Probing Neutral Majorana Fermion Edge Modes with Charge Transport

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(Received 13 March 2009; published 28 May 2009)

We propose two experiments to probe the Majorana fermion edge states that occur at a junction between a superconductor and a magnet deposited on the surface of a topological insulator. Combining two Majorana fermions into a single Dirac fermion on a magnetic domain wall allows the neutral Majorana fermions to be probed with charge transport. We will discuss a novel interferometer for Majorana fermions, which probes their Z_2 phase. This setup also allows the transmission of neutral Majorana fermions through a point contact to be measured. We introduce a point contact formed by a superconducting junction and show that its transmission can be controlled by the phase difference across the junction. We discuss the feasibility of these experiments using the recently discovered topological insulator Bi_2Se_3.

DOI: 10.1103/PhysRevLett.102.216403 PACS numbers: 71.10.Pm, 03.67.Lx, 74.45.+c, 74.90.+n

Majorana fermions have attracted interest in condensed matter physics because their exotic non-Abelian quantum statistics [1] form the basis for topological quantum computation [2,3]. Potential electronic systems hosting Majorana fermions include the $\nu = 5/2$ quantum Hall state [1,4], the p-wave superconductor Sr_2RuO_4 [5], and topological insulator-superconductor structures [6–8]. In the $\nu = 5/2$ quantum Hall state, a Majorana bound state is associated with the charge $e/4$ quasiparticle, and gapless chiral Majorana fermions form the neutral sector of the edge states. The $e/4$ charge allows the quasiparticle's non-Abelian statistics to be probed by measuring charge transport in the edge states [9–11]. Recent experiments have shown evidence for the quasiparticle charge $e/4$ [12,13], and there are now intense efforts to prove or disprove their non-Abelian nature.

Detecting Majorana fermions in superconductors is more challenging because they are electrically neutral. Tunneling experiments provide an indirect probe [14–16]. Here we propose interference experiments to probe neutral Majorana fermion edge states in superconductor-magnet-topological insulator structures [6]. Our basic setup, shown in Fig. 1, involves a grounded superconductor surrounded by two insulating magnets with opposite out-of-plane magnetization deposited on the surface of a topological insulator. The magnetic domain wall gives rise to 1D chiral *Dirac* fermions on the surface that play the role of "leads" connecting the superconductor to a source and drain. An electron incident from the source splits into two Majorana fermions which take different paths around the edge of the superconductor and then recombine before going to the drain. We will show the source-drain conductance probes the *interference* of the Majorana fermions, forming a novel "Z_2 interferometer." We will also show that the *transmission* of Majorana fermions through a "point contact" formed by a Josephson junction between two superconductors can be measured, and that the transmission can be tuned by controlling the phase difference across the junction.

A topological insulator [17,18] has gapless surface states that are topologically protected in the absence of time reversal or gauge symmetry breaking fields. Breaking time reversal symmetry either by an applied magnetic field or by depositing an insulating magnetic material can open an energy gap leading to a novel surface quantum Hall effect with $\sigma_{xy} = \pm e^2/2h$ [19,20]. Depositing a superconductor on the surface leads, via the proximity effect, to a surface superconducting state that hosts Majorana fermions [6]. In view of the recent experimental discoveries of topological insulator phases in Bi_xSb_{1-x} [21,22] and Bi_2Se_3 [23], and the earlier experimental evidence of good contact between superconducting Nb and Bi_xSb_{1-x} [24], the experimental study of these novel gapped phases is now possible.

The superconducting and magnetic phases of the surface states, as well as the gapless states at interfaces between

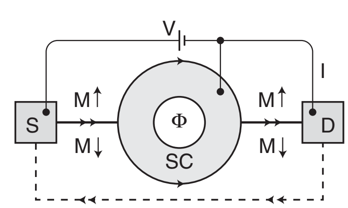

FIG. 1. An interferometer for Majorana fermions. Magnetic (M) and superconducting (SC) materials are deposited on a topological insulator. Chiral Majorana fermion edge states (denoted by a single arrow) circle the outer boundary of the superconductor, and chiral Dirac fermion edge states (denoted by the double arrow) are confined to the magnetic domain wall connected to a source (S) and drain (D). A return path between the drain and source is shown with the dashed line. When a voltage is applied to the source electrons are split into two Majorana fermions, allowing their Z_2 interference phase ± 1 to be probed by measuring the current in the drain.

