectangle. We consider the form

$$\kappa(q_y, q_z) = K [1 + \lambda_1 \cos(q_y) + \lambda_1 \cos(q_z) + \lambda_2 \cos(q_y) \cos(q_z)]^2. \quad (2.51)$$

$\lambda_1$  and  $\lambda_2$  are adjusted such that  $\kappa$  has a minimum  $K\Delta^2$  at  $q_y = q_z = q_0$ . This gives

$$\lambda_1 = -\frac{(1-\Delta)}{\cos(q_0)}, \\ \lambda_2 = \frac{(1-\Delta)}{\cos^2(q_0)}. \quad (2.52)$$

Close to  $(q_y, q_z) = (q_0, q_0)$ , we can expand  $\kappa$  as



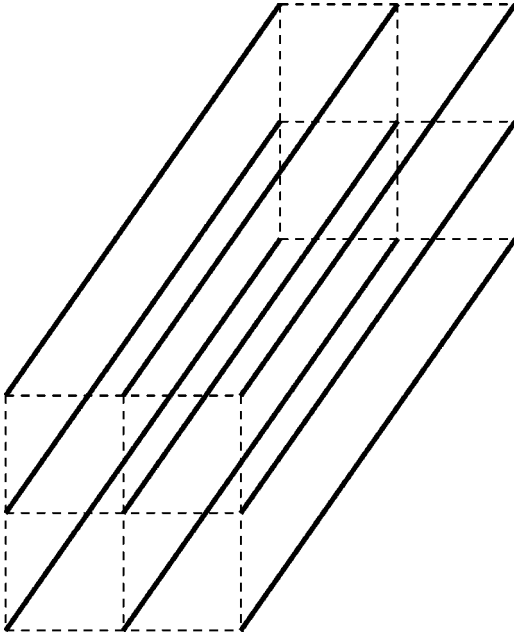


FIG. 5. A three-dimensional array of quantum wires.

$$\kappa \simeq K[\Delta + \lambda_2(q_y - q_0)(q_z - q_0)]^2. \quad (2.53)$$

As before  $\Delta_{\text{SC},\mathbf{m}} = K a_{\mathbf{m}}$  and  $\Delta_{\text{CDW},\mathbf{n}} = K b_{\mathbf{n}}$ , where  $\mathbf{m}$  and  $\mathbf{n}$  are now vectors. The sliding phase is stable provided

$$\beta \equiv \frac{K_{\text{CDW}}}{K_{\text{SC}}} = \frac{a_{\mathbf{m}} b_{\mathbf{n}}}{4} \Big|_{\min. \text{ wrt. } \mathbf{m} \& \mathbf{n}} > 1. \quad (2.54)$$

For  $\Delta = 10^{-2}$  (or smaller) there is a large range of  $q_0$ , where the sliding phase is stable.

One could also consider the highly anisotropic form

$$\begin{aligned} \kappa(q_y, q_z) = K \{ & [1 + \lambda_1 \cos(q_y) + \lambda_2 \cos(2q_y)]^2 \\ & + \lambda_3 [1 + \cos(q_z)] \}. \end{aligned} \quad (2.55)$$

Again, for any  $\lambda_3$ , one can adjust  $\lambda_1$  and  $\lambda_2$  to produce a stable sliding phase. We conclude by noting that in three dimensions, obtaining a sliding phase, requires an even finer adjustment of parameters than in 2D. At finite temperature, as before, the region of phase space controlled by the smectic-metal fixed point is expected to expand considerably.

### III. CROSSED SLIDING LUTTINGER LIQUID PHASE

Having established regions of stability of the sliding metal phases formed from arrays of quantum wires, we now turn to the investigation of sliding phases formed from crossed arrays of wires. We consider two basic configurations: one a two-dimensional system formed from two coupled 2D sliding phases (1:1 arrays) oriented at right angles to each other and the other a 3D system formed by stacking the crossed two-dimensional system. The latter three-dimensional system can be constructed from two interpenetrating 3D anisotropic sliding phases (2:1 arrays), of the type discussed in Sec 2 D, oriented at right angles to each other. Both the 2D and 3D systems have  $C_{4v}$  symmetry. As a result, their in-

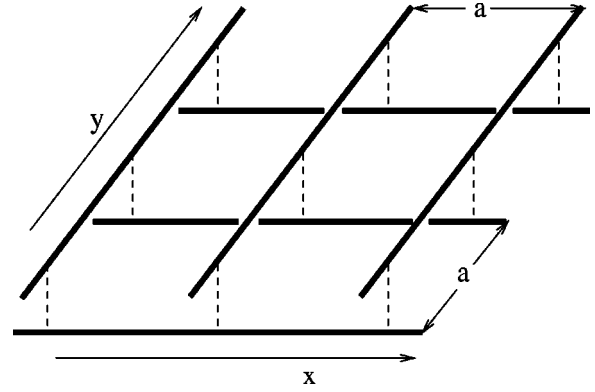


FIG. 6. Schematic depiction of a two-dimensional crossed array.

plane conductivities at finite temperature are isotropic at long wavelengths. In this section we demonstrate the existence of a sliding phase in the crossed arrays that is stable if the sliding phase in the constituent arrays is stable. The correlation functions in this phase exhibit power-law decay along the planes, and the electric conductivity diverges as a power law in temperature  $T$  as  $T \rightarrow 0$ .

#### A. Crossed two-dimensional sliding phase

We consider now a square grid of wires, starting again with the spin-gapped case. The system consists of two arrays of quantum wires, the X- and Y- arrays running, respectively, parallel to the  $x$  and  $y$  directions. Each wire sees a periodic one-electron potential from the array of wires crossing it. For simplicity we assume that this periodicity is commensurate with bands in the wire. This leads to a new band structure with new band gaps. It is assumed that the Fermi surface is between gaps so that the wires are conductors in the absence of further interactions. By removing degrees of freedom with wavelengths smaller than the inverse wire separation, we obtain a new effective theory whose form is identical to the theory before the periodic potential was introduced. Thus, in the absence of two-particle interactions between crossed arrays, the system could be in a phase consisting of two crossed, noninteracting smectic-metal states.

We will now demonstrate the existence of a stable sliding phase in the crossed arrays. In addition to the interwire couplings within each array, we need to consider Coulomb interactions between wires on the X array and wires on the Y array. These interarray couplings are marginal and should be included in the fixed point. They do not, however, change the dimensions of the operators, except by renormalizing  $\kappa(q_{\perp})$ . For a stable sliding phase, additional interactions between the two arrays, such as the Josephson and CDW couplings, have to be irrelevant. We will show that it is possible to tune  $\kappa(q_{\perp})$  such that this is indeed the case (see Fig. 6).

