Electron Interactions and Nanotube Fluorescence Spectroscopy

C.L. Kane & E.J. Mele

Large radius theory of optical transitions in semiconducting nanotubes derived from low energy theory of graphene

Phys. Rev. Lett. in press
cond-mat/ 0403153

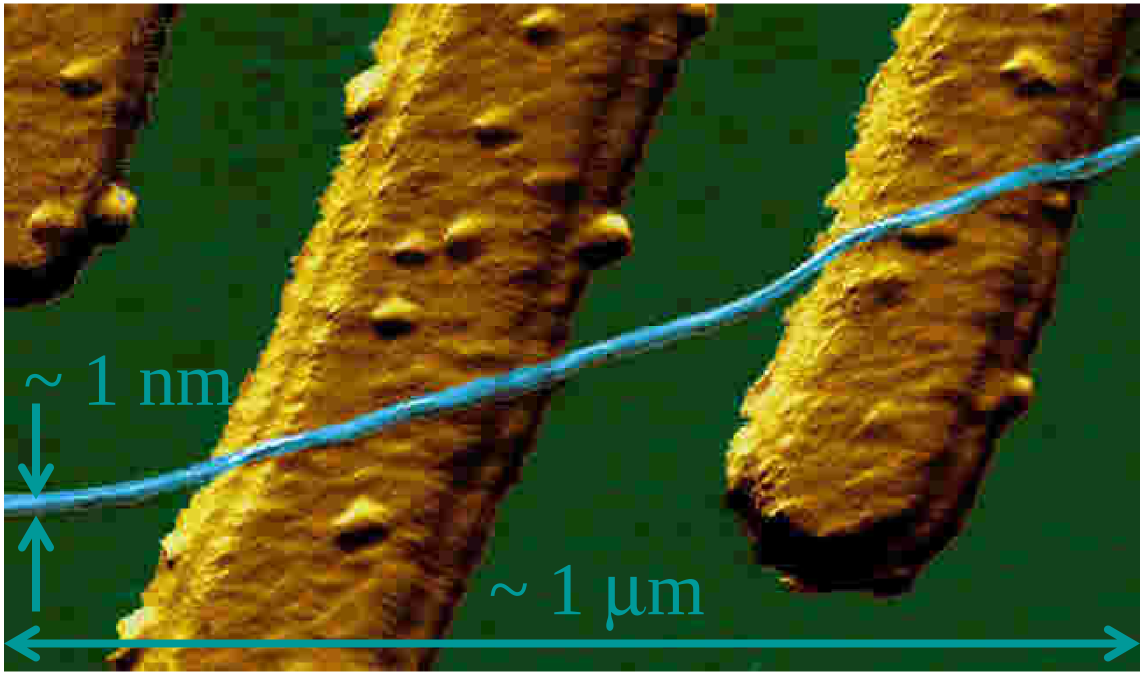
- Brief Introduction to nanotubes
- Independent electron model for optical spectra
- 2D interactions: nonlinear scaling with 1/R
- 1D interactions: excitons
- Short Range Interactions: exciton fine structure

Penn
University of Pennsylvania

LRSM
the University of Pennsylvania's Materials Research Science and Engineering Center

Carbon Nanotubes as Electronic Materials

Gate Source Drain



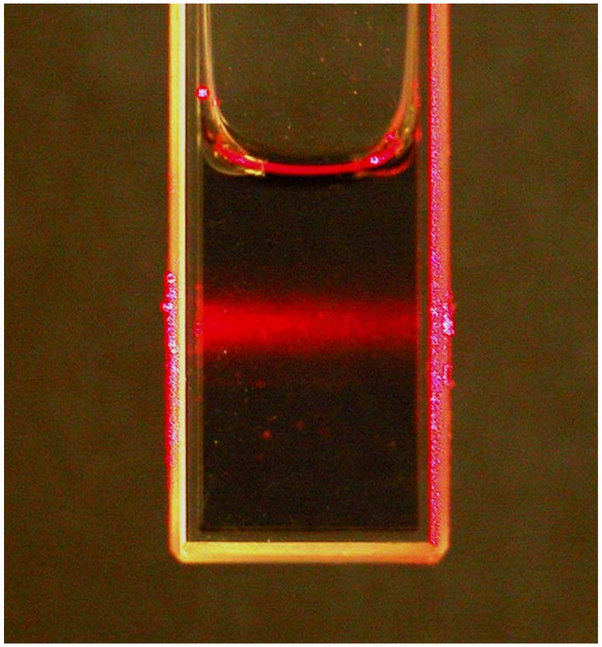
A Molecular Quantum Wire

Tans et al. (Nature 1998)

- Ballistic Conductor
- Field Effect Transistor
- Logic Gates

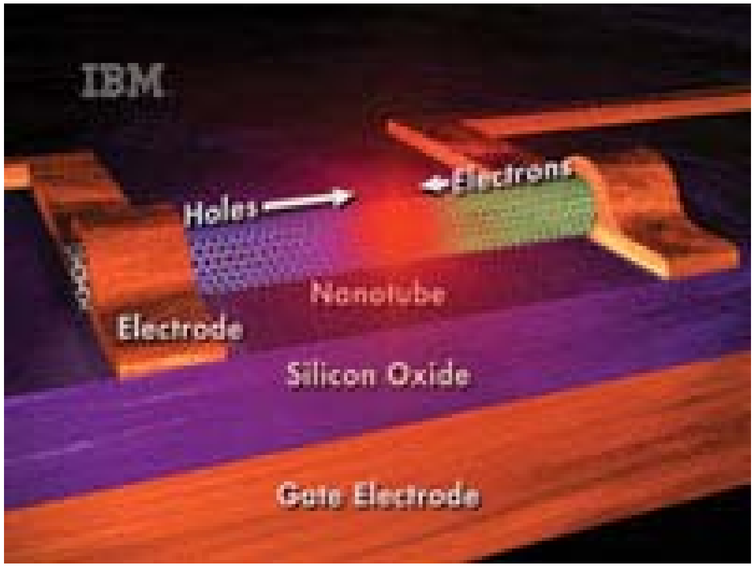
Carbon Nanotubes as Optical Materials

Photoluminescence



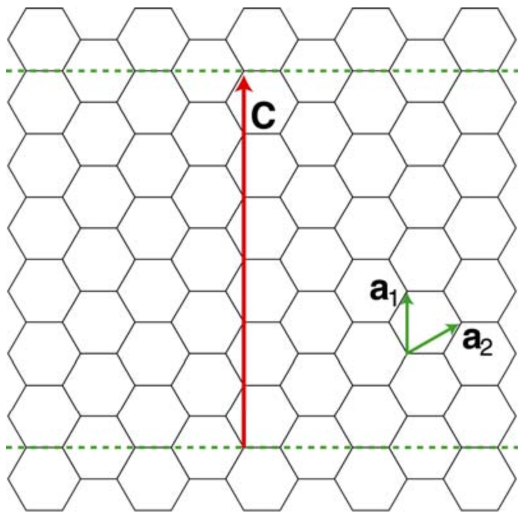
- Nanotubes in surfactant micelles
  Bachillo et al. (2002).
- Photoluminescence from individual suspended nanotubes
  Lefebvre et al. (2003).

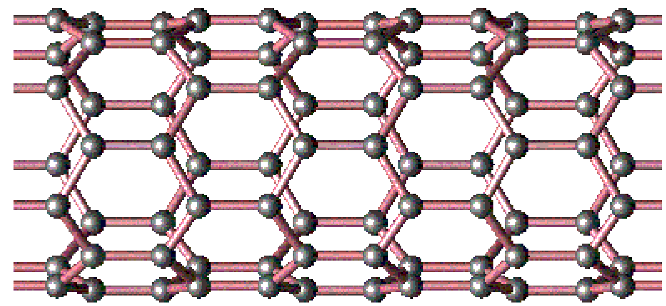
Electroluminescence & Photoconductivity



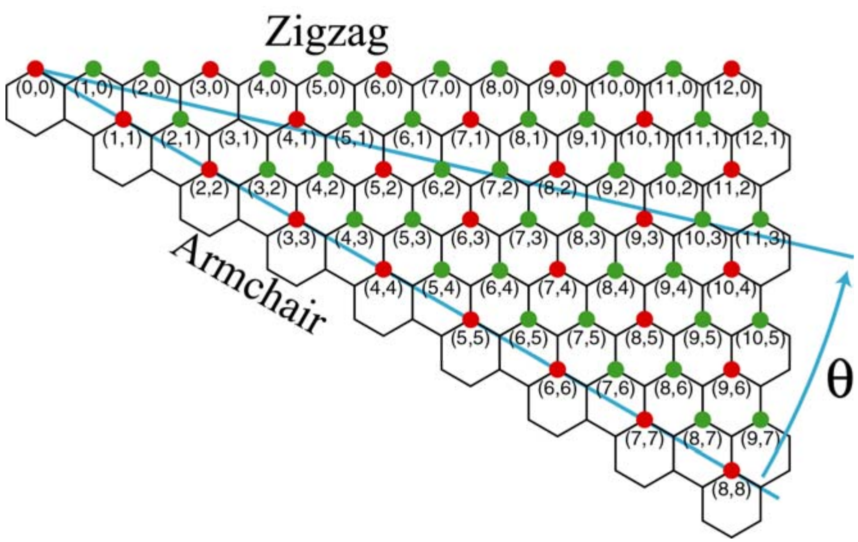
- Infrared Emission, photoconductivity in individual nanotube field effect devices   Freitag et al., (IBM) 2004