them, can be described with the Bogoliubov–de Gennes (BdG) formalism. The Hamiltonian is [19] $H = \Psi^\dagger \mathcal{H} \Psi/2$, where $\Psi = [(\psi_\uparrow, \psi_\downarrow), (\psi_\downarrow^\dagger, -\psi_\uparrow^\dagger)]^T$ and

$$\mathcal{H} = \tau^z[v_F \hat{z} \cdot \vec{\sigma} \times (-i\nabla - e\mathbf{A}\tau_z) - \mu] + (\Delta\tau^+ + \Delta^*\tau^-) + M\sigma_z. \quad (1)$$

Here $\psi_{\uparrow,\downarrow}$ are electron operators of the surface states which are Kramers degenerate at $\mathbf{k} = \mathbf{0}$. $\vec{\sigma}$ and $\vec{\tau}$ are Pauli matrices in spin space and particle-hole space, and $\tau^\pm = (\tau_x \pm i\tau_y)/2$. v_F is the Fermi velocity, and μ is the chemical potential. The first line in $\mathcal{H}$ describes free surface states coupled to a vector potential $\mathbf{A}$. $\Delta\psi_\uparrow^\dagger\psi_\downarrow^\dagger$ + H.c. describes the superconducting proximity effect. Spatially uniform Δ gives a gapped excitation spectrum $E_\mathbf{k}^s = \sqrt{(\pm v_F|\mathbf{k}| - \mu)^2 + \Delta^2}$. $M\psi^\dagger\sigma_z\psi$ describes the Zeeman splitting due to the magnet. Spatially uniform M gives $E_\mathbf{k}^z = \sqrt{v_F^2|\mathbf{k}|^2 + M^2} \pm \mu$, which is gapped when $M > \mu$. The BdG Hamiltonian has particle-hole symmetry, expressed by $\{\Xi, \mathcal{H}\} = 0$ where the particle-hole operator is $\Xi\xi = \sigma^y\tau^y\xi^*$. The eigenstates $\xi_{\pm E}$ with energy $\pm E$ obey $\xi_{-E} = \Xi\xi_E$, and only the $E \geq 0$ half of the spectrum represents independent excitations.

An interface between two half planes ($y > 0$ and $y < 0$) with different mass terms gives rise to gapless 1D domain-wall states. First consider a superconductor-magnet interface modeled by $\Delta = \Delta_0\Theta(y)$ and $M = M_0\Theta(-y)$. Solving (1), we find one chiral branch of bound states with a four component wave function $\xi_k(x, y)$ localized near $y = 0$. $\xi_{k=0}$ has zero energy and satisfies $\Xi\xi_0 = \xi_0$, which fixes its phase up to a $\pm$ sign. Using $k \cdot p$ theory the eigenstates for small k are $\xi_k(x, y) = \exp(ikx)\xi_0(y)$ with energy $E(k) = \hbar v_M k$, where $v_M = v_F\langle\xi_0|\tau_z\sigma_y|\xi_0\rangle = v_F\sqrt{1 - \mu^2/M_0^2}/(1 + \mu^2/\Delta_0^2)$. These define Bogoliubov operators $\gamma_k = \int dxdy\xi_k(x, y)^\dagger\Psi(x, y)$ which satisfy $\gamma_k^\dagger = \gamma_{-k}$. The continuum operators $\gamma(x) \sim \int dk\gamma_k e^{ikx}$ are Majorana fields, $\gamma^\dagger(x) = \gamma(x)$ obeying the low energy Hamiltonian $H = -i\hbar v_M\gamma\partial_x\gamma$.

To model a magnetic domain wall we take $M = M_0\,\mathrm{sgn}(y)$. We find a gapless branch of chiral edge states between $\sigma_{xy} = \pm e^2/2h$. When expressed in the BdG formalism, *two* chiral branches of bound states with energy $E(k) \sim \hbar v_D k$ appear due to the double counting. For $E(k) > 0$, the two states have the form $f_k \otimes |\tau_z = 1\rangle$ and $\Theta f_{-k} \otimes |\tau_z = -1\rangle$, where $f_k(x, y)$ is a two component wave function in the σ_z sector and $\Theta f = \sigma_y f^*$ is the time reversal operator. These correspond to the electron operators $c_{k\alpha}^\dagger$ and $c_{-k\alpha}$, respectively, where the spin state α is an eigenstate of σ_y.

To analyze the device in Fig. 1, we employ the BCS mean-field theory to calculate the transport current due to *quasiparticles*. This is justified because the superconducting order parameter at the surface inherits its phase from the bulk 3D superconductor, which behaves classically at low temperature. When the source is biased at a subgap voltage $V \ll \Delta_0$ the quasiparticles involved are exclusively the gapless Majorana fermion edge states.