The Coulomb interactions between electrons on intersecting wires give rise to a term in the Hamiltonian of the form  $V_{m,n}^c(x,y) \rho_{x,m}(x) \rho_{y,n}(y)$ , where  $\rho_{x,m}(x)$  [ $\rho_{y,m}(y)$ ] is the electron density on the  $m$ th wire in the X(Y)-array at position  $x$  ( $y$ ). We expect  $V_{m,n}^c(x,y)$  to have the form  $V^c(x$

$-na, y-ma$ ), where  $a$  is the distance between parallel wires. Thus, we represent the interaction between the  $X$  and  $Y$  array as

$$\int dx dy [\partial_x \phi_{x,m} V^c(x-na, y-mb) \partial_y \phi_{y,n}].$$

If all parameters for the  $X$  and  $Y$  arrays are the same, the crossed-grid action as a functional of the  $\theta$  and  $\phi$  variables, can be written as

$$\begin{aligned} S = \frac{1}{2} \int \frac{d\omega dq_x dq_y}{(2\pi)^3} [ & V^\theta(q_y) q_x^2 |\theta_x|^2 + V^\theta(q_x) q_y^2 |\theta_y|^2 \\ & + V^\phi(q_y) q_x^2 |\phi_x|^2 + V^\phi(q_x) q_y^2 |\phi_y|^2 \\ & + V^c(q_x, q_y) q_x q_y \{ \phi_x \phi_y^* + \text{c.c.} \} - i\omega q_x \{ \theta_x^* \phi_x + \text{c.c.} \} \\ & - i\omega q_y \{ \theta_y^* \phi_y + \text{c.c.} \}] \end{aligned} \quad (3.1)$$

with obvious definitions for  $\phi_x = \phi_x(\omega, q_x, q_y)$ ,  $\phi_y, \theta_x$ , and  $\theta_y$ . It should be noted that this is an effective theory with  $-\pi/a < q_x, q_y < \pi/a$ . Integrating out the  $\phi$  variables, we are left with an effective action, which is conveniently expressed in matrix form as

$$S_\theta = \frac{1}{2} \int d^2k d\omega \theta_a (\mathbf{G}^{-1})_{ab} \theta_b^*, \quad (3.2)$$

where  $a=x, y$  and  $b=x, y$ . Here

$$\mathbf{G}^{-1} = \begin{pmatrix} \frac{1}{\kappa_x} \left( \frac{\omega^2}{v_x} + v_x q_x^2 \right) & -V_R^c \omega^2 \\ -V_R^c \omega^2 & \frac{1}{\kappa_y} \left( \frac{\omega^2}{v_y} + v_y q_y^2 \right) \end{pmatrix}, \quad (3.3)$$

where

$$\begin{aligned} \kappa_x(\mathbf{q}) &= \sqrt{\frac{\gamma(\mathbf{q})}{V^\phi(q_x) V^\theta(q_y)}}, \\ v_x(\mathbf{q}) &= \sqrt{\frac{V^\theta(q_y) \gamma(\mathbf{q})}{V^\phi(q_x)}}, \end{aligned}$$

$$V_R^c(\mathbf{q}) = \frac{V^c(\mathbf{q})}{\gamma(\mathbf{q})},$$

$$\gamma(\mathbf{q}) = V^\phi(q_x) V^\theta(q_y) - [V^c(\mathbf{q})]^2, \quad (3.4)$$

and  $\kappa_y(\mathbf{q}) = \kappa_x(P\mathbf{q})$ ,  $v_y(\mathbf{q}) = v_x(P\mathbf{q})$  where  $P\mathbf{q} = P(q_x, q_y) = (q_y, q_x)$ . From Eq. (3.3) for  $\mathbf{G}^{-1}$ , we can calculate

$$\mathbf{G} = \frac{1}{D} \begin{pmatrix} \frac{1}{\kappa_y} \left( \frac{\omega^2}{v_y} + v_y q_y^2 \right) & V_R^c \omega^2 \\ V_R^c \omega^2 & \frac{1}{\kappa_x} \left( \frac{\omega^2}{v_x} + v_x q_x^2 \right) \end{pmatrix}, \quad (3.5)$$

where

$$D = \frac{1}{\kappa_x \kappa_y} \left( \frac{\omega^2}{v_x} + v_x q_x^2 \right) \left( \frac{\omega^2}{v_y} + v_y q_y^2 \right) - (V_R^c)^2 \omega^4 \quad (3.6)$$

is the determinant of  $\mathbf{G}^{-1}$ .

In order to determine the dimensions of operators, we calculate the leading dependence of correlation functions such  $\langle \theta_x^2(\mathbf{r}, t) \rangle$  on system size  $L$ . Thus we can consider the function

$$\langle \theta_x^2(\mathbf{r}, t) \rangle = \int \frac{dq_x dq_y d\omega}{(2\pi)^3} \frac{\left( \frac{\omega^2}{v_y} + v_y q_y^2 \right)}{\kappa_y D}. \quad (3.7)$$

The leading  $L$  dependence is related to the infrared divergence of the integral just introduced. This infrared divergence comes purely from the integration over  $q_x, \omega$ . We can write the integrand as  $\kappa_x / (\omega^2/v_x + v_x q_x^2)$  plus a remaining part. The integral of the remaining part is free of infrared singularities (see Appendix C for details). Thus, it is easy to see that the leading  $L$  dependence goes as

$$\langle \theta_x^2 \rangle \sim \pi \ln(L) \int \frac{dq_y}{2\pi} \kappa(q_y), \quad (3.8)$$

where  $\kappa(q_y) = \kappa_x(0, q_y)$ . Notice that this is precisely what we had for a single array of parallel wires. A similar analysis yields  $\langle \theta_y^2 \rangle = \langle \theta_x^2 \rangle$ . Also note that cross correlations of the form  $\theta_x \theta_y$  are finite as  $L$  goes to infinity.

We also need to consider correlation functions in the  $\phi$  variables. To do so, we start with the action of Eq. (3.1), and integrate out the  $\theta$  variables. The effective action as a functional of the  $\phi$  variables is

$$S_\phi = \frac{1}{2} \int \frac{d\omega dq_x dq_y}{2\pi^3} \phi_a (\mathbf{G}_\phi^{-1})_{ab} \phi_b, \quad (3.9)$$

with  $a=x, y$  and  $b=x, y$ . Here

$$\mathbf{G}_\phi^{-1} = \begin{pmatrix} \bar{\kappa}_x \left( \frac{\omega^2}{v_x} + \bar{v}_x q_x^2 \right) & V^c q_x q_y \\ V^c q_x q_y & \bar{\kappa}_y \left( \frac{\omega^2}{v_y} + \bar{v}_y q_y^2 \right) \end{pmatrix}, \quad (3.10)$$

where

$$\bar{\kappa}(q_y) = [V^\phi(q_y)/V^\theta(q_y)]^{1/2},$$

$$\bar{v}(q_y) = [V^\phi(q_y) V^\theta(q_y)]^{1/2}. \quad (3.11)$$

Note that  $\bar{\kappa}$  is different from  $\kappa$  defined for the  $\theta$  correlation functions. From  $\mathbf{G}_\phi^{-1}$ , we calculate

$$\mathbf{G}_\phi = \frac{1}{D} \begin{pmatrix} \bar{\kappa}_y \left( \frac{\omega^2}{v_y} + \bar{v}_y q_y^2 \right) & -V^c q_x q_y \\ -V^c q_x q_y & \bar{\kappa}_x \left( \frac{\omega^2}{v_x} + \bar{v}_x q_x^2 \right) \end{pmatrix}, \quad (3.12)$$