Carbon Nanotube : Wrapped Graphene





$\mathbf{C} = n_1\, \mathbf{a}_1 + n_2\, \mathbf{a}_2$



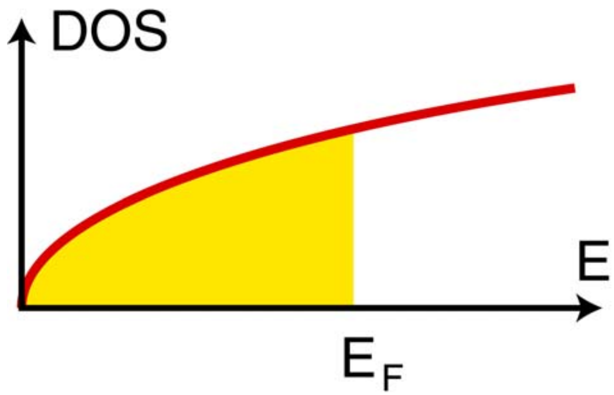
Tubes characterized by $[n_1, n_2]$ or

- Radius : $R = |\mathbf{C}|/2\pi$
- Chiral Angle : $0 < \theta < 30^o$
- Chiral Index : $\nu = n_1 - n_2 \bmod 3$
  $= 0, 1, -1$

Electronic Structure

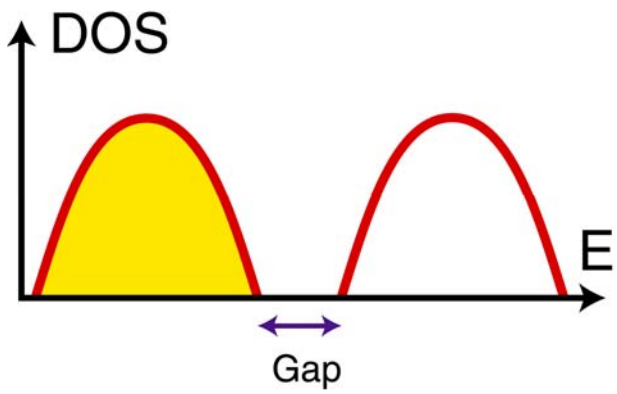
Metal

- Finite Density of States (DOS) at Fermi Energy



Semiconductor

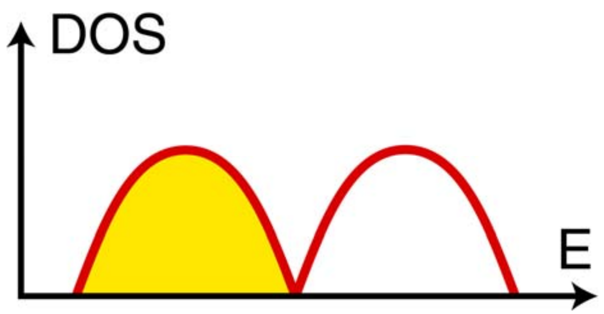
- Gap at Fermi Energy



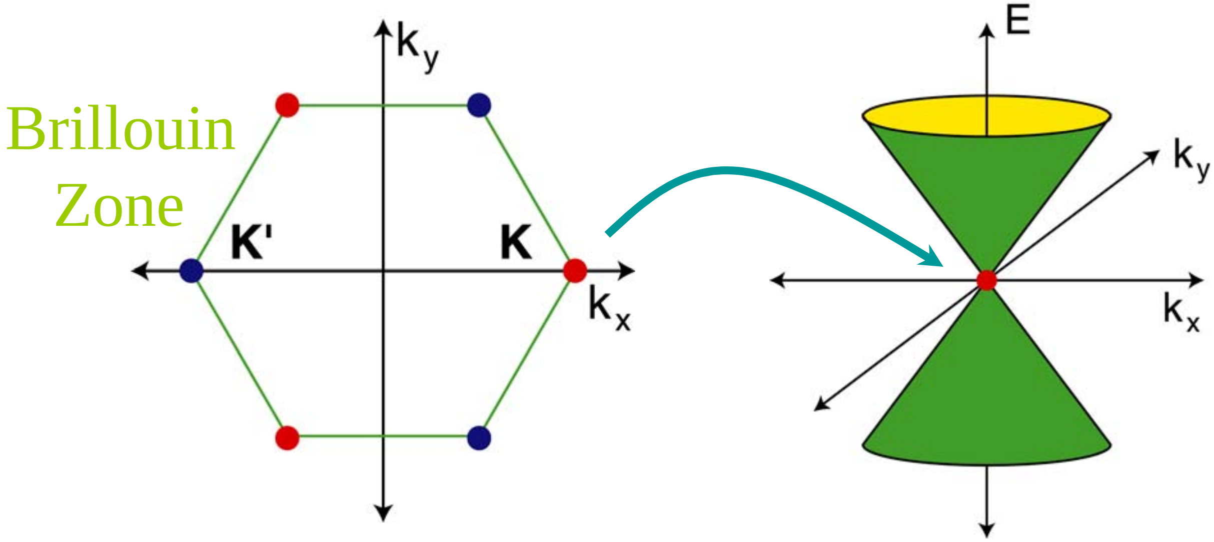
Graphene

- Zero Gap Semiconductor

- Zero DOS metal



Low Energy Theory of Graphene



"Effective Mass" Model: Massless Dirac Hamiltonian

$$H_{eff} = \hbar v_F \psi^\dagger \frac{\sigma \cdot \nabla}{i} \psi$$

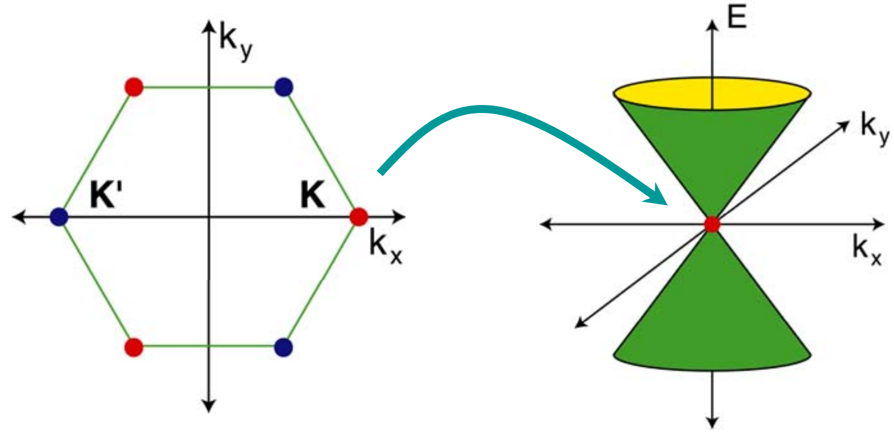
$$E(q) = \pm \hbar v_F \,|\, q \,| \qquad\qquad \hbar v_F = 0.53 \text{ eV nm}$$

Tight Binding model: $\hbar v_F = \sqrt{3}\gamma_0 a / 2$; $\gamma_0 = 2.5$ eV

Wrap it up……

- Flat Graphene:

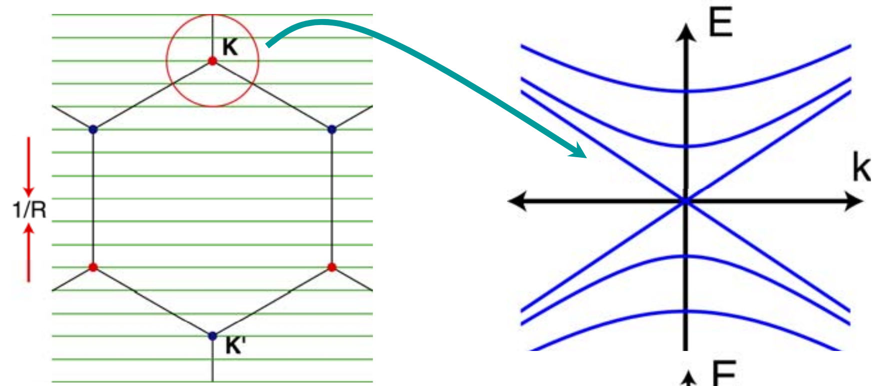
  A zero gap semiconductor



- Periodic boundary conditions on cylinder:

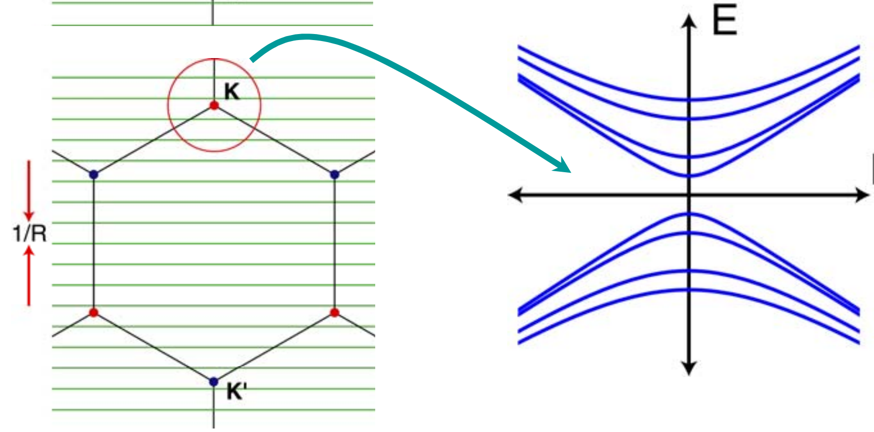
n_1-n_2 = 0 mod 3

1D Metal



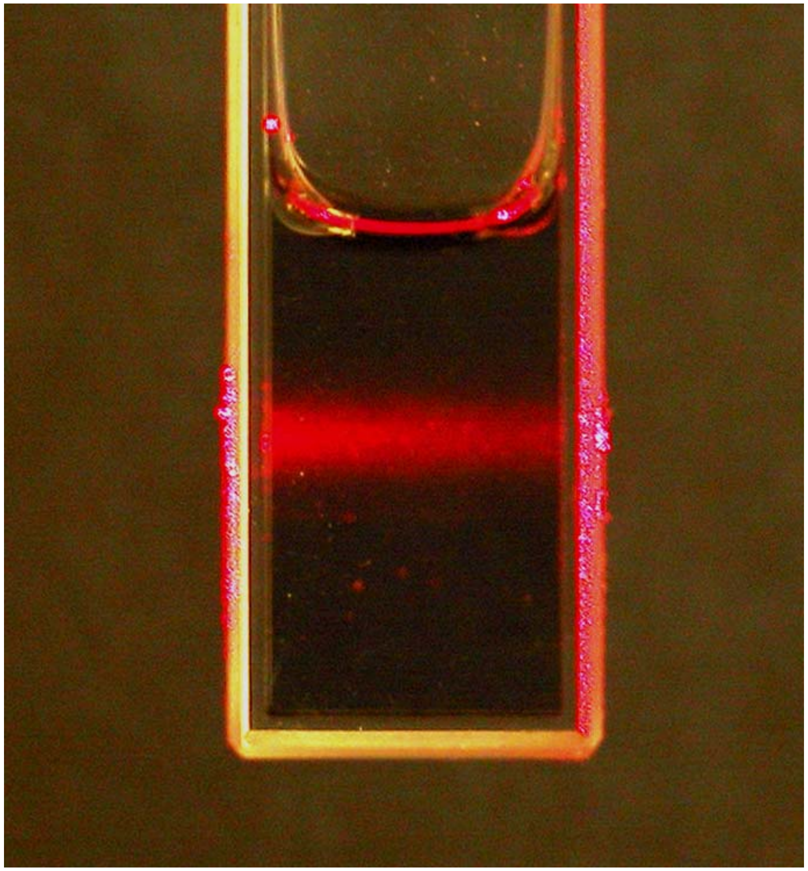
n_1-n_2 = +/-1 mod 3

Semiconductor

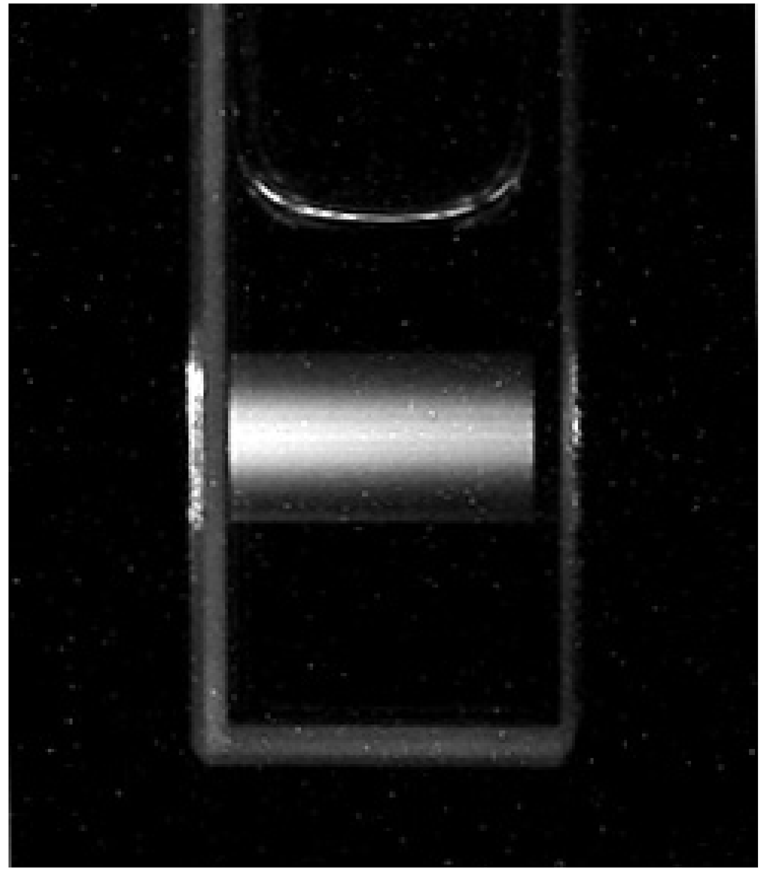


Near-infrared Photoluminescence from Single-wall Carbon Nanotubes

O’Connel et al. (Science 02)
Bachillo et al. (Science 02)



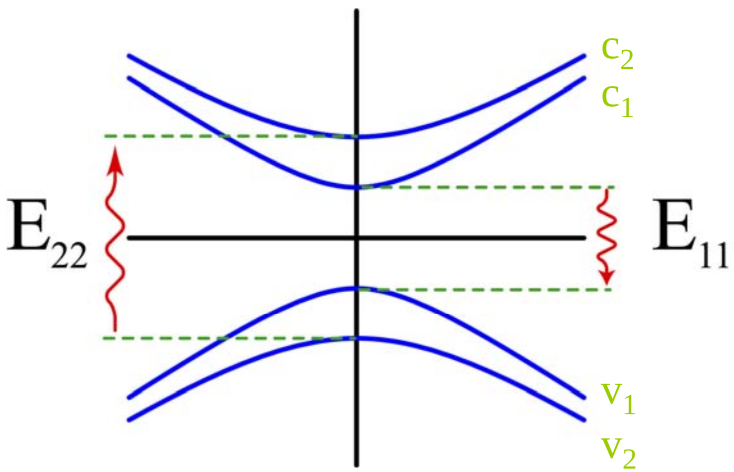
Excitation (661 nm)

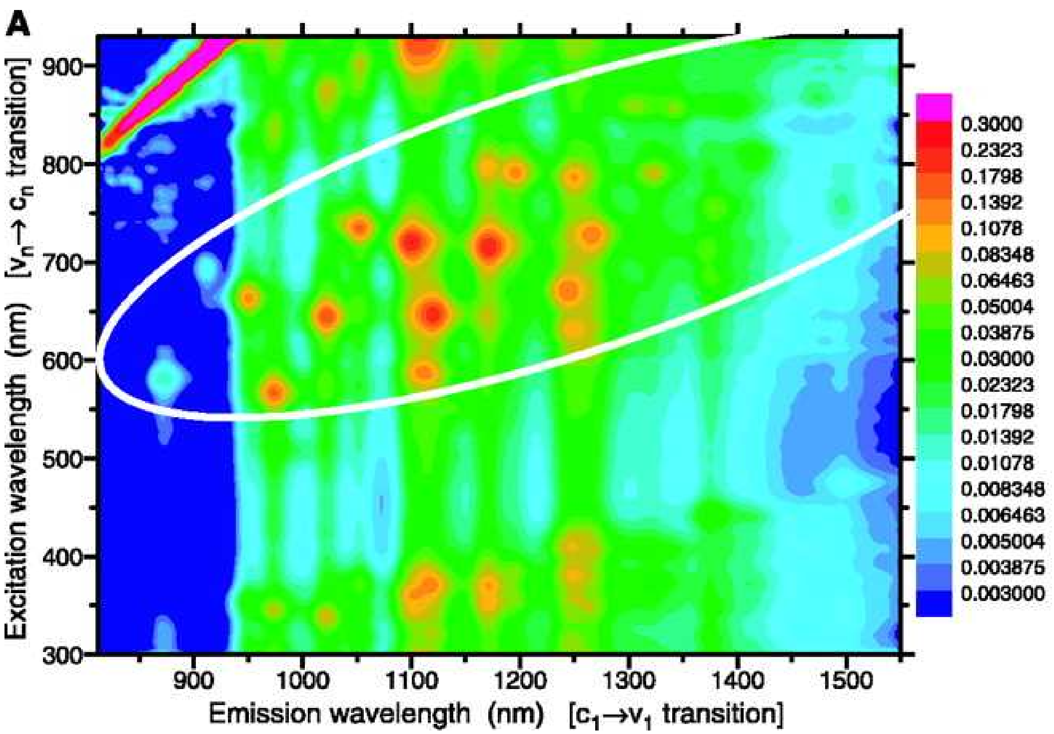


Emission (> 850 nm)

Nanotube Fluorescence Spectroscopy

O'Connel et al. (Science 02)
Bachillo et al. (Science 02)





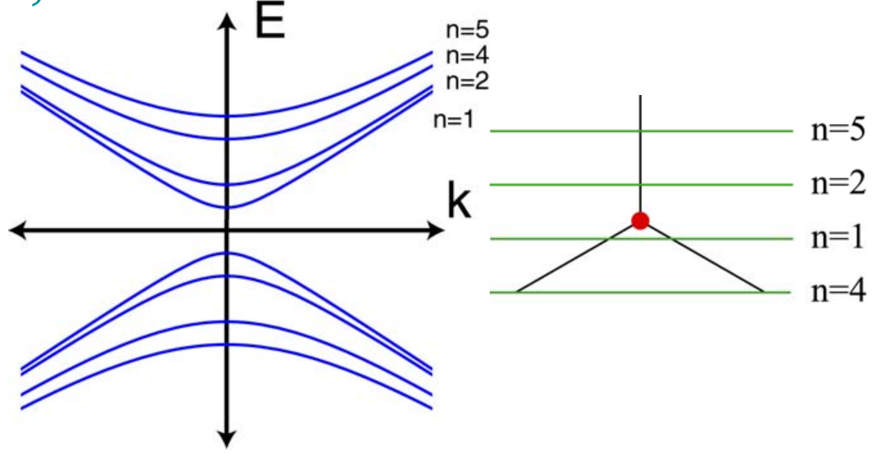
Each peak in the correlation plot corresponds to a particular species $[n_1,n_2]$ of semiconducting nanotube

GOAL:

Understand observed transition energies in terms of low energy properties of an ideal 2 dimensional graphene sheet.

Free Electron Theory of Nanotube Bandgaps

Systematic expansion for large radius, R



- Zeroth order:

$$E_n^0 = \frac{2\hbar v_F}{3}\frac{n}{R} \qquad (n = 1, 2, 4, 5, \ldots)$$

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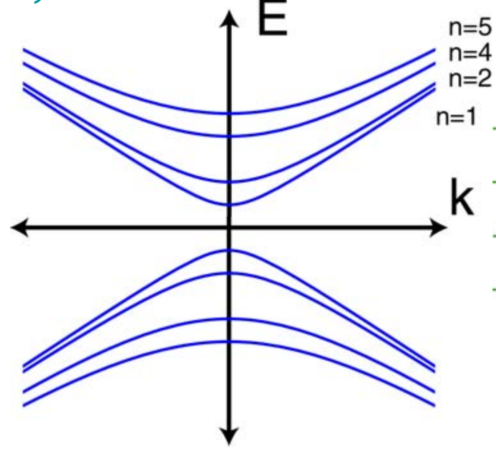
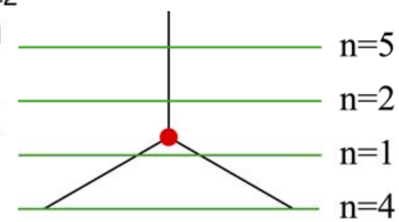


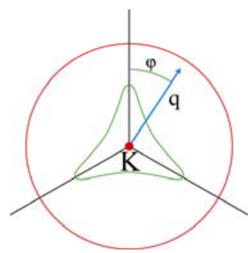


- Zeroth order:

$$E_n^0 = \frac{2\hbar v_F}{3}\frac{n}{R} \qquad (n = 1, 2, 4, 5, \ldots)$$

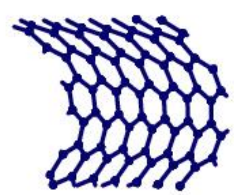
- Trigonal Warping Correction

$$\Delta E_n^{T.W.} \propto (-1)^n \nu \frac{n^2 \sin 3\theta}{R^2}$$



- Curvature Correction

$$\Delta E_n^{C} \propto (-1)^n \nu \frac{\sin 3\theta}{R^2}$$



Curvature and Trigonal Warping:

- Vary as $1/R^2$
- Alternate with band index n
- Alternate with chiral index ν
- Vanish for armchair tubes, θ=0

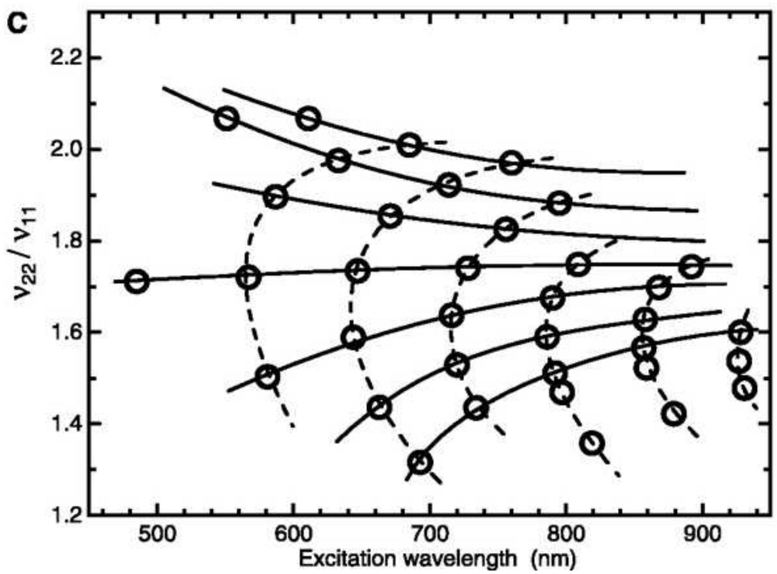
Different dependence on n

The large R limit is most accurate for nearly armchair tubes: θ ~ 0

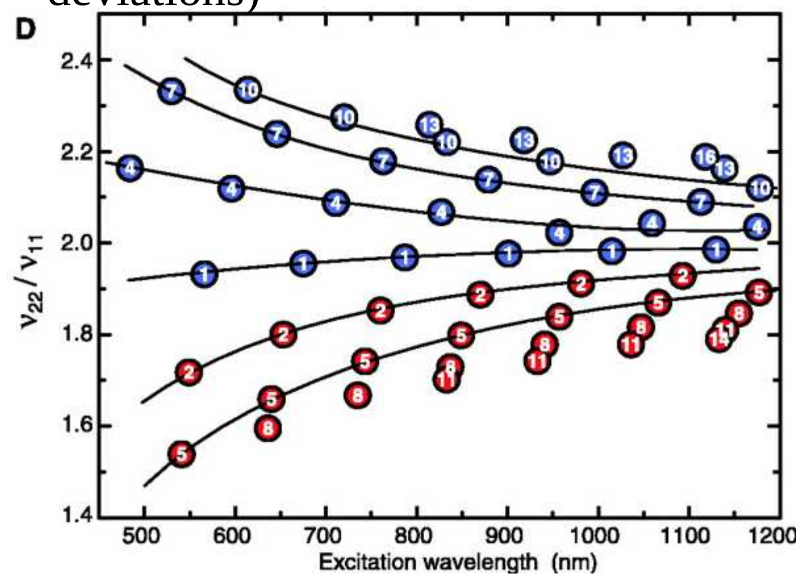
E_n^0 describes tight binding gaps accurately for R > .5 nm

Nanotube Assignments from Pattern of sin 3θ/R² Deviations

Experimental "Ratio Plot"



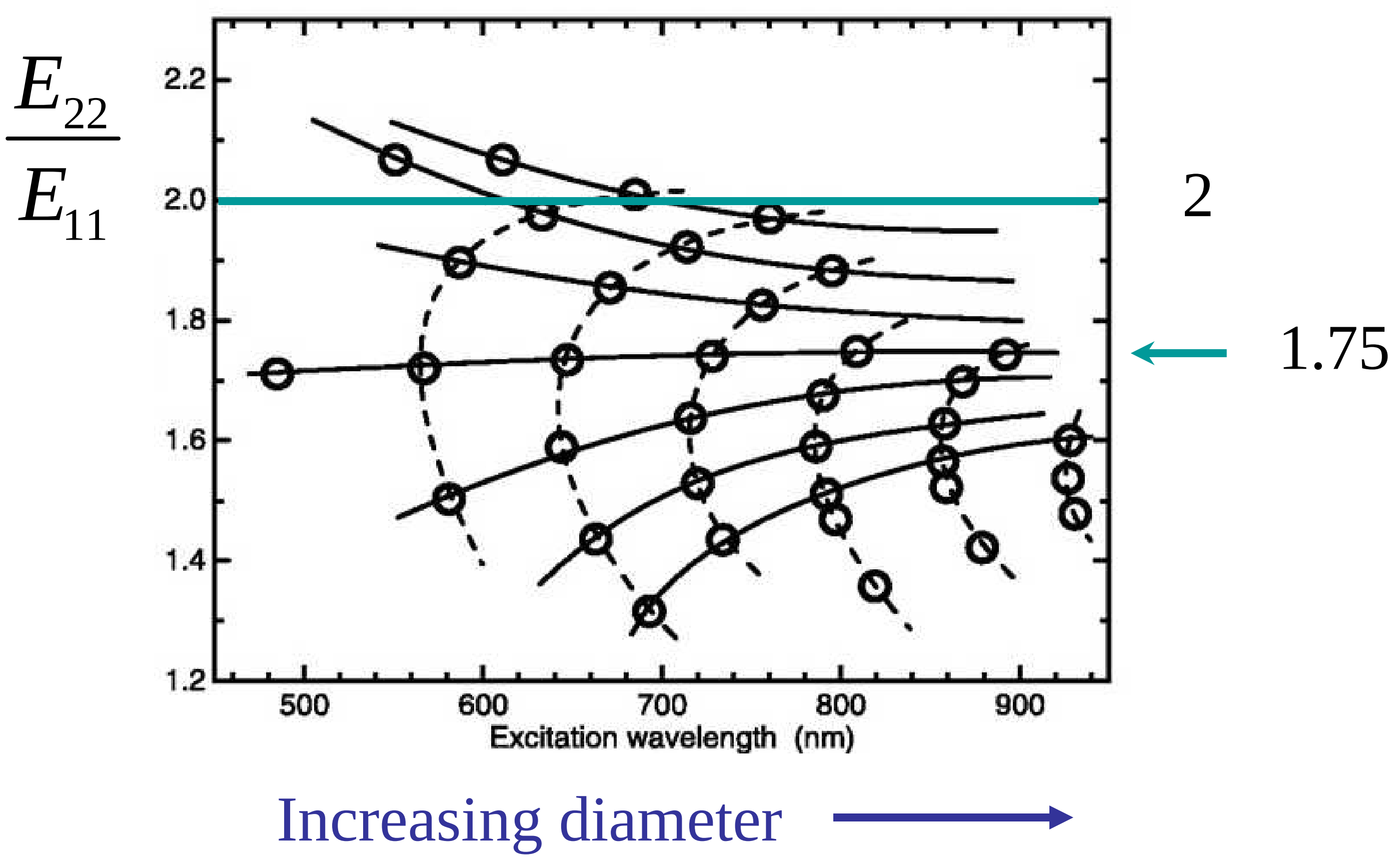
Theory (includes sin3θ/R² deviations)



- By comparing the experimental and theoretical ratio plots the $[n_1,n_2]$ values (and hence R and θ) for each peak can be identified.
- Corroborated by Raman spectroscopy of the radial breathing mode.

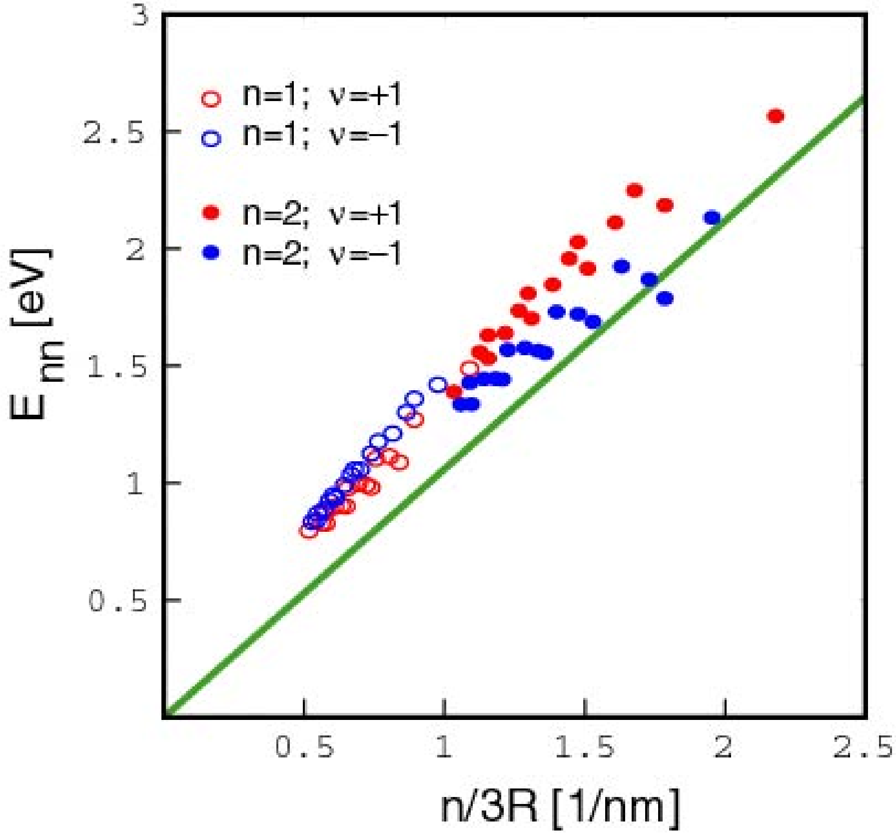
The Ratio Problem

- Free electron theory predicts $E_{22} / E_{11} \to 2$ for $R \to \infty$
- Consequence of linear dispersion of graphene



Scaling of Optical Transition Energies

(Kane,Mele '04)



— Free electrons for θ=0

$$E_{nn}^{0}(R) = 2\hbar v_F n / 3R$$

- ν sin 3θ / R^2 deviations are clear
- Separatrix between ν=+1 and ν=−1describes nearly armchair tubes with θ=0, where sin 3θ/R^2 deviations vanish.

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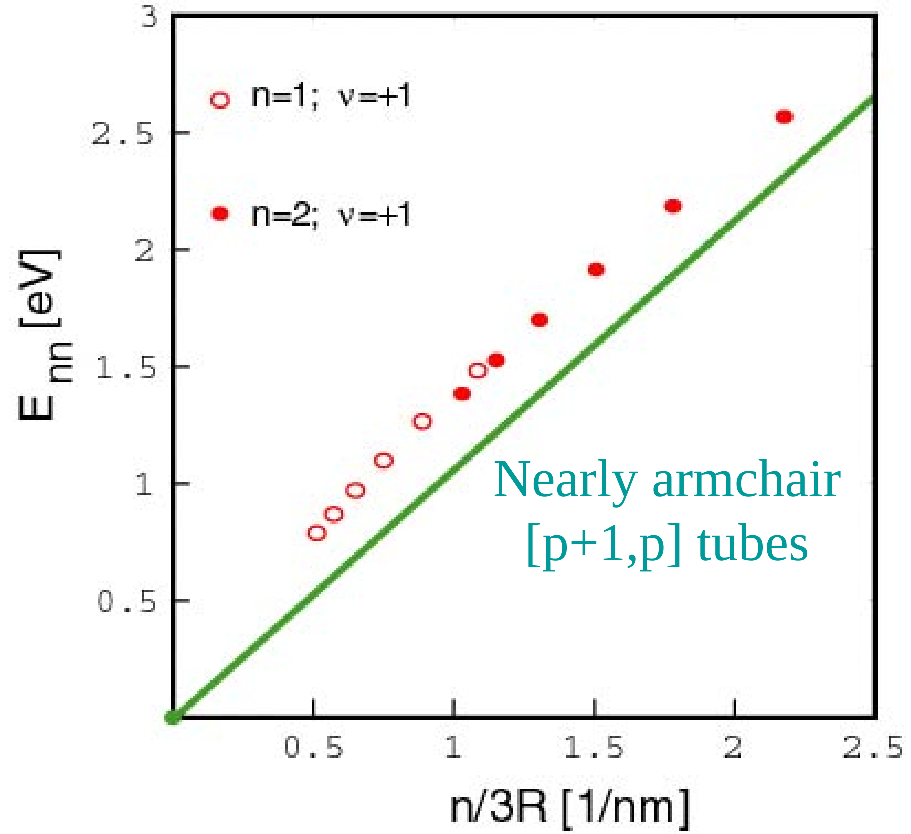

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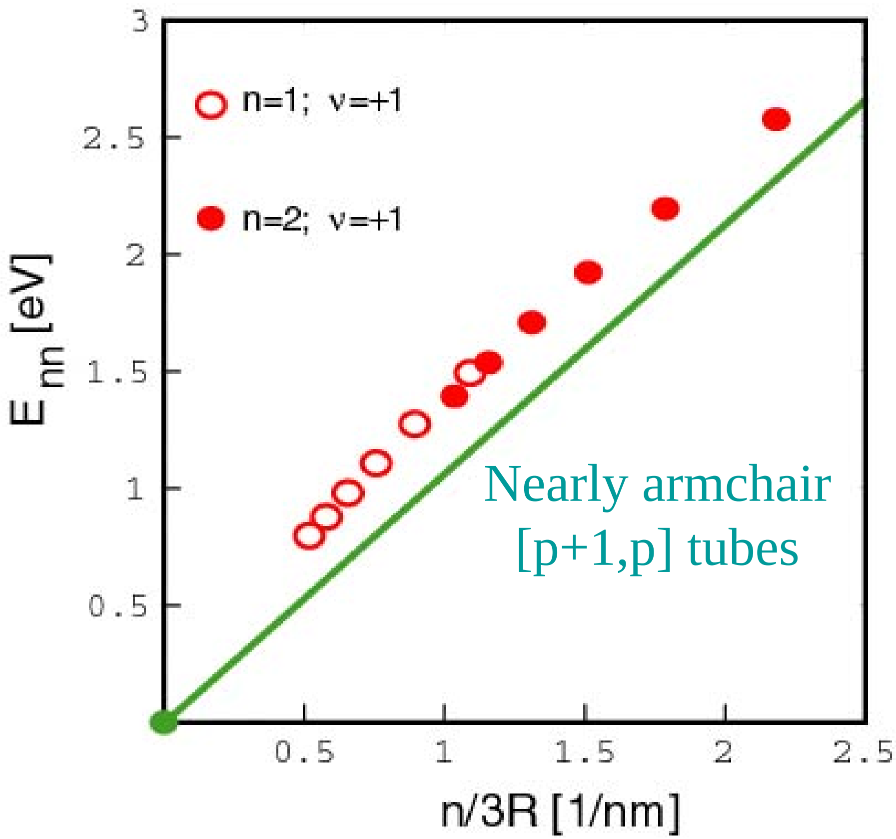


— Free electrons for θ=0

$$E^0_{nn}(R) = 2\hbar v_F n/3R$$

Ratio Problem:

$$E_{22}/E_{11} < 2$$

Blue Shift Problem:

$$E_{nn}(R) > E^0_{nn}(R)$$

Worse for large R

Nonlinear scaling $E_{nn}(R)=E(q_n=n/3R)$ accounts for both effects.

Electron Interactions in large radius tubes

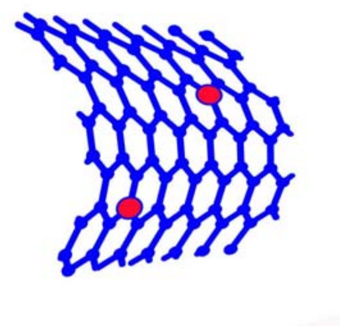
For $2\pi R \gg a$ electron interactions can be classified into three regimes, which lead to distinct physical effects.

- Long Range Interaction : ($r > 2\pi R$)

One Dimensional in character

→ Strongly bound excitons

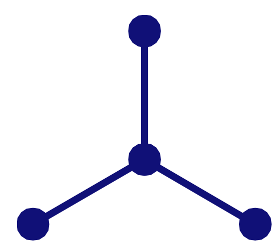
- Intermediate Range Interaction : ($a < r < 2\pi R$)



Two Dimensional in character

→ Nonlinear Scaling with n/R

- Short Range Interaction : ($r \sim a$)

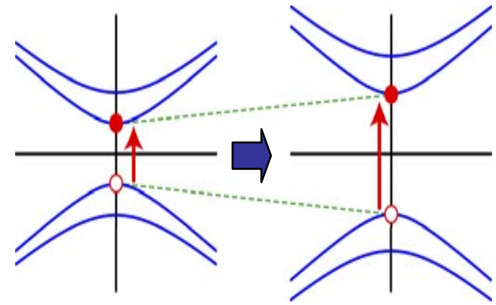


Atomic in character

→ Exciton "Fine Structure"

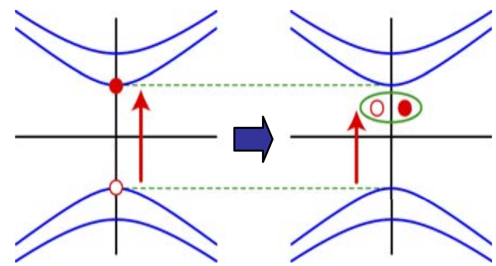
Long Range Interaction : (r > 2πR)

$$V(z) = \frac{e^2}{\varepsilon |z|}$$



- Renormalize Single Particle Gap

  Increase observed energy gap



- Leads to exciton binding

  Decrease observed energy gap

- Single Particle and Particle hole gaps both scale linearly with 1/R :

$$E_G^0 \sim \hbar \mathrm{v}_F / R$$

$$\hbar^2 / 2m^* \sim \hbar \mathrm{v}_F R$$

$$E(R) \sim (\hbar \mathrm{v}_F / R)\, f\,(e^2 / \varepsilon \hbar \mathrm{v}_F)$$

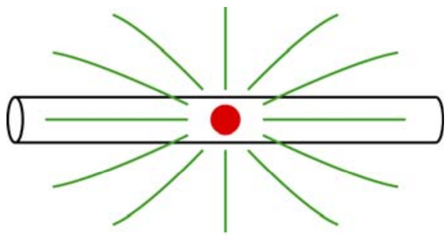
- Gap renormalization and exciton binding largely cancel each other.

Cancellation of gap renormalization and exciton binding:

- Single Particle excitation:

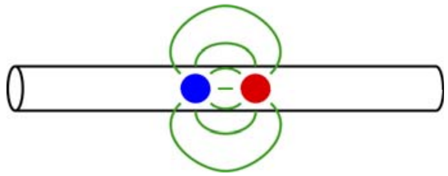
  Self energy ~ $e^2/\varepsilon R$

  Depends on dielectric environment



- Particle-hole excitation:

  Bound exciton is unaffected by the long range part of the interaction.



The cancellation is exact for an infinite range interaction

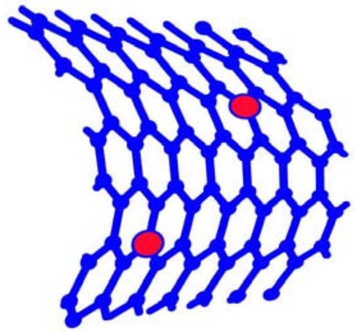
- Coulomb Blockade Model :

  Bare gap: 2Δ Interaction energy: $U N^2/2$

- Single particle gap $2\Delta + U$
- Particle-hole gap 2Δ

Intermediate Range Interaction: ($a < r < 2\pi R$)

- Leads to nonlinear $q \log q$ dispersion of graphene.
- Responsible for nonlinear scaling of $E_{11}(n/R)$.



Short Range Interaction: ($r \sim a$)

- Leads to "fine structure" in the exciton spectrum: S=0,1, etc.
- Splittings $\sim e^2 a / R^2$

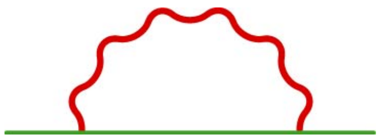
Interactions in 2D Graphene

Gonzalez, Guinea, Vozmediano, PRB 99

$$H = \hbar \mathrm{v}_{\mathrm{F}} \int d^2 r \psi^\dagger \frac{\sigma \cdot \nabla}{i} \psi + e^2 \int d^2 r d^2 r' \frac{n(r) n(r')}{2|r-r'|}$$

- Renormalized Quasiparticle Dispersion:

$$E(q) = \hbar \mathrm{v}_F q \left(1 + \frac{g}{4} \log \frac{\Lambda}{q} \right) \qquad g = e^2 / \hbar \mathrm{v}_{\mathrm{F}}$$



Singularity due to long range Coulomb interaction $V(q) = 2\pi e^2/q$.

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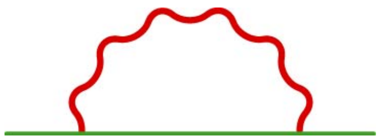

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- "Dielectric Screening" in 2 Dimensions

$g_{screened} = g/\varepsilon \qquad \Pi_{static}(q) = q/4v_F \qquad \varepsilon_{static} = 1+g\pi/2$

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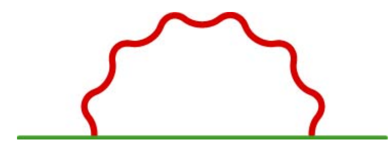

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- “Dielectric Screening” in 2 Dimensions

$g_{\mathrm{screened}} = g/\varepsilon \qquad \Pi_{\mathrm{static}}(q) = q/4\mathrm{v}_{\mathrm{F}} \qquad \varepsilon_{\mathrm{static}} = 1 + g\pi/2$

- Scaling Theory $\quad g = g(\Lambda)\ ; \quad \mathrm{v}_F = \mathrm{v}_F(\Lambda)$

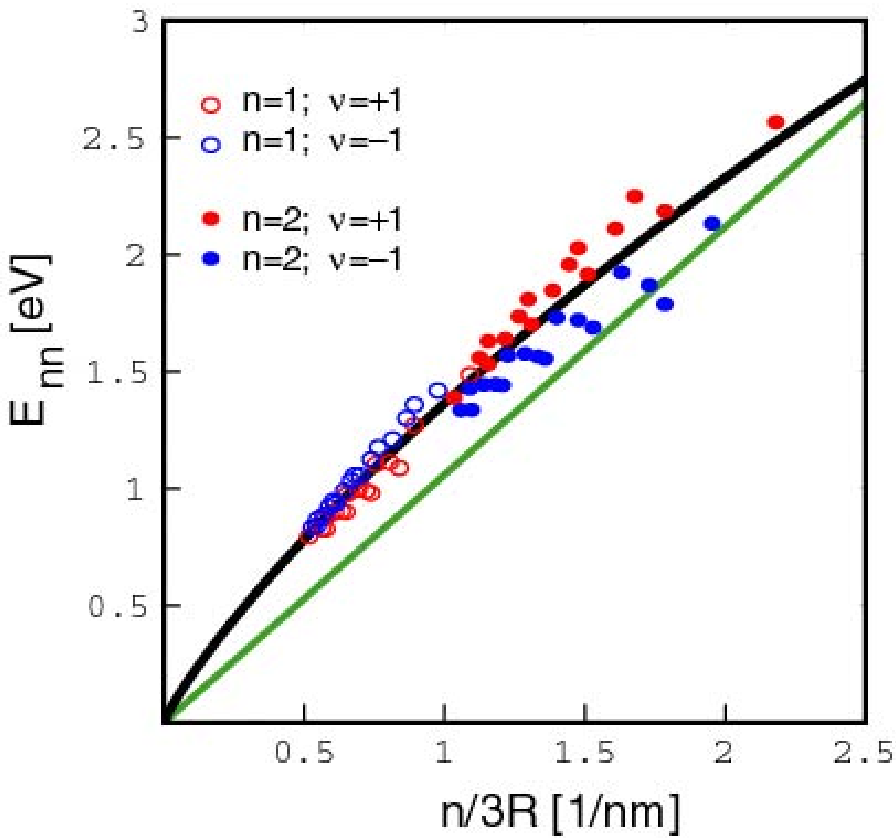
$$\frac{dg}{d \ln \Lambda} = -\frac{1}{4} g^2$$

Marginally Irrelevant

Marginal Fermi Liquid

- q ln q correction is exact for q⌘0

Compare 2D Theory with Experiment



Free electron Theory

2D Interacting Theory

$$E_{nn} = E(q_n = n/3R)$$

$$E(q) = 2\hbar v_F q\left(1 + \frac{g}{4}\log\frac{\Lambda}{q}\right)$$

$$\hbar v_F = .47 \text{ eV nm}$$

$$g = \frac{e^2}{\varepsilon \hbar v_F} = 1.2 \quad \Rightarrow \quad \varepsilon \sim 2.5$$

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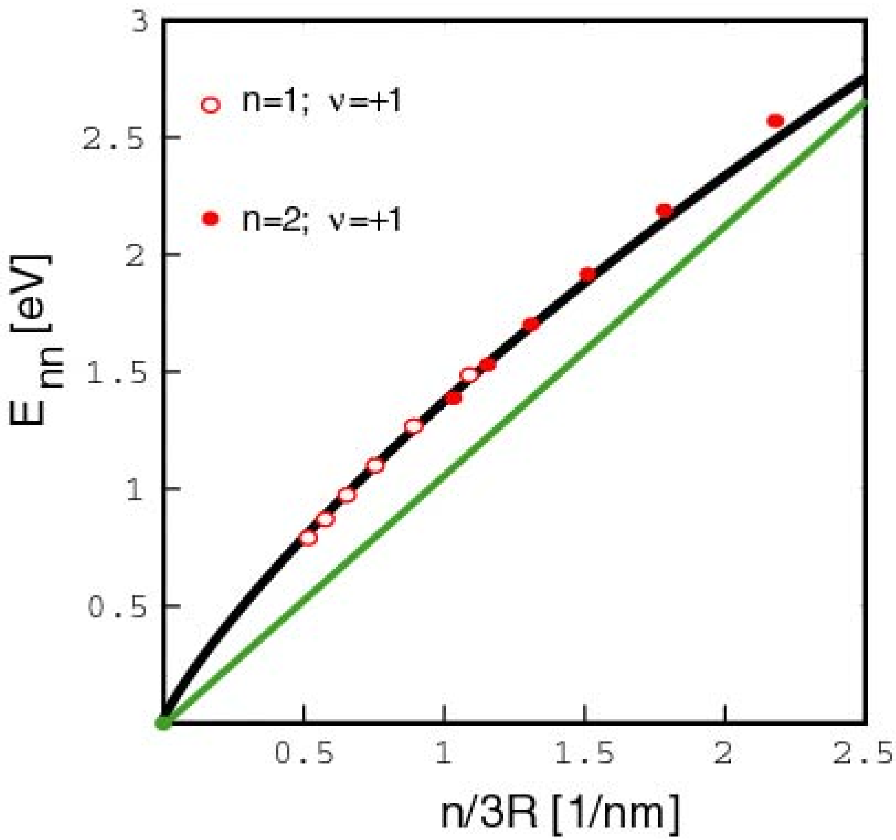

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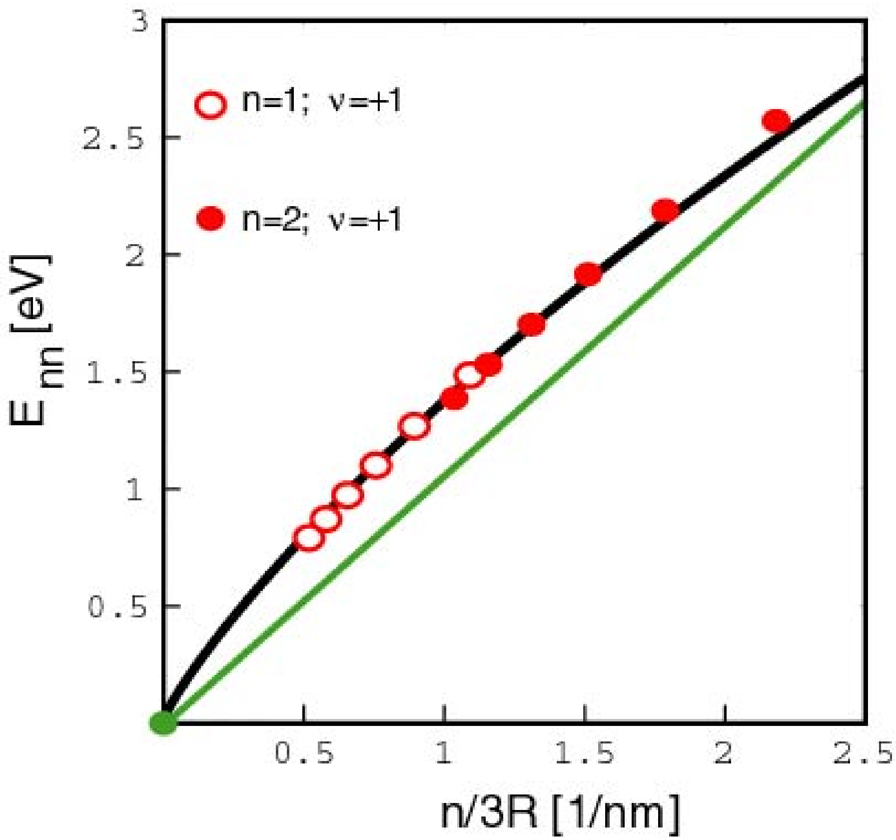

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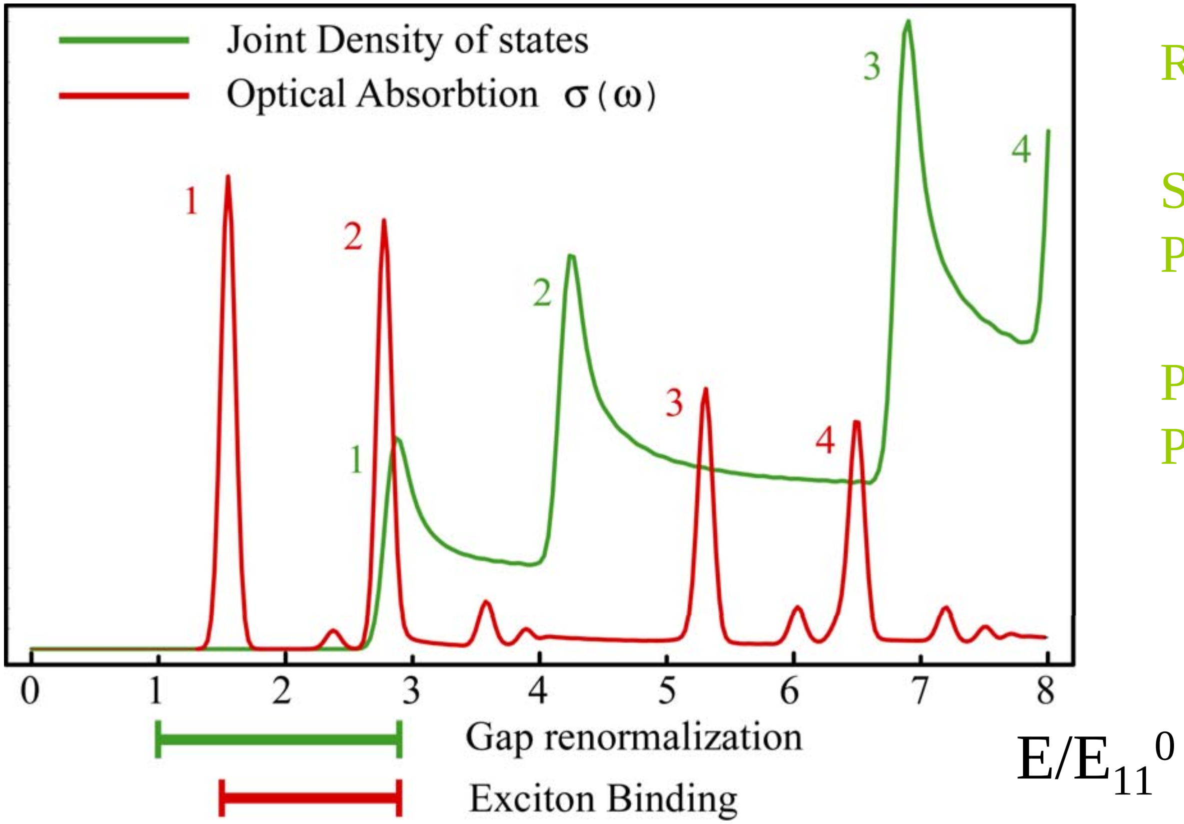
$$g = \frac{e^2}{\varepsilon \hbar v_F} = 1.2 \qquad \Rightarrow \qquad \varepsilon \sim 2.5$$

The optical spectra reflects the finite size scaling
of the 2D Marginal Fermi Liquid

Exciton effects: Compute particle-hole binding due to statically screened interaction (similar to Ando '97).



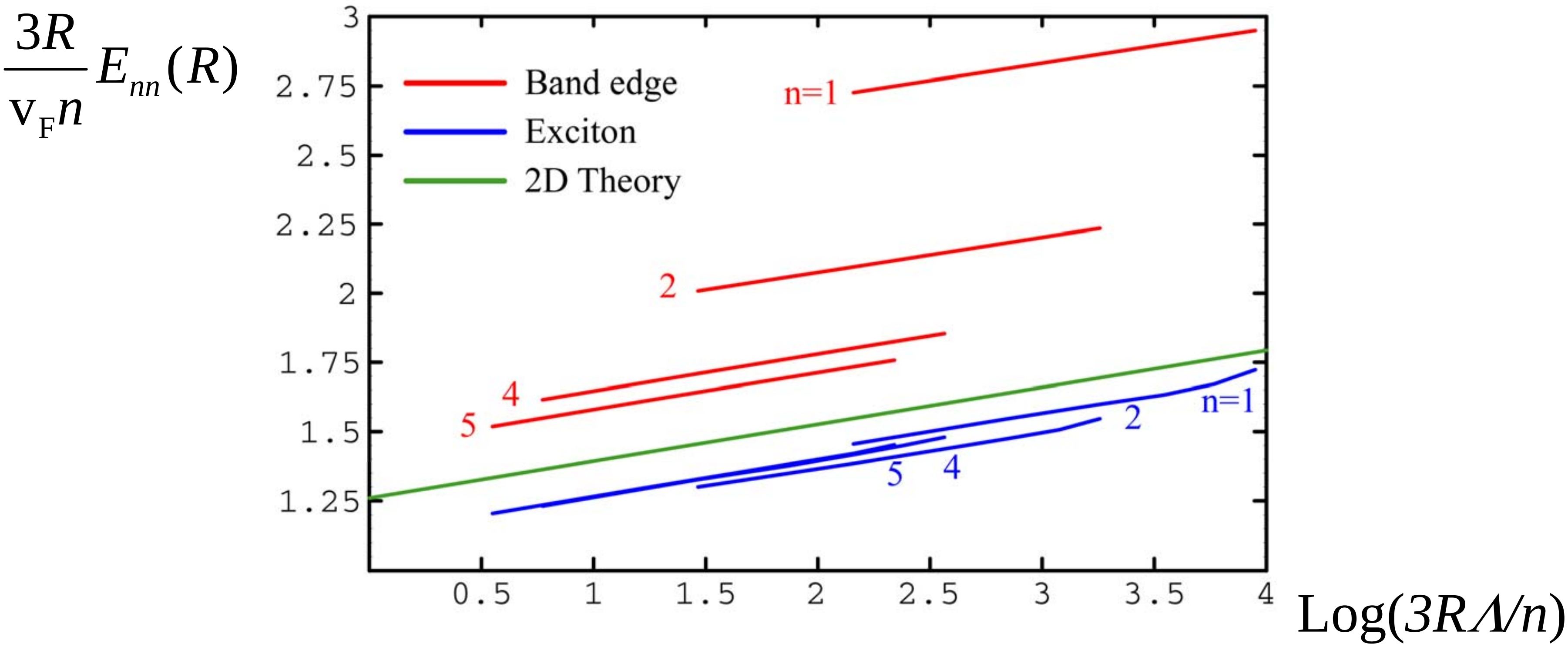
Related Work:

Spaturu et al (Berkeley) PRL 03

Perebeinos et al (IBM) PRL 04

- Lowest exciton dominates oscillator strength for each subband.
- Lineshape for absorption is not that of van Hove singularity.
- Large bandgap renormalization mostly cancelled by exciton binding.

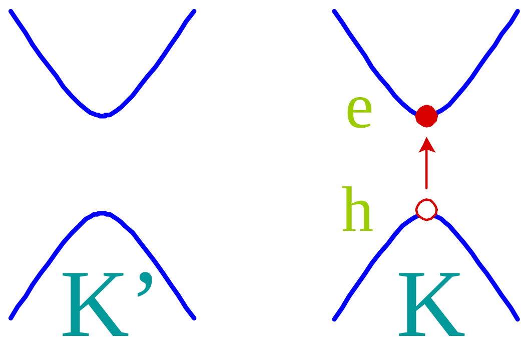
Scaling behavior: $E_n(R) = E(q_n \stackrel{?}{=} n/3R)$



$$E_{exciton}^{nn}(R) = \frac{\mathrm{v_F} n}{3R}\left[c_n + \frac{1}{4}\frac{e^2}{\varepsilon\hbar \mathrm{v_F}}\log\frac{3R\Lambda}{n}\right]$$

c_n ~ independent of n

Exciton Fine Structure

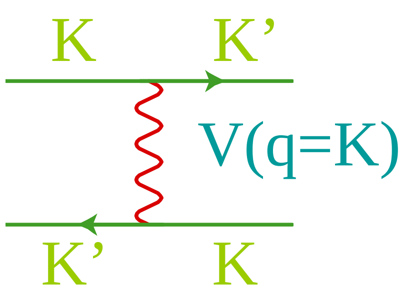


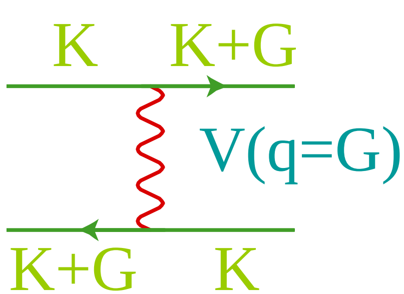
Degenerate exciton states:

$$\left.\begin{array}{l} \text{e}:\; k = K \text{ or } K' \;;\quad s = \uparrow \text{ or } \downarrow \\ \text{h}:\; k = K \text{ or } K' \;;\quad s = \uparrow \text{ or } \downarrow \end{array}\right\}$$

16 states

Degeneracy lifted by short range (q~1/a) interactions:





Effective 2D Contact Interaction:

$$H_C = \int d^2 r U^{\alpha\beta\gamma\delta}_{abcd} \psi^\dagger_{a\alpha}(\vec{r})\psi_{b\beta}(\vec{r})\psi^\dagger_{c\gamma}(\vec{r})\psi_{d\delta}(\vec{r})$$

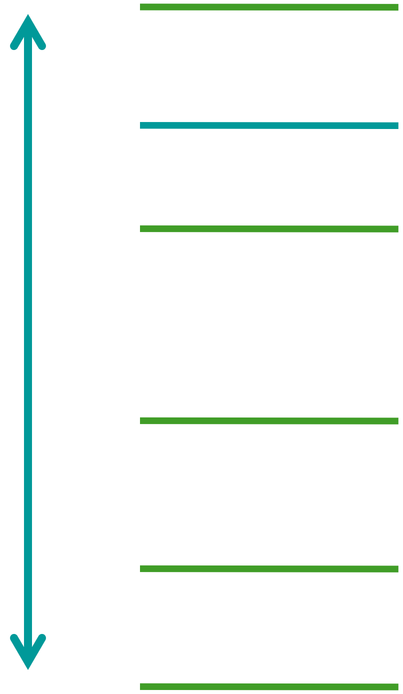
$$U \sim e^2 a$$

$$\langle H_C \rangle \sim \frac{e^2 a}{4\pi R \xi}$$

ξ ~ exciton size ~ $2\pi R$

Exciton Eigenstates:

Classify by momentum, spin, parity under C_2 rotation



~30 meV
(R~.5nm)

$q = \pm K \;\; ; \;\; S = 0 \; ;$

$q = 0 \;\; ; \;\; S = 0$; odd — Optically Allowed

$q = 0 \;\; ; \;\; S = 0$; even

$q = 0 \;\; ; \;\; S = 1$; odd

$q = \pm K \;\; ; \;\; S = 1 \; ;$

$q = 0 \;\; ; \;\; S = 1$; even

"Dark States"

See also Zhao, Mazumdar PRL 04

Conclusion

Fluorescence spectroscopy data for nearly armchair tubes
is well described by a systematic large radius theory.

- 2D interactions:

    - q log q renormalization of graphene dispersion.
    - Non linear scaling with 1/R.
    - Explains ratio problem and blue shift problem.

- 1D interactions

    - Lead to large gap enhancement AND large exciton binding
    - Largely cancels in optical experiments revealing 2D effects.

- Short Range interactions

    -Lead to fine structure in exciton levels
    -Dark Ground State

Experiments: measure single particle energy gap

- Tunneling (complicated by screening)
- Photoconductivity
- Activated transport