An electron incident from the source can be transmitted to the drain as an electron, or converted to a hole by an Andreev process in which charge $2e$ is absorbed into the superconducting condensate. Before solving the general source to drain transmission problem we will show that the behavior at $E = 0$ follows from a simple argument. Scattering at the left trijunction, where the incident Dirac fermion meets the superconductor, must transform an incident $E = 0$ electron $c_L^\dagger$ into a fermion ψ built from the Majorana operators γ_1 and γ_2. The arbitrary sign of $\gamma_{1,2}$ allows us to choose $\psi = \gamma_1 + i\gamma_2$. Likewise, scattering at the right trijunction transforms ψ into a fermion in the right lead. This must be *either* $c_R^\dagger$ or c_R. A superposition of the two is not allowed because it is not a fermion operator. To determine which occurs, we observe that when the size of the superconductor shrinks continuously to zero, the left and right lead seamlessly connect to each other. Adiabatic continuity thus dictates that an incident $E = 0$ electron is transmitted as an electron, $c_L^\dagger \to c_R^\dagger$. When the ring encloses a quantized flux $\Phi = nh/2e$, this adiabatic argument breaks down. Instead, odd n introduces a branch cut for one of the Majorana modes, i.e., $\gamma_1 \to -\gamma_1$. Thus, when the ring encloses an odd number of flux quanta, $c_L^\dagger \to c_R$. An incident $E = 0$ electron is converted to a hole.

To obtain the scattering probabilities at finite energy $0 < E \ll \Delta$, we use the BdG formalism to solve the scattering problem in the limit that the size of the ring L is much larger than the decay length of the Majorana edge states into the bulk, which is of order $\max(\hbar v_F/\Delta_0, \hbar v_F/M_0)$. First consider the scattering at the left trijunction. A 2×2 scattering matrix $S(E)$ relates the two incoming states in the left lead $|\tau_z = \pm 1\rangle$, which we denote e and h (for electron and a hole), to the two outgoing Majorana edge states ξ_1 and ξ_2 on the top and bottom of the ring, $(\xi_1, \xi_2)^T = S(E)(e, h)^T$. To simplify the notation, we have used the channel label to denote the amplitude of the scattering states in the corresponding channel. Particle-hole symmetry implies that $S(E) = S^*(-E)\tau_x$. At $E = 0$, this property, along with unitary $S^\dagger S = 1$, allows S to be chosen as

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}, \quad (2)$$

so that $c^\dagger \to (\gamma_1 - i\gamma_2)/\sqrt{2}$. For $E \neq 0$ there will be corrections to (2), which will be small when $E \ll (v_M/v_F)\min(\Delta_0, M_0)$. These corrections vanish when the junction has a mirror symmetry, so that $\mathcal{H}(-y) = \mathcal{M}^{-1}\mathcal{H}(y)\mathcal{M}$ with $\mathcal{M} = i\sigma_y$. The electron and hole channels are then eigenstates of $\mathcal{M}$ with eigenvalue $\pm i$, whereas $\mathcal{M}$ interchanges the Majorana edge states. This leads to (2) at any energy. In the following we will assume $S(E)$ is well described by its low energy limit (2).

Next we study the propagation of the chiral Majorana fermion. When there is no magnetic flux the wave function at $E \ll \Delta_0$ can be approximated by $\xi(l,s) = \xi_0(s) \times \exp(ik(E)l)$, where $l(s)$ is the length along (perpendicular to) the interface, and $k(E) = E/v_M$. In the presence of flux $\Phi = nh/2e$, the superconducting phase ϕ winds by $2\pi n$ around the ring accompanied by a vector potential $\mathbf{A} = \nabla\phi$. It is convenient to choose a gauge in which the spatial variation of ϕ is concentrated near the middle of the upper semicircle. Away from this "scattering region," the wave function is the same as before. This problem can be solved with a $U(2)$ gauge transformation that eliminates the spatial variation of ϕ and the nonzero $\mathbf{A}$. The wave function is then simply the undisturbed wave function multiplied by $\exp[i\tau_z\phi(l)/2]$. For $\Phi = nh/2e$ the chiral Majorana edge mode γ_1 thus acquires an *additional* phase shift $n\pi$ across the junction.