where

$$\bar{\mathcal{D}} = \bar{\kappa}_x \bar{\kappa}_y \left( \frac{\omega^2}{\bar{v}_x} + \bar{v}_x q_x^2 \right) \left( \frac{\omega^2}{\bar{v}_y} + \bar{v}_y q_y^2 \right) - (V^c)^2 q_x^2 q_y^2 \quad (3.13)$$

is the determinant of  $\mathbf{G}_\phi^{-1}$ .  $\mathbf{G}_\phi$  can be used to calculate dimensions of operators involving  $\phi$ . For example, the expectation value

$$\langle \phi_x^2(\mathbf{r}, t) \rangle = \int \frac{dq_x dq_y d\omega}{(2\pi)^3} \frac{\bar{\kappa}_x \left( \frac{\omega^2}{\bar{v}_y} + \bar{v}_y q_y^2 \right)}{\bar{\mathcal{D}}}, \quad (3.14)$$

where again we take  $\pi/L < |q_x|, |q_y|, |\omega|$ . In the integral, as before, the infrared divergence comes purely from the integral over  $q_x, \omega$ . Once more, the infrared divergent part goes as

$$\langle \phi_x^2 \rangle \sim \pi \ln(L) \int \frac{dq_y}{2\pi} \frac{1}{\kappa(q_y)}, \quad (3.15)$$

where  $\kappa(q_y) = \kappa_x(0, q_y)$  is the same function appearing in  $\langle \theta_x^2 \rangle$ , Eq. (3.8).

Thus, correlation functions for  $\theta_x$  and  $\theta_y$  can be calculated directly from Eq. (3.2).  $\theta_x$ - $\theta_y$  cross correlations are nonsingular, whereas,  $\theta_x$ - $\theta_x$  and  $\theta_y$ - $\theta_y$  correlations have singular parts with exactly the same functional forms as they have in the absence of coupling between layers, but with the  $\kappa(q)$  function in expressions for the scaling exponents replaced by

$$\kappa(q_\perp) = \kappa_x(0, q_\perp) = \kappa_y(q_\perp, 0). \quad (3.16)$$

The same holds for  $\phi$ - $\phi$  correlation functions. Thus correlation functions within a given array have the same functional form as for  $V^c = 0$  but with different definitions of  $\kappa$ . Other than renormalizing  $\kappa(q)$ , the coupling  $V_{m,n}^c$  between the two arrays leaves the dimensions of all operators *unchanged*. This means that it is possible to choose interchain interactions within the  $X$  and  $Y$  grids so that these grids form 2D anisotropic sliding phases even in the presence of the intergrid coupling  $V_{m,n}^c$ . Equations (3.2) and (3.16) define a 2D non-Fermi-liquid with scaling properties to be discussed in the next section.

First, however, we must verify that it is possible to choose potentials so that this 2D non-Fermi liquid is stable with respect to perturbations. All pairwise couplings within a given array, i.e.,  $S_{SC,n}^X$ ,  $S_{CDW,n}^X$ ,  $S_{SC,n}^Y$  and  $S_{CDW,n}^Y$  defined as obvious generalizations of Eqs. (2.19), can be rendered irrelevant by choosing  $\kappa(q_\perp)$ , as in the case of an individual array. We must also consider Josephson and CDW couplings between the two arrays, which operate at the points of crossing  $(x, y) = (na, ma)$  of wire  $m$  in the  $X$  array and wire  $n$  of the  $Y$  array, respectively. These take the form

$$S_{SC}^{XY} = \sum_{m,n} \int d\tau \mathcal{J}^{XY} \cos\{\sqrt{2\pi}[\theta_{x,m}(na) - \theta_{y,n}(ma)]\},$$

$$S_{CDW}^{XY} = \sum_{m,n} \int d\tau \mathcal{V}^{XY} \cos\{\sqrt{2\pi}[\phi_{x,m}(na) - \phi_{y,n}(ma)] + 2k_F(ma - na)\}. \quad (3.17)$$

The dimensions of the cosine operators in the integrands are, respectively,

$$\Delta_{SC,\infty} \equiv \int_{-\pi}^{\pi} \frac{dq}{2\pi} \kappa(q) = K,$$

$$\Delta_{CDW,\infty} \equiv \int_{-\pi}^{\pi} \frac{dq}{2\pi} \frac{1}{\kappa(q)} \approx \frac{1}{K} \frac{1}{\sqrt{C\Delta}}, \quad (3.18)$$

where we assume that  $\kappa(q)$  has the form given by Eq. (3.17),  $\Delta$  is defined as before, and  $C \equiv \kappa''(k_0)/2K$ . If  $\kappa$  is chosen such that Eq. (3.16) is satisfied for each array, then  $S_{SC}^{XY}$  and  $S_{CDW}^{XY}$  are automatically irrelevant. Thus, we do not need any further fine tuning of  $\kappa$  to get a stable CSLL phase.

Having established a region of stability of the CSLL phase, we now investigate the nature of the correlation functions. Consider once more the correlation function

$$G_\phi^X(x, y = ma) \equiv \langle \exp[i\phi_{x,m}(x, \tau) - i\phi_{x,0}(0, \tau) + 2k_F x] \rangle + \text{c.c.}, \quad (3.19)$$

which corresponds to the component of the density-density correlation function modulated at  $2k_F$ . In the absence of terms such as  $S_{CDW}^{XY}$ , this correlation function vanishes for  $y \neq 0$ . Thus, though irrelevant, the presence of  $S_{CDW}^{XY}$  changes the nature of the correlation functions. In its presence, to lowest order in  $\mathcal{V}^{XY}$ , we obtain

$$\begin{aligned} G_\phi^X(x, y) &\approx \frac{(\mathcal{V}^{XY})^2}{4} e^{i2k_F(x+y)} \\ &\times \int dx_1 [\langle \exp\{i\sqrt{2\pi}[\phi_x(x, y = ma) - \phi_x(x_1, y = ma)]\} \rangle_0 \langle \exp\{i\sqrt{2\pi}[\phi_y(x_1, y) - \phi_y(x_1, 0)]\} \rangle_0 \langle \exp\{i\sqrt{2\pi}[\phi_x(x_1, 0) - \phi_x(0, 0)]\} \rangle_0 ] + \text{c.c.}, \end{aligned} \quad (3.20)$$

where  $\langle \rangle_0$  is the expectation value with respect to the CSLL fixed point. We need the asymptotic form of the correlation function for large  $x, y$ . Following Eq. (2.12), we obtain, for example,

$$\langle \exp\{i\sqrt{2\pi}[\phi_x(x, 0) - \phi_x(0, 0)]\} \rangle_0 \approx \frac{A}{x^{\Delta_{CDW,\infty}}}. \quad (3.21)$$

Similarly

$$\langle \exp\{i\sqrt{2\pi}[\phi_y(x,y) - \phi_x(x,0)]\} \rangle_0 \sim \frac{1}{y^{\Delta_{\text{CDW},\infty}}} \quad (3.22)$$

Note that in the sliding phase,  $\Delta_{\text{CDW},n}$  is greater than unity and thus in Eq. (3.20), the largest contribution to the integral comes from  $x_1$  close to 0 and to  $x$ . It thus follows that for large  $x$  and  $y$ ,  $G_\phi$  goes as

$$G_\phi^x \approx \frac{C \cos[2k_F(x+y)]}{(xy)^{\Delta_{\text{CDW},\infty}}}, \quad (3.23)$$

where  $C$  is a constant that is proportional to  $(\mathcal{V}^{XY})^2$ . Thus the correlation function  $G_\phi$  decays as a power law in all directions. Notice that  $G_\phi$  is not isotropic but exhibits a square  $C_{4v}$  symmetry.

