Elasticity and Dynamics of LC Elastomers

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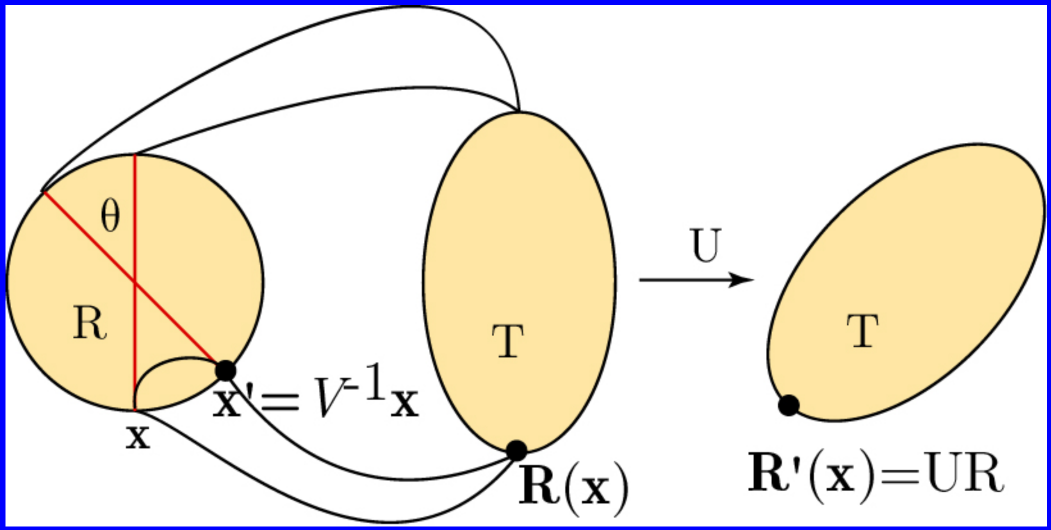
Outline

- Review of Elasticity of Nematic Elastomers
  - Soft and Semi-Soft Strain-only theories
  - Coupling to the director
- Phenomenological Dynamics
  - Hydrodynamic
  - Non-hydrodynamic
- Phenomenological Dynamics of NE
  - Soft hydrodynamic
  - Semi-soft with non-hydro modes

Strain



Displacements

$$\mathbf{R}(\mathbf{x}) = \mathbf{x} + \mathbf{u}(\mathbf{x})$$

Cauchy Deformation Tensor
(A "tangent plane" vector)

$$\Lambda_{i\alpha} = \frac{\partial R_i}{\partial x_\alpha} = \delta_{i\alpha} + \eta_{i\alpha}$$

α,β = Ref. Space
i,j = Target space

Displacement strain

$$\eta_{i\alpha} = \partial_\alpha u_i$$

Invariances

$$\mathbf{R} \rightarrow \underline{U}\mathbf{R};\ \mathbf{x} \rightarrow \underline{V}^{-1}\mathbf{x}$$

TCL, Mukhopadhyay, Radzihovsky, Xing, *Phys. Rev. E* **66**, 011702/1-22(2002)

Isotropic and Uniaxial Solid

Isotropic: free energy density f has two harmonic elastic constants

$$\tilde{u}_{\alpha\beta} = u_{\alpha\beta} - \tfrac{1}{3}\delta_{\alpha\beta}u_{\gamma\gamma}$$

$$f = f(\underline{\Lambda}) = f(\underline{U}\Lambda\underline{V}^{-1})$$

$$f = f(\underline{u}) = f(\underline{V}\,\underline{u}\,\underline{V}^{-1})$$

$$= \tfrac{1}{2}Bu_{\alpha\alpha}^2 + \mu \mathrm{Tr}\underline{\tilde{u}}^2 - C\mathrm{Tr}\underline{\tilde{u}}^3 + D\left(\mathrm{Tr}\underline{\tilde{u}}^2\right)^2$$

Invariant under

$$\mathbf{R}(\mathbf{x}) \to \underline{\mathrm{U}}\mathbf{R}(\underline{\mathrm{V}}\mathbf{x})$$

Uniaxial: five harmonic elastic constants

$$f = \tfrac{1}{2}C_1 u_{zz}^2 + C_2 u_{zz}u_{\nu\nu} + \tfrac{1}{2}C_3 u_{\nu\nu}^2 + C_4 u_{\nu\tau}^2 + C_5 u_{\nu z}^2;$$

$$\mathbf{x}_\alpha = (\mathbf{x}_\nu, x_z)$$

Invariant under

$$\mathbf{R}(\mathbf{x}) \to \underline{\mathrm{U}}\mathbf{R}(\underline{\mathrm{V}_{\mathrm{uni}}}\mathbf{x})$$

Nematic elastomer: uniaxial. Is this enough?

Nonlinear strain

Green – Saint Venant strain tensor- **Physicists' favorite – invariant under** ***U;***

$$dR^2 - dx^2 = 2u_{\alpha\beta}dx_\alpha dx_\beta$$

$$\boxed{\begin{aligned} \underline{u} &= \tfrac{1}{2}(\underline{\Lambda}^T\underline{\Lambda} - \underline{\delta}) \approx \tfrac{1}{2}(\underline{\eta} + \underline{\eta}^T) \\ u_{\alpha\beta} &= \tfrac{1}{2}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha + \partial_\alpha u_k \partial_\beta u_k\right) \end{aligned}}$$

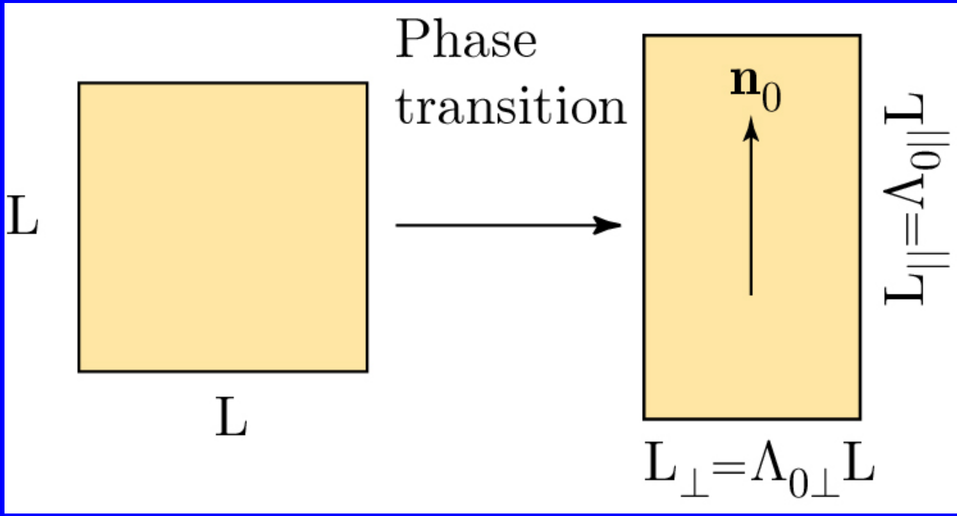
$$\mathbf{R} \rightarrow \underline{U}\mathbf{R};\ \mathbf{x} \rightarrow \underline{V}^{-1}\mathbf{x}$$

$$\underline{\Lambda} \rightarrow \underline{U}\,\underline{\Lambda}\,\underline{V}^{-1};\ \underline{u} \rightarrow \underline{V}\,\underline{u}\,\underline{V}^{-1}$$

$\underline{u}$ is a tensor is the reference space, and a scalar in the target space

Spontaneous Symmetry Breaking

Phase transition to anisotropic state as μ goes to zero



$$\underline{u}_0 = \tfrac{1}{2}\left(\underline{\Lambda}_0^T \underline{\Lambda}_0 - \underline{\delta}\right)$$

$$\underline{\Lambda}_0 = \sqrt{\underline{\delta} + 2\underline{u}_0}$$

Direction of $\mathbf{n}_0$ is arbitrary

$$u_{\alpha\alpha} \sim \Psi^2$$

$$\tilde{u}_{\alpha\beta} = \tilde{u}_{0\alpha\beta}$$
$$= \Psi\left(n_\alpha^0 n_\beta^0 - \tfrac{1}{3}\delta_{\alpha\beta}\right)$$

Symmetric-Traceless part

Golubovic, L., and Lubensky, T.C.,, *PRL* **63**, 1082-1085, (1989).

Strain of New Phase

$$R_i(\mathbf{x}) = \Lambda_{0ij} x_j + \delta u_i(\mathbf{x})$$
$$= x'_i + u'_i(\mathbf{x}')$$

$\underline{u}'$ is the strain relative to the new state at points x'

$$\Lambda_{ij} = \frac{\partial R_i}{\partial x_j} = \frac{\partial R_i}{\partial x'_k}\frac{\partial x'_k}{\partial x_j} = \Lambda'_{ik}\Lambda_{0kj}$$

$$\delta\underline{u} = \underline{u} - \underline{u}_0$$
$$= \tfrac{1}{2}\left(\underline{\Lambda}^T\underline{\Lambda} - \underline{\Lambda}_0^T\underline{\Lambda}_0\right)$$
$$= \underline{\Lambda}_0^T\,\underline{u}'\,\underline{\Lambda}_0$$

$\delta\underline{u}$ is the deviation of the strain relative to the original reference frame R from $\underline{u}_0$

$$\underline{u}' = \tfrac{1}{2}\left(\underline{\Lambda}'^T\underline{\Lambda}' - \underline{\delta}\right) \approx \tfrac{1}{2}\left(\underline{\eta}' + \underline{\eta}'^T\right)$$

$\delta\underline{u}$ is linearly proportional to $\underline{u}'$

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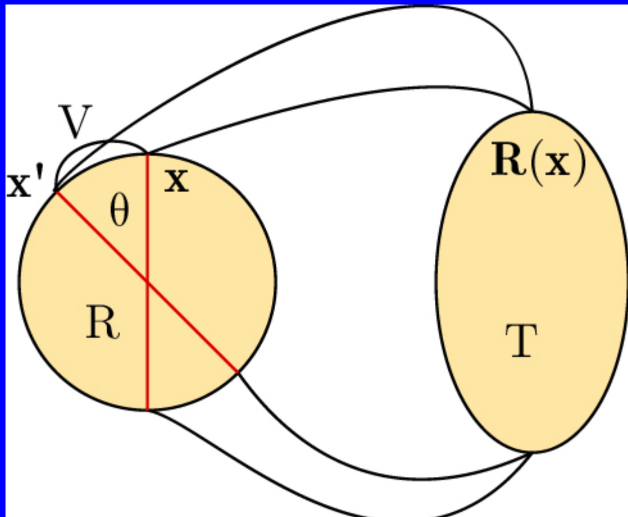
Elasticity of New Phase



Rotation of anisotropy direction costs no energy

$$r = \frac{\Lambda_{0||}^2}{\Lambda_{0\perp}^2}$$

$$\underline{u}' = (\underline{\Lambda}_0^T)^{-1}\left(\underline{V}\,\underline{u}_0\underline{V}^{-1} - \underline{u}_0\right)\underline{\Lambda}_0^{-1}$$

$$= \tfrac{1}{4}(r-1)\begin{pmatrix} 1-\cos 2\theta & \frac{1}{\sqrt{r}}\sin 2\theta \\ \frac{1}{\sqrt{r}}\sin 2\theta & -\frac{1}{r}(1-\cos 2\theta) \end{pmatrix}$$

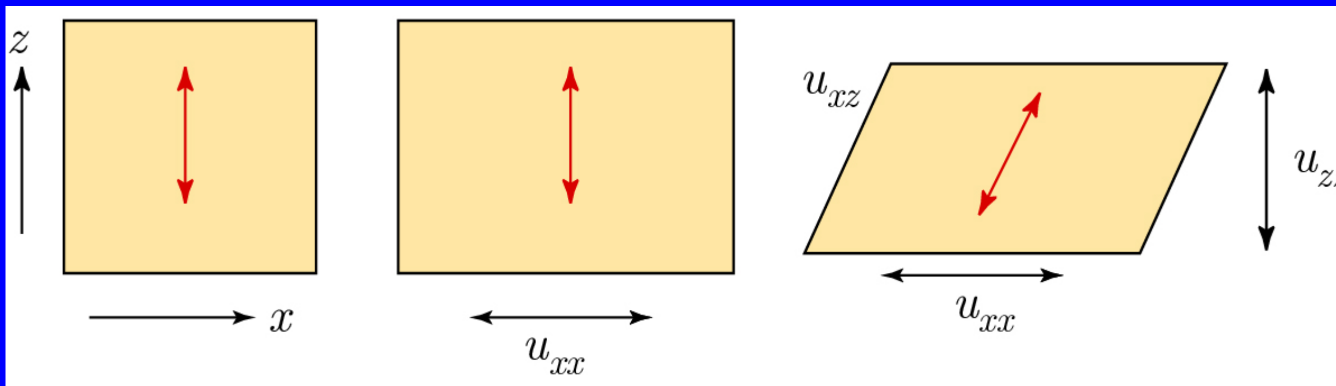
$$\boxed{u'_{xz} \sim \frac{(r-1)}{4\sqrt{r}}\theta}$$

C_5=0 because of rotational invariance

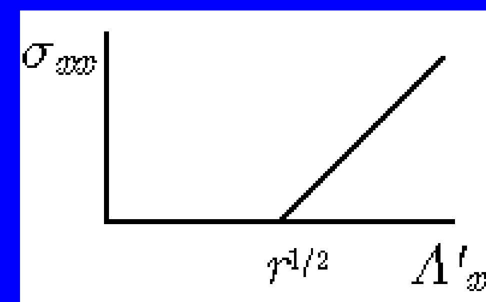
$$f_{\mathrm{el}} = \tfrac{1}{2}C_1 u_{zz}^{\prime 2} + C_2 u'_{zz}u'_{\nu\nu} + \tfrac{1}{2}C_3 u'_{\nu\nu}u'_{\nu\nu} + C_4 u'_{\nu\tau}u'_{\nu\tau} + \cancel{C_5 u'_{z\nu}u'_{z\nu}}$$

This 2nd order expansion is invariant under all U but only infinitesimal V

Soft Extensional Elasticity



$$\underline{u} = \frac{1}{4}(r-1)\begin{pmatrix} 1-\cos 2\theta & \frac{1}{\sqrt{r}}\sin 2\theta \\ \frac{1}{\sqrt{r}}\sin 2\theta & -\frac{1}{r}(1-\cos 2\theta) \end{pmatrix}$$



$$u_{zz} = -\frac{1}{r}u_{xx}$$

$$u_{xz} = \frac{1}{\sqrt{2r}}\sqrt{u_{xx}(r-1-2u_{xx})}$$

Strain u_{xx} can be converted to a zero energy rotation by developing strains u_{zz} and u_{xz} until u_{xx} =(r-1)/2

Frozen anisotropy: Semi-soft

System is now uniaxial – why not simply use uniaxial elastic energy? This predicts linear stress-stain curve and misses lowering of energy by reorientation:

$$f = \tfrac{1}{2}C_1 u_{zz}^2 + C_2 u_{zz}u_{\nu\nu} + \tfrac{1}{2}C_3 u_{\nu\nu}^2 + C_4 u_{\nu\tau}^2 + C_5 u_{\nu z}^2$$

Model Uniaxial system: Produces harmonic uniaxial energy for small strain but has nonlinear terms – reduces to isotropic when h=0

$$f^h(\underline{u}) = f(\underline{u}) - hu_{zz}$$

$f\,(u)$: isotropic

Rotation $$u \rightarrow u' = u + \theta \begin{pmatrix} -2u_{xz} & u_{xx} - u_{zz} \\ u_{xx} - u_{zz} & 2u_{xz} \end{pmatrix}$$

$$f^h(\underline{u}') = f(\underline{u}) - h(u_{zz} + 2\theta u_{xz})$$

Semi-soft stress-strain

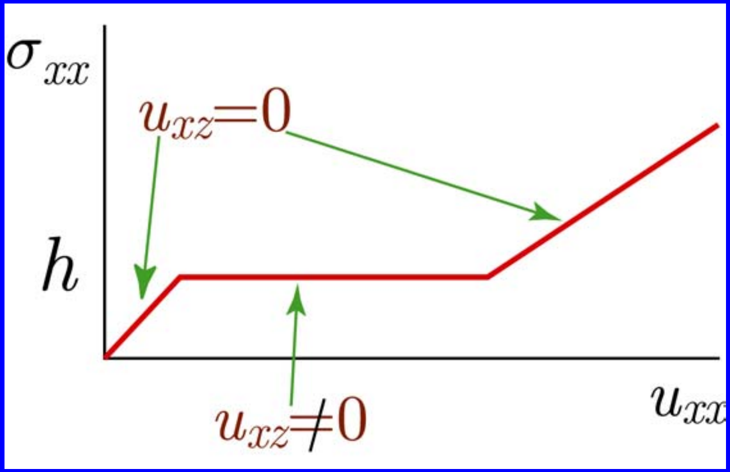
Ward Identity

$$\frac{df^h}{d\theta} = -2hu_{xz} = 2\sigma_{xz}(u_{xx} - u_{zz}) + 2(\sigma_{zz} - \sigma_{xx})u_{xz}$$

$$\sigma_{xz} = \frac{(\sigma_{xx} - h)u_{xz}}{u_{xx} - u_{zz}} \Rightarrow u_{xz} = 0 \text{ or } \sigma_{xx} = h$$

$$\sigma_{\alpha\beta} = \frac{\partial f^h}{\partial u_{\alpha\beta}}$$

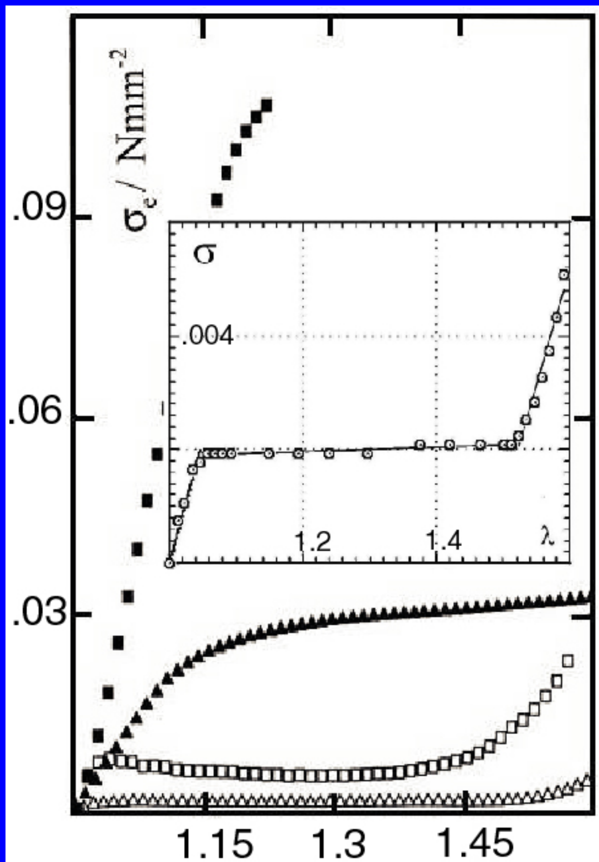
Second Piola-Kirchoff stress tensor; not the same as the familiar Cauchy stress tensor



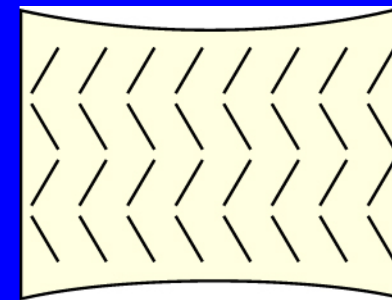
Ranjan Mukhopadhyay and TCL: in preparation

Semi-soft Extensions

Break rotational symmetry



Stripes form in real systems: semi-soft, BC



Not perfectly soft because of residual anisotropy arising from crosslinking in the the nematic phase - **semi-soft.** length of plateau depends on magnitude of spontaneous anisotropy r.

Warner-Terentjev

Note: Semi-softness only visible in nonlinear properties

Finkelmann, et al., J. Phys. II **7**, 1059 (1997); Warner, J. Mech. Phys. Solids **47**, 1355 (1999)

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Soft Biaxial SmA and SmC

Free energy density for a uniaxial solid (SmA with locked layers)

$$f = \tfrac{1}{2} C_1 u_{zz}^2 + C_2 u_{zz} u_{\nu\nu} + \tfrac{1}{2} C_3 u_{\nu\nu}^2 + C_4 \hat{u}_{\nu\tau}^2 + C_5 u_{\nu z}^2$$
$$+ g_1 u_{zz} \hat{u}_{\nu\tau}^2 + g_2 u_{\alpha\alpha} \hat{u}_{\nu\tau}^2 + g_3 (\hat{u}_{\nu\tau}^2)^2$$
$$+ v_1 u_{zz} u_{\nu z}^2 + v_2 u_{\alpha\alpha} u_{\nu z}^2 + v_3 \hat{u}_{\nu\tau} u_{\nu z} u_{\tau z}$$

$$\hat{u}_{\nu\tau} = u_{\nu\tau} - \tfrac{1}{2} \delta_{\nu\tau} u_{\sigma\sigma}$$

Red: Corrections for transition to biaxial SmA
Green: Corrections for trtansition to SmC

C_4=0: Transition to Biaxial Smectic with soft in-plane elasticity
C_5=0: Transition to SmC with a complicated soft elasticity

Olaf Stenull, TCL, PRL **94**, 081304 (2005)

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Coupling to Nematic Order

- **Strain** $u_{\alpha\beta}$ transforms like a **tensor** in the **ref. space** but as a **scalar** in the **target space**.
- The **director** n_i and the **nematic order parameter** Q_{ij} transform as **scalars** in the **ref. space** but , respectively, as a **vector** and a **tensor** in the **target space**.
- How can they be coupled? – Transform between spaces using the **Polar Decomposition Theorem**.

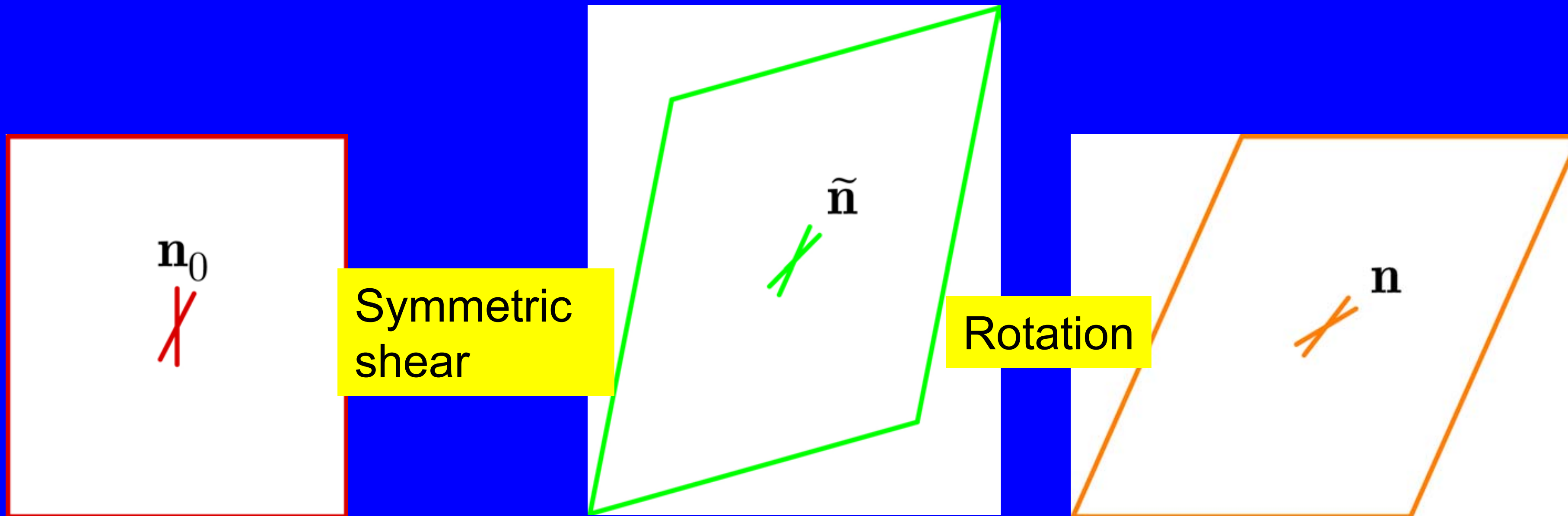
$$\underline{\Lambda} = \underline{\Lambda}(\underline{\Lambda}^{\mathrm{T}}\underline{\Lambda})^{-1/2}(\underline{\Lambda}^{\mathrm{T}}\underline{\Lambda})^{1/2} \equiv \underline{O}\,\underline{M}$$

$$\underline{O} = \underline{\Lambda}(\underline{\Lambda}^{\mathrm{T}}\underline{\Lambda})^{-1/2} = \text{ Rotation Matrix}$$

$$\underline{M} = (\underline{\Lambda}^{\mathrm{T}}\underline{\Lambda})^{1/2} = (\underline{1} + 2\underline{u})^{1/2} = \text{ Symmetric}$$

Ref->target $n_i = O_{i\alpha}\tilde{n}_\alpha ; \qquad \tilde{n}_\alpha = O^{\mathrm{T}}_{\alpha i} n_i$ Target->ref

Strain and Rotation



$\tilde{\mathrm{n}}$ is a reference space vector; it is equal to the target space vector that is obtained when $\underline{\Lambda}$ is symmetric

$$O_{i\alpha} \approx \delta_{i\alpha} + \tfrac{1}{2}(\partial_\alpha u_i - \partial_i u_\alpha)$$
$$\approx \delta_{i\alpha} - \varepsilon_{i\alpha k}\Omega_k$$

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Softness with Director

$\boldsymbol{N}_\alpha =$ unit vector along uniaxial direction in reference space; layer normal in a locked SmA phase

$$\tilde{\mathbf{n}}_\alpha = (\tilde{n}_\nu, \tilde{n}_z)$$

$$\tilde{n}_\nu^2 = 1 - (N_\alpha \cdot \tilde{n}_\alpha)^2 \equiv c_\nu^2; \quad u_{zz} = N_\alpha u_{\alpha\beta} N_\beta, \text{ etc.}$$

Red: SmA-SmC transition

$$\begin{aligned} f &= \tfrac{1}{2} C_1 u_{zz}^2 + C_2 u_{zz} u_{\nu\nu} + \tfrac{1}{2} C_3 u_{\nu\nu}^2 + C_4 u_{\nu\tau}^2 + \lambda_1 \tilde{n}_\nu^2 u_{zz} \\ &\quad + C_5 u_{\nu z}^2 + D_2 \tilde{n}_\nu \tilde{n}_z u_{\nu z} + \tfrac{1}{2} D_1 \tilde{n}_\nu^2 + \tfrac{1}{4} g \tilde{n}_\nu^4 + \lambda_2 \tilde{n}_\nu^2 u_{\tau\tau} + \cdots \\ &= \tfrac{1}{2} C_1 u_{zz}^2 + C_2 u_{zz} u_{\nu\nu} + \tfrac{1}{2} C_3 u_{\nu\nu}^2 + C_4 u_{\nu\tau}^2 \\ &\quad + \tfrac{1}{2} D_1 [\tilde{n}_\nu + (D_2 / D_1) u_{\nu z}]^2 + [C_5 - \tfrac{