The scattering amplitude of the ring is found by composing the scattering matrices:

$$\begin{bmatrix} e \\ h \end{bmatrix}_R = S^{-1}\begin{pmatrix} e^{i\pi n + ikl_1} & 0 \\ 0 & e^{ikl_2} \end{pmatrix} S \begin{bmatrix} e \\ h \end{bmatrix}_L. \qquad (3)$$

The current in the drain when the source is biased at voltage V and the superconductor and drain are grounded is

$$I = (-1)^n \frac{e}{h}\int_0^\infty dE[f(E - eV) - f(E + eV)]\cos\theta(E), \qquad (4)$$

where f is the Fermi-Dirac distribution function and $\theta = k(l_1 - l_2) \equiv E\delta L/v_M$ is the relative phase between two paths of different lengths. Evaluating the integral we find

$$I = (-1)^n \frac{e}{h}\frac{\pi k_B T \sin(eV\delta L/v_M)}{\sinh(\pi k_B T\delta L/v_M)}, \qquad k_B T, eV \ll \Delta_0. \qquad (5)$$

At fixed bias, the current "oscillates" as a function of the *discrete* magnetic flux $nh/2e$, reflecting the Aharonov-Bohm phase for Majorana fermions, which takes values ± 1. Our device thus functions as a "Z_2 interferometer" for Majorana fermions. The "visibility" of these oscillations is suppressed below a temperature scale $k_B T_{\delta L} \equiv \hbar v_M/\delta L$ due to thermal averaging. In addition, at finite bias voltage the current oscillates as a function of V with a period $2\pi k_B T_{\delta L}/e$ due to the energy dependence of the relative phase. That the oscillation persists to high bias voltages without any damping is due to the absence of dephasing in our calculation. A similar situation occurs in the electronic Mach-Zehnder interferometer: the decay of the magnitude of interference oscillation at high bias voltage is attributed to dephasing processes [25]. Sources of dephasing in our system include coupling of Majorana fermions with other degrees of freedom, as well as interactions between Majorana fermions. Since Majorana fermions are neutral, we expect environmental coupling is weak. In addition, the lowest order *local* interaction term within the Majorana fermions is $\gamma(x)\partial_x\gamma(x)\partial_x^2\gamma(x)\partial_x^3\gamma(x)$, which involves spatial derivatives at sixth order and will be strongly suppressed at low temperature. Thus there is reason to expect the low temperature dephasing rate for the Majorana fermion edge states will be smaller than that of ordinary electrons.

We next study the transmission of Majorana fermions across a Josephson junction between two superconductors, shown in Fig. 2(a). The junction plays the role of a *point contact* for Majorana fermions and can be characterized by a scattering matrix relating incoming and outgoing Majorana modes, $\gamma_i^{\text{out}} = S_{ij}^{\text{pc}}(E)\gamma_j^{\text{in}}$. Each superconductor is connected to a source and drain by chiral electron modes at magnetic domain walls. An incident electron from S_1 splits into two Majorana modes. One of the two is scattered by the junction, and has amplitude $t = S_{11}^{\text{pc}}$ to be transmitted before recombining with its partner and going to D_1. Following the previous procedure, we calculate the scattering matrix relating an incident fermion at S_1 to an outgoing fermion at D_1 to obtain the current flowing to D_1 when S_1 is at voltage V and the other leads are grounded.

$$I = e\int_0^\infty dE[f(E - eV) - f(E + eV)]\,\text{Re}[t(E)e^{i\theta(E)}], \qquad (6)$$

where $\theta(E)$ is the same as in (4). At $E = 0$ particle-hole symmetry constrains S^{pc} to be a real $O(2)$ matrix describing the transmission $t = \cos\delta$ and reflection $r = \sin\delta$ such that $\gamma_1^{\text{out}} + i\gamma_2^{\text{out}} = e^{i\delta}(\gamma_1^{\text{in}} + i\gamma_2^{\text{in}})$. At $T = 0$ the conductance $G = I_{D1}/V_{S1} = te^2/h$ directly measures the transmission of the neutral Majorana fermions.

The transmission amplitude t can be controlled by adjusting the phase difference ϕ of the Josephson junction. Consider a simple model for $t(\phi)$.

$$H = (\gamma_1, \gamma_2)[-iv_M\tau^z\partial_x + \lambda(x)\cos(\phi/2)\tau_y](\gamma_1, \gamma_2)^T. \qquad (7)$$

When $\lambda(x) = \lambda\delta(x)$ and $\lambda/v_M \ll 1$, H describes superconductors weakly coupled by single electron tunneling at a point [7,26,27]. When $\lambda(x) = \Delta_0$ for $x \in [0, L]$ and 0 otherwise, H becomes the low energy theory of a line