The stability of the CSLL phase for the gapless case follows along the same lines. If there are no marginal interarray spin-dependent coupling terms, then  $\kappa_\sigma = 1$ , and we define a renormalized  $\kappa_\rho(q_\perp)$ . The stability of the CSLL phase is identical to the stability of the smectic phase on a single array, with a fixed-point action described by the renormalized function  $\kappa_\rho(q_\perp)$ . Also, proceeding as in Eqs. (3.20) to (3.23), we now expect the single-electron correlation functions to exhibit power-law decay in all directions.

### B. Crossed 2:1 array

The above analysis can also be extended quite easily to a three-dimensional stack of alternate 2D  $X$  and  $Y$  arrays. We could also think of such a stack as a three-dimension array of wires running along the  $X$  axis, intermeshed with a 3D array of wires running in the  $Y$  direction. Thus the fixed point action would be of the form  $S_X + S_Y + S_{XY}$  where  $S_X$  and  $S_Y$  are the actions for the 3D arrays formed by wires running along the  $X$  axis and  $Y$  axis, respectively, where  $S_{XY}$  represents the interarray Coulomb interactions. Thus the fixed-point action is

$$\begin{aligned} S = & \frac{1}{2} \int \frac{d\omega dq_x dq_y dq_z}{(2\pi)^4} [V^\theta(q_y, q_z) q_x^2 |\theta_x|^2 \\ & + V^\theta(q_x, q_z) q_y^2 |\theta_y|^2 + V^\phi(q_y, q_z) q_x^2 |\phi_x|^2 \\ & + V^\phi(q_x, q_z) q_y^2 |\phi_y|^2 + \{V^{XY}(q_x, q_y, q_z) q_x q_y \phi_x \phi_y^* \\ & + \text{c.c.}\} - i\omega q_x \{\theta_x^* \phi_x + \text{c.c.}\} - i\omega q_y \{\theta_y^* \phi_y + \text{c.c.}\}], \end{aligned} \quad (3.24)$$

where  $\phi_{x,y}$  and  $\theta_{x,y}$  are functions of  $\omega$ ,  $q_x$ ,  $q_y$ , and  $q_z$ , and  $V^{XY}(q_x, q_y, q_z)$  represents the interactions between the  $X$  and  $Y$  arrays. For interactions only between nearest-neighbor layers, we obtain  $V^{XY}(q_x, q_y, q_z) = V^c(q_x, q_y)(1 + e^{iq_z})$ . Integrating out the  $\phi$  variables, we are left with an effective action

$$\begin{aligned} S_\theta = & \frac{1}{2} \int \frac{d\omega dq_x dq_y}{(2\pi)^3} \left[ \frac{1}{\kappa_x(\mathbf{q})} \left( v_x(\mathbf{q}) q_x^2 + \frac{\omega^2}{v_x(\mathbf{q})} \right) |\theta_x|^2 \right. \\ & + \frac{1}{\kappa_y(\mathbf{q})} \left( v_y(\mathbf{q}) q_y^2 + \frac{\omega^2}{v_y(\mathbf{q})} \right) |\theta_y|^2 \\ & \left. - \{V_R^{XY}(\mathbf{q}) \omega^2 \theta_x \theta_y^* + \text{c.c.}\} \right], \end{aligned} \quad (3.25)$$

where

$$\kappa_x(\mathbf{q}) = \sqrt{\frac{\gamma(\mathbf{q})}{V^\theta(q_y, q_z) V^\phi(q_x, q_z)}},$$

$$v_x(\mathbf{q}) = \sqrt{\frac{V^\theta(q_y, q_z) \gamma(\mathbf{q})}{V^\phi(q_x, q_z)}},$$

$$V_R^{XY}(\mathbf{q}) = \frac{V^{XY}(\mathbf{q})}{\gamma(\mathbf{q})},$$

$$\gamma(\mathbf{q}) = V^\phi(q_x) V^\phi(q_y) - |V^{XY}|^2 \quad (3.26)$$

and  $\kappa_y(\mathbf{q}) = \kappa_x(P\mathbf{q})$ ,  $v_y(\mathbf{q}) = v_x(P\mathbf{q})$ , where  $P\mathbf{q} = P(q_x, q_y, q_z) = (q_y, q_x, q_z)$ . Proceeding exactly as in the previous case, we find that

$$\langle \theta_x^2 \rangle \sim \pi \ln(L) \int \frac{dq_y dq_z}{(2\pi)^2} \kappa_x(0, q_y, q_z) \quad (3.27)$$

and

$$\langle \phi_x^2 \rangle \sim \pi \ln(L) \int \frac{dq_y dq_z}{(2\pi)^2} \frac{1}{\kappa_x(0, q_y, q_z)}. \quad (3.28)$$

The stability of the three-dimensional crossed stack is precisely the same as the stability of a three-dimensional stack of parallel quantum wires with the Luttinger liquid parameter  $\kappa(q_y, q_z)$  set equal to  $\kappa_x(0, q_y, q_z)$  of the crossed stack. As before, there are no additional singularities due to the coupling between the crossed arrays.

## IV. TRANSPORT PROPERTIES

We now investigate the transport properties of the sliding Luttinger liquid phases. The conductivities of an array of parallel wires has been considered by Emery *et al.*<sup>12</sup> In a pure system, the conductivity along a wire is infinite. In the presence of impurities, the resistivity along the wires vanishes as:<sup>25</sup>

$$\rho_{\parallel} \sim T^{\alpha_{\parallel}}, \quad (4.1)$$

with

$$\alpha_{\parallel} = \Delta_{\text{CDW},\infty} - 2. \quad (4.2)$$

The conductivity perpendicular to the wires for an array of parallel wire can be calculated<sup>12,27</sup> using the Kubo formula, giving us

$$\sigma_{\perp} \sim T^{\alpha_{\perp}} \quad (4.3)$$

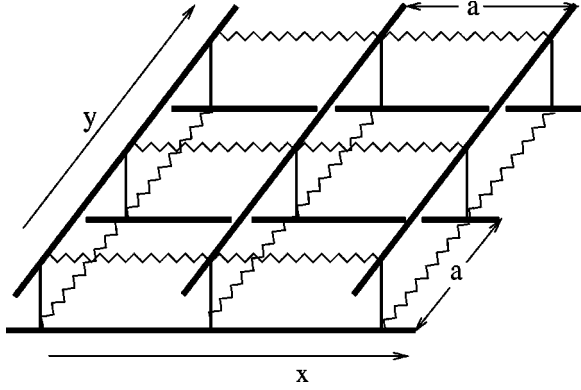


FIG. 7. A schematic depiction of the 2D non-Fermi liquid as a resistor network, with two parallel arrays of wire running along the  $x$  and  $y$  axes, with nodes in the  $z$  direction.

with (Ref. 28)  $\alpha_{\perp} = 2\Delta_{\text{SC}} - 3$ , where  $\Delta_{\text{SC}}$  is the minimum of  $\Delta_{\text{SC},1}$  and  $\Delta_{\text{SC},2}$ . (For details see Appendix D.) The conductance  $\sigma_c$ , arising from the Josephson coupling at the contact between the crossed wires, can be calculated similarly using the Kubo formula, and satisfies

$$\sigma_c \sim T^{\alpha_c}, \quad (4.4)$$

where  $\alpha_c = 2\Delta_{\text{SC},\infty} - 3$ . In this section we focus on the gapped case. In the gapless case,  $\rho_{\parallel}$ ,  $\sigma_{\perp}$ , and  $\sigma_c$  still exhibit power-law behavior even though the major contribution to perpendicular conductivities may come from single-particle hopping.

Thus we can model our 2D non-Fermi liquid as the resistor network depicted in Fig. 7 with nodes at the vertical Josephson junctions between the arrays at  $(x, y) = (na, ma)$ . The nodes of the  $X(Y)$  array are connected by nearest-neighbor resistors with conductances  $\sigma_{\parallel} = \rho_{\parallel}^{-1}$ , if they are parallel to the  $x(y)$  axis and  $\sigma_{\perp}$ , if they are perpendicular to the  $x$  axis ( $y$  axis). Nearest-neighbor nodes of the  $X$  and  $Y$  arrays are connected by resistors of conductance  $\sigma_c$ . In the continuum limit, the 2D current densities in the plane of the  $\alpha$  grids ( $\alpha = X, Y$ ) is  $J_i^{\alpha} = \sigma_{ij}^{\alpha} E_j^{\alpha}$ , where

$$\sigma^X = \begin{pmatrix} \sigma_{\parallel} & 0 \\ 0 & \sigma_{\perp} \end{pmatrix}, \quad (4.5)$$

$$\sigma^Y = \begin{pmatrix} \sigma_{\perp} & 0 \\ 0 & \sigma_{\parallel} \end{pmatrix}, \quad (4.6)$$

and  $\mathbf{E}^{\alpha}$  is the in-plane electric field in plane  $\alpha$ . The current per unit area passing between the planes is  $J_n = (\sigma_c/a^2) \times (V^X - V^Y)$ , where  $V$  is the local voltage. In this limit, the local voltages satisfy

$$\begin{aligned} -\sigma_{ij}^X \partial_i \partial_j V^X + \frac{\sigma_c}{a^2} (V^X - V^Y) &= T^X, \\ -\sigma_{ij}^Y \partial_i \partial_j V^Y - \frac{\sigma_c}{a^2} (V^X - V^Y) &= T^Y, \end{aligned} \quad (4.7)$$

where  $T^X$  and  $T^Y$  are current densities (current/area) injected, respectively, into the  $X$  and  $Y$  grids. If no currents are injected, then this equation is solved by  $V^X = V^Y = -\mathbf{E} \cdot \mathbf{x}$  to produce a total in-planar current density

$$J_i \equiv J_i^X + J_i^Y = (\sigma_{ij}^X + \sigma_{ij}^Y) E_j = (\sigma_{\parallel} + \sigma_{\perp}) E_i. \quad (4.8)$$

Thus under a uniform electric field, the double layer behaves like an isotropic 2D material with in-plane conductivity  $\sigma = \sigma_{\parallel} + \sigma_{\perp} \simeq \sigma_{\parallel}$ , or equivalently with an isotropic resistivity that vanishes as  $\rho_{\parallel} \sim T^{\alpha_{\parallel}}$ .

We could also consider currents that are spatially nonuniform, as they are, for example, when current is inserted at one point and extracted from another. In that case, there is a crossover from isotropic to anisotropic behavior at length scale

$$l = a \sqrt{\frac{\sigma_{\parallel}}{\sigma_c}} \sim T^{-(\alpha_{\parallel} + \alpha_c)/2} \quad (4.9)$$

that diverges as  $T \rightarrow 0$ . To illustrate this crossover, we calculate explicitly the case where a current  $I$  is inserted at a point  $\mathbf{r}_1$  on the  $X$  array and extracted at another point  $\mathbf{r}_2$  on the  $X$  array. Then

$$\begin{aligned} T^X &= I[\delta(\mathbf{r} - \mathbf{r}_1) - \delta(\mathbf{r} - \mathbf{r}_2)], \\ T^Y &= 0. \end{aligned} \quad (4.10)$$

Using Eqs. (4.7) and (4.10), one can solve for the resistance between these two points:

$$R = \frac{V^X(\mathbf{r}_1) - V^X(\mathbf{r}_2)}{I} = 2 \int \frac{d^2 q}{(2\pi)^2} \frac{1 - e^{i\mathbf{q}(\mathbf{r}_2 - \mathbf{r}_1)}}{g(\mathbf{q})}, \quad (4.11)$$

where

$$\begin{aligned} g(\mathbf{q}) &= \frac{\frac{\sigma_c}{a^2} [(\sigma_{\parallel} + \sigma_{\perp}) q_x^2 + (\sigma_{\parallel} + \sigma_{\perp}) q_y^2]}{\frac{\sigma_c}{a^2} - \sigma_{\perp} q_x^2 - \sigma_{\parallel} q_y^2} \\ &\quad - \frac{(\sigma_{\perp} q_x^2 + \sigma_{\parallel} q_y^2)(\sigma_{\parallel} q_x^2 + \sigma_{\perp} q_y^2)}{\frac{\sigma_c}{a^2} - \sigma_{\perp} q_x^2 - \sigma_{\parallel} q_y^2}. \end{aligned} \quad (4.12)$$

For

$$\sigma_c/a^2 \gg \sigma_{\parallel} q^2, \quad (4.13)$$

$g(\mathbf{q})$  takes the simple form  $(\sigma_{\parallel} + \sigma_{\perp}) q^2$ . If  $|\mathbf{r}_1 - \mathbf{r}_2| \gg l$ , with  $l$  defined in Eq. (4.9), then the integral over  $q$  in Eq. (4.11) is dominated by small  $q$  satisfying Eq. (4.13). Thus for  $|\mathbf{r}_1 - \mathbf{r}_2| \gg l$ , the system has approximately the same resistance as an *isotropic* conductor with conductivity  $\sigma_{\parallel}$ . If we inserted current at a point on the  $X$  array and extracted it from the  $Y$  array, we would have the form Eq. (4.11) for the resistance with a different function  $g'(q)$  whose small  $q$  limit is still given by  $(\sigma_{\parallel} + \sigma_{\perp}) q^2$ . Thus for  $|\mathbf{r}_1 - \mathbf{r}_2| \gg l$ , the resis-

tance is approximately independent of whether the current is inserted into (or extracted from) the  $X$  or the  $Y$  array. More generally, in a region where  $T=0$ , by inspection of Eq. (4.7) it can be seen that inhomogeneities in the voltage (or difference between  $V^X$  and  $V^Y$ ) would heal over the lengthscale  $l$  as defined in Eq. (4.9), and at longer lengthscales the system would behave isotropically. This length diverges as  $T \rightarrow 0$  and at  $T=0$ , current can only be carried along the wires; the resistance between wires in a grid or between grids is infinite.

We could, in addition, investigate the frequency-dependent zero-temperature conductivity. By arguments similar to those used with Eqs. (4.1) to (4.4), we obtain

$$\begin{aligned}\rho_{\parallel}(\omega) &\sim \omega^{\alpha_{\parallel}}, \\ \sigma_{\perp}(\omega) &\sim \omega^{\alpha_{\perp}}, \\ \sigma_c(\omega) &\sim \omega^{\alpha_c}.\end{aligned}\quad (4.14)$$

$\alpha_{\parallel}$ ,  $\alpha_{\perp}$ , and  $\alpha_c$  are the same as before, though the coefficients are now different (and complex, in general). At finite  $\omega$ , the long-wavelength resistivity is isotropic as before, and vanishes as  $\rho_{\parallel}(\omega) \sim \omega^{\alpha_{\parallel}}$ .

We could also consider extensions of these calculations to three-dimensional stacks of crossed arrays. As we saw in the previous section, it is possible to get a stable sliding phase in such a system. The conductivity now has a three-dimensional character, with conductivity along the planes given by  $\sigma_{\parallel}/d$ ,  $d$  being the separation between adjacent  $X$  arrays, but with conductivity in the third direction given by  $\sigma_c/a$ . Thus the conductivity along the planes is much larger than the perpendicular conductivity.

## V. CONCLUSION

In conclusion, we have demonstrated the existence of non-Fermi metallic phases in two and three dimensions, that are stable with respect to a wide class of perturbations. We consider both spin-gapped systems and gapless systems that exhibit spin-charge separation. Our central results pertain to the stability and properties of the CSSL phase. This is a remarkable phase, which could be identified as a two-dimensional Luttinger liquid. The correlation functions in this phase exhibit power-law decay along the planes, and the finite-temperature long-wavelength electric conductivity, which is isotropic along the planes, diverges as a power law in temperature  $T$  as  $T \rightarrow 0$ . The importance of this paper is that it provides a perturbative access to non-Fermi-liquid fixed points in two- and three-dimensional systems, something that has proven to be quite difficult in the past.<sup>7</sup> This paper could be of significant relevance for higher-dimensional strongly correlated electron systems in general, and to the normal conducting phases of the cuprates in particular.

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## APPENDIX A

In this appendix we sketch out the steps leading to the asymptotic form for the correlation function  $G_{\phi}(x, \tau=0)$  for large  $x$  [see Eqs. (2.12) and (2.13)].

$$\begin{aligned}G_{\phi}(x, 0) &= \langle \exp\{i\sqrt{2\pi}[\phi_j(x, 0) - \phi(0, 0)] + i2k_F x\} \rangle + \text{c.c.} \\ &= \exp\{-2\pi\langle [\phi_j(x) - \phi_j(0)]^2 \rangle\} \cos(2k_F x).\end{aligned}\quad (A1)$$

It can be easily checked that

$$\begin{aligned}\langle [\phi_j(x) - \phi_j(0)]^2 \rangle \\ = 2 \int \frac{dq_{\parallel} dq_{\perp} d\omega}{(2\pi)^3} \frac{1 - \cos(xq_{\parallel})}{\kappa(q_{\perp}) \left[ v(q_{\perp})q_{\parallel}^2 + \frac{\omega^2}{v(q_{\perp})} \right]}.\end{aligned}\quad (A2)$$

Next we carry out the integration over  $q_{\parallel}$  and  $\omega$  obtaining, for large  $x$ ,

$$\int dq_{\parallel} d\omega \frac{1 - \cos(xq_{\parallel})}{\kappa(q_{\perp}) \left[ v(q_{\perp})q_{\parallel}^2 + \frac{\omega^2}{v(q_{\perp})} \right]} \simeq \frac{\pi \log(x)}{\kappa(q_{\perp})} + F(q_{\perp}),\quad (A3)$$

where  $F(q_{\perp})$  is some function of  $q_{\perp}$ , which depends on  $\kappa$ ,  $v(q_{\perp})$ , and the momentum cutoff. From Eqs. (A1) and (A3) it follows that

$$\langle [\phi_j(x) - \phi_j(0)]^2 \rangle \simeq \left[ \int \frac{dq_{\perp}}{2\pi} \frac{1}{\kappa(q_{\perp})} \right] \log(x) + \text{const.}\quad (A4)$$

Using this, we obtain

$$G_{\phi}(x, 0) \simeq \frac{A_1 \cos(2k_F x)}{x^{\Delta_{\text{CDW}, \infty}}},\quad (A5)$$

where

$$\Delta_{\text{CDW}, \infty} = \int_{-\pi}^{\pi} \frac{dq_{\perp}}{2\pi} \frac{1}{\kappa(q_{\perp})}.\quad (A6)$$

Equations (2.14) and (2.15) follow along similar lines.

## APPENDIX B

Here we outline how the integral

$$I_n = \int_{-\pi}^{\pi} \frac{dq}{2\pi} \frac{(1 - \cos nq)}{f(q)},\quad (B1)$$

needed to calculate  $\Delta_{\text{CDW}, n}$ , can be solved exactly. Here

$$f(q) = 1 + \lambda_1 \cos(q) + \lambda_2 \cos(2q),\quad (B2)$$

using expression (2.30) for  $\lambda_1$  and  $\lambda_2$ . We can rewrite

$$f(q) = 2\lambda_2 (\cos q + u^+) (\cos q + u^-),\quad (B3)$$

where

$$u^\pm = \frac{1}{2} \left[ \frac{\lambda_1}{2\lambda_2} \pm iD \right], \quad (\text{B4})$$

and

$$D^2 = -\frac{\lambda_1^2}{4\lambda_2^2} + \frac{2}{\lambda_2} - 2 = \frac{2\Delta}{\lambda_2}. \quad (\text{B5})$$

It is easy to check that

$$I_n = -\frac{1}{\sqrt{2\Delta\lambda_2}} \text{Im} J_n^+ \quad (\text{B6})$$

with

$$J_n^+ = \int_0^{2\pi} \frac{dq}{2\pi} \frac{1 - \cos(nq)}{u^+ + \cos q} = \frac{1}{\pi i} [J_{1,n} + J_{2,n}],$$

$$J_{1,n} = \oint \frac{dz}{1 + 2u^+z + z^2},$$

$$J_{2,n} = -\frac{1}{2} \oint \frac{dz}{z^n} \frac{[1 + z^{2n}]}{1 + 2u^+z + z^2}, \quad (\text{B7})$$

where  $z = e^{iq}$ , and the  $z$  integral is over the unit circle centered about the origin. The integrands have poles at  $z = (0, z_0^+, z_0^-)$ :

$$z_0^\pm = -u^+ \pm i\sqrt{1 - (u^+)^2}. \quad (\text{B8})$$

Using Eq. (B8), it is easy to check that  $z_0^+ \times z_0^- = 1$ . Thus, either  $z_0^+$  or  $z_0^-$  lies inside the contour of integration (the unit circle). Using the method of residues, it is now straightforward to calculate the integrals. We simply need to sum over the residues of the poles enclosed within the contour of integration. In order to express our results, we distinguish two cases:

(1)  $|z_0^+| < 1$ . Then

$$J_n^+ = \frac{2}{(z_0^+ - z_0^-)} \left[ 1 - \frac{(1 + (z_0^+)^{2n})}{2(z_0^+)^n} \right] - \sum_{m=0}^{n-1} \frac{1}{(z_0^+)^{m+1} (z_0^-)^{n-m}}$$

$$= \frac{2}{(z_0^+ - z_0^-)} [1 - (z_0^-)^n]. \quad (\text{B9})$$

(2)  $|z_0^-| < 1$ . In this case

$$J_n^+ = \frac{2}{(z_0^- - z_0^+)} \left[ 1 - \frac{(1 + (z_0^-)^{2n})}{2(z_0^-)^n} \right] - \sum_{m=0}^{n-1} \frac{1}{(z_0^+)^{m+1} (z_0^-)^{n-m}}$$

$$= \frac{2}{(z_0^- - z_0^+)} [1 - (z_0^+)^n]. \quad (\text{B10})$$

## APPENDIX C

Here we consider the infrared divergence of the integral

$$I = \int \frac{dq_x dq_y d\omega}{(2\pi)^3} \frac{(\omega^2/v_y + v_y q_y^2)}{\kappa_y \mathcal{D}}, \quad (\text{C1})$$

where

$$\mathcal{D} = \frac{1}{\kappa_x \kappa_y} \left( \frac{\omega^2}{v_x} + v_x q_x^2 \right) (\omega^2/v_y + v_y q_y^2) - (V_R^c)^2 \omega^4. \quad (\text{C2})$$

At first sight, it may appear that the integral is divergence free, since by power counting, there are two powers of  $Q$  [where  $\mathbf{Q} = (\omega, q_x, q_y)$ ] in the numerator multiplying  $d^3Q$ , and four powers of  $Q$  in the denominator. This seems to indicate that the integral is finite as  $L \rightarrow \infty$ . Notice, however, that if  $V_R^c$  is set equal to zero in  $\mathcal{D}$ , the integral can be written as

$$I = \int \frac{dq_x dq_y d\omega}{2\pi^3} \frac{\kappa_x(q_x, q_y)}{\omega^2/v_x + v_x q_x^2}, \quad (\text{C3})$$

which is clearly infrared divergent. This divergence comes purely from the integration over  $q_x, \omega$ . It turns out that even in the presence of  $V_R^c$ , the divergence comes purely from the integration over  $q_x, \omega$ . To obtain the infrared divergent part we write

$$\frac{(\omega^2/v_x + v_x q_x^2)}{\kappa_x \mathcal{D}} = \frac{\kappa_x(0, q_y)}{\omega^2/v_x + v_x q_x^2} + R, \quad (\text{C4})$$

where  $R$  is the remaining piece, and our task is to show that its integral has no infrared divergence. Let us write  $R = R_1 + R_2$ , where

$$R_1 = \frac{-\kappa_x(q_x, q_y) - \kappa_x(0, q_y)}{\omega^2/v_y + v_y q_y^2},$$

$$R_2 = \frac{(\omega^2/v_y + v_y q_y^2)}{\kappa_y} \left[ \frac{1}{\mathcal{D}} - \frac{\kappa_x \kappa_y}{(\omega^2/v_x + v_x q_x^2)(\omega^2/v_y + v_y q_y^2)} \right]. \quad (\text{C5})$$

The integral of  $R_1$  has no infrared divergence. To check that this is true for  $R_2$  as well, we note that in the expression for  $R_2$  the term in the square brackets can be written as

$$\frac{\kappa_x \kappa_y V_R^c \omega^4}{\mathcal{D}(\omega^2/v_x + v_x q_x^2)(\omega^2/v_y + v_y q_y^2)}. \quad (\text{C6})$$

Now, by noticing the powers of  $\omega, q_y$ , it is easy to see that the integration of  $R_2$  has no divergence. Thus the infrared divergent part of  $I$  can be written as

$$\int \frac{dq_x dq_y d\omega}{(2\pi)^3} \frac{\kappa_x(0, q_y)}{\omega^2/v_x + v_x q_x^2}. \quad (\text{C7})$$

Equation (3.8) now follows easily.

**APPENDIX D**

We demonstrate explicitly the leading dependence of the perpendicular conductivity on temperature. According to the Kubo formula, the transverse conductivity is given by

$$\sigma_{\perp}(\omega) = \frac{i}{\omega} \left[ \Pi_{\perp}(\omega) + \frac{n_0 e^2}{m} \right], \quad (\text{D1})$$

where the first term represents the paramagnetic contribution with

$$\begin{aligned} \Pi_{\perp}(\omega) = & -i \sum_j \int dx \int_{-\infty}^{\infty} dt \Theta(t) e^{i\omega t} \\ & \times \langle [J_{\perp}(x, j, t), J_{\perp}(0, 0, 0)] \rangle \end{aligned} \quad (\text{D2})$$

being the retarded current-current correlator, and the second term represents the diamagnetic contribution. The step function  $\Theta(t)$  may be written as  $[1 + \text{sign}(t)]/2$ , where  $\text{sign}(t)$  is  $+1$  for positive  $t$ , and  $-1$  for negative  $t$ . In the spin-gapped case, the contribution to the paramagnetic part comes from superconducting pair hopping. The paramagnetic and diamagnetic terms can be combined to give

$$\begin{aligned} \sigma_{\perp}(\omega) = & \frac{i}{\omega} \left[ \int dx \int dt e^{i\omega t} [1 + \text{sign}(t)] [\Pi^{\>}(x, t) \right. \\ & \left. - \Pi^{\<}(x, t)] - (\omega=0) \right], \end{aligned} \quad (\text{D3})$$

where

$$\begin{aligned} \Pi^{\>} = & -i \sum_j \langle J(x, j, t) J(0, 0, 0) \rangle, \\ \Pi^{\<} = & -i \sum_j \langle J(0, 0, 0) J(x, j, t) \rangle. \end{aligned} \quad (\text{D4})$$

Since  $\Pi^{\>} - \Pi^{\<}$  is odd in  $t$ , the real part of the conductivity is given by

$$\sigma'_{\perp}(\omega) = \frac{i}{\omega} \left[ \int dx \int dt e^{i\omega t} [\Pi^{\>}(x, t) - \Pi^{\<}(x, t)] \right]. \quad (\text{D5})$$

Note that the dc transverse conductivity is purely real, and can be obtained from  $\sigma'_{\perp}(\omega)$  by taking the limit  $\omega \rightarrow 0$ .  $\Pi^{\>}$  is related by analytic continuation to the Matsubara correlator  $\Pi_M(x, \tau)$  in the upper-half-plane of complex  $t$  space, and  $\Pi^{\<}$  is related to  $\Pi_M(x, \tau)$  in the lower-half-plane. Thus we could view the integral in Eq. (D5) as an integral over the Keldyish contour shown in Fig. 8(a). This contour can be distorted to the contour shown in Fig. 8(b). Note that  $\Pi^{\>}(t + i\beta/2) = \Pi^{\<}(t - i\beta/2)$ , where  $\beta = 1/k_B T$ . Thus, we obtain

$$\sigma'_{\perp}(\omega) = \frac{1}{\omega} \int dx \int_{-\infty}^{\infty} dt \left[ e^{i\omega t} \sinh\left(\frac{\omega\beta}{2}\right) \Pi^{\>}\left(x, t + \frac{i\beta}{2}\right) \right]. \quad (\text{D6})$$

The next step is to calculate the Matsubara correlator  $\Pi_M(x, \tau)$ . To begin with, we only consider nearest-neighbor hoppings between wires. To lowest order in  $\mathcal{J}$  and  $\mathcal{T}$ :

$$\Pi_M(x, \tau) = \sum_j \langle J_{\perp}(x, j, \tau) J_{\perp}(0, 0, 0) \rangle, \quad (\text{D7})$$

where

$$\begin{aligned} J(x, j, \tau) = & a \mathcal{J}_1 \{ \sin[\theta_j(x, \tau) - \theta_{j-1}(x, \tau)] \\ & + \sin[\theta_{j+1}(x, \tau) - \theta_j(x, \tau)] \}. \end{aligned} \quad (\text{D8})$$

Here  $a$  is the distance between adjacent wires, and the expectation value is taken with respect to the sliding fixed-point Lagrangian. The correlator can be written as

$$\begin{aligned} \Pi_M(x, \tau) = & \mathcal{J}^2 a^2 \langle e^{i\{[\theta_1(x, \tau) - \theta_1(0, 0)] + [\theta_0(x, \tau) - \theta_0(0, 0)]\}} \rangle \\ = & \mathcal{J}^2 a^2 \exp[-f(x, \tau)], \end{aligned} \quad (\text{D9})$$

where

$$\begin{aligned} f(x, \tau) = & \langle [\theta_1(x, \tau) - \theta_1(0, 0)]^2 - [\theta_1(x, \tau) - \theta_1(0, 0)] \\ & \times [\theta_0(x, \tau) - \theta_0(0, 0)] \rangle \\ = & \frac{1}{\beta} \sum_{\omega} \int dq_{\perp} dq_x \{ \langle \theta(q_x, q_{\perp}, \omega) \theta^*(q_x, q_{\perp}, \omega) \rangle \\ & \times [1 - \cos(q_x x + \omega \tau)] \times (1 - \cos q_{\perp}) \}. \end{aligned} \quad (\text{D10})$$

Let us first consider a simpler case, where the velocity  $v_s$  has no dependence with  $q_{\perp}$ . Then

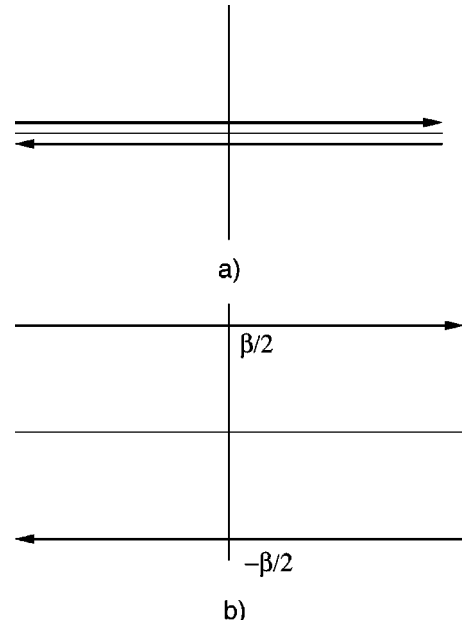


FIG. 8. (a) The Keldyish contour in complex  $t$ -plane, with real time along the X axis. In (b) we depict how the contour is deformed in order to evaluate the integral.



$$\begin{aligned} \Pi_M(x, \tau) &= a^2 \mathcal{J}_1^2 \frac{(\pi T a_x / v)^{2\eta}}{\{\sinh[\pi T(x/v + i\tau)] \sinh[\pi T(x/v - i\tau)]\}^\eta}, \\ & \quad (D11) \end{aligned}$$

where  $\eta = \delta_{\text{SC},1}$ , and  $a_x$  is the spatial cutoff along  $x$ . Thus we may write

$$\begin{aligned} \sigma'_\perp &= a_y^2 \mathcal{J}_1^2 \frac{\sinh(\omega\beta/2)}{\omega} (\pi T a_x / v)^{2\eta} (\pi T)^{-2} v \int d\tilde{x} d\tilde{t} \\ & \quad \times \frac{e^{i\tilde{t}(\omega/\pi T)}}{[\cosh(\tilde{x} + \tilde{t}) \cosh(\tilde{x} - \tilde{t})]^\eta}, \\ & \quad (D12) \end{aligned}$$

where  $\tilde{x} = \pi x T / v$  and  $\tilde{t} = \pi t T$  (Fig. 8). By introducing new variables  $\tilde{x} + \tilde{t}$  and  $\tilde{x} - \tilde{t}$ , we carry out the above integrals, giving

$$\begin{aligned} \sigma'_\perp &= \mathcal{J}_1^2 a_y^2 (v a_x)^{2\eta} \frac{\sinh(\omega/2T)}{8\omega/T} \pi v (\pi T)^{2\eta-3} \\ & \quad \times \frac{\Gamma^2(\eta/2 + i\omega/4\pi T) \Gamma^2(\eta/2 - i\omega/4\pi T)}{\Gamma^2(\eta)}, \\ & \quad (D13) \end{aligned}$$

where  $\eta = \Delta_{\text{SC},1}$ . The  $\omega \rightarrow 0$  limit of the above expression yields

$$\sigma'_\perp(\omega=0) \sim T^{(2\Delta_{\text{SC},1}-3)}. \quad (D14)$$

When the velocity  $v$  is a function of  $q_\perp$ , the integral can no longer be solved exactly. However, the leading  $T$  dependence of the conductivity remains unchanged. To check this, we follow the previous set of steps and arrive at the expression

$$\begin{aligned} \sigma'_\perp &= a_y^2 \mathcal{J}_1^2 \frac{\sinh(\omega/2T)}{\omega} (\pi T a_x)^{2\eta} (\pi T)^{-2} \\ & \quad \times \int d\tilde{x} d\tilde{t} \left[ e^{i\tilde{t}(\omega/\pi T)} \right. \\ & \quad \left. \times \prod_{q_\perp} \frac{1}{[v(q_\perp) \cosh(\tilde{x}/v + \tilde{t}) \cosh(\tilde{x}/v - \tilde{t})]^{\Delta_{q_\perp}}} \right], \\ & \quad (D15) \end{aligned}$$

where  $\tilde{x} = \pi x T$ ,  $\tilde{t} = \pi t T$ , and  $\sum_{q_\perp} \Delta_{q_\perp} = \eta = \Delta_{\text{SC},1}$ . The result can be expressed in the scaling form

$$\sigma'_\perp(\omega, T) = T^\eta F(\omega/T). \quad (D16)$$

In the limit  $\omega \rightarrow 0$ , the integral is finite and  $T$  independent, implying  $F(0)$  is finite. Thus  $\sigma'_\perp(\omega=0) \sim T^\eta$ , where  $\eta = \Delta_{\text{SC},1}$  for nearest-neighbor hopping. In exactly the same manner, we can calculate the contribution to  $\sigma'_\perp$  from next-nearest neighbor pair hopping. It has the same scaling form as before with  $\Delta_{\text{SC},1}$  replaced by  $\Delta_{\text{SC},2}$ .

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