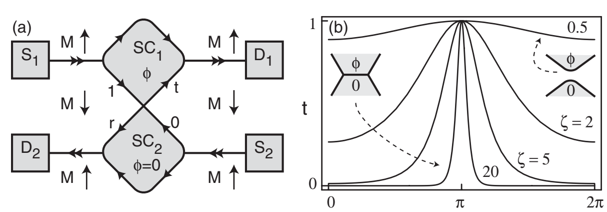

FIG. 2. (a) A point contact for neutral Majorana fermions characterized by reflection and transmission amplitudes t and r formed by a junction between two superconductors. Each superconductor is connected to a source and drain by chiral fermions at a magnetic domain wall, allowing t to be measured with charge transport. (b) Zero energy transmission of the point contact as a function of phase ϕ for different coupling strengths. The insets indicate the limits of a weakly coupled point contact (right) and a long line junction (left).

junction [6]. The transmission amplitude at $E = 0$ in this model is

$$t(\phi) = 1/\cosh[\zeta \cos(\phi/2)], \quad (8)$$

with $\zeta = \int dx \lambda(x)/2v_M$. Figure 2(b) shows $t(\phi)$ for different values of ζ. At $\phi = \pi$, the transmission is perfect. This is guaranteed by gauge invariance. When $\phi \to \phi + 2\pi$ one of the Majorana edge modes changes sign [7] so $r(\phi) = -r(\phi + 2\pi)$. Thus, $r(\phi) = 0$ and $t(\phi) = 1$ for some $\phi \in [0, 2\pi]$. For a symmetric junction this occurs at $\phi = \pi$.

For a weakly coupled point contact [Fig. 2(b), right-hand inset], $t(\phi)$ is energy independent, but is only weakly dependent on ϕ. For a long line junction [Fig. 2(b), left-hand inset], $t(\phi)$ varies over a wide range of values between 0 and 1, but has a very narrow peak $\delta\phi \sim \hbar v_M/\Delta_0 L$. In addition, near the peak the transmission will be strongly energy dependent due to the small gap when $\phi \sim \pi$. It is desirable to engineer the size and geometry of the Josephson junction in between these two limits, so that $t(\phi)$ has a well defined peak which can be probed by the low temperature conductance.

It is worthwhile to compare the superconducting point contact for Majorana fermions studied here with a point contact in the $\nu = 5/2$ quantum Hall effect. Our point contact is precisely equivalent to the *neutral sector* of the $\nu = 5/2$ point contact, which has been described in terms of the Ising boundary conformal field theory [28]. For $\nu = 5/2$, however, the physics is dominated by the backscattering of charge $e/4$ quasiparticles, which is analogous to quantum tunneling *vortices* across the superconductor in our system. Since the superconducting phase is essentially a classical variable, this process is strongly suppressed in a superconducting point contact. Thus, unlike the $\nu = 5/2$ problem, vortex backscattering does *not* lead to a crossover to the weak tunneling limit.

The recently discovered topological insulator Bi_2Se_3 [23,29], with bulk gap $\sim$0.35 eV, is a promising material to probe these states. Unlike $Bi_{1-x}Sb_x$, its surface states have a small Fermi surface that encloses a single Dirac point. Photoemission experiments reveal a Fermi velocity $\hbar v_F \sim 0.3$ eV nm and a Fermi energy $\mu \sim 0.3$ eV relative to the Dirac point. The current materials are unintentionally doped, with the bulk Fermi energy in the conduction band. If the material can be compensated either by doping or gating, it is likely that the surface Fermi energy can be made much closer to the Dirac point. This is important because achieving the magnetic gapped state requires a field $M > \mu$. Moreover, the $k \cdot p$ theory predicts that the Majorana velocity v_M is suppressed when $\Delta_0 \ll \mu$, reducing the temperature scale $T_{\delta L}$ required to observe the signature of Majorana fermions. Our model calculation gives $v_M \sim v_F(\Delta_0/\mu)^2$. Assuming a superconductor can be found that gives a proximity induced gap $\Delta_0 \sim 0.1$ meV, we require size $L > \hbar v_F/\Delta_0 \sim 3$ μm. If $\mu \sim 1$ meV and $\delta L \sim 1$ μm, then $T_{\delta L} \sim 30$ mK. $T_{\delta L}$ can be larger if the path difference δL can be finely tuned.

To conclude, we have proposed experiments to probe the interference and transmission of neutral Majorana fermions with charge transport. We hope they offer a first step towards the more ambitious goal [6] of detecting the non-Abelian statistics of individual Majorana bound states and using them for quantum computation.

We thank Carlo Beenakker for an insightful discussion. This work was supported by NSF Grant No. DMR-0605066 and ACS PRF Grant No. 44776-AC10.

Note added.—Recently, Akhmerov *et al.* [30] independently studied an interferometer similar to Fig. 1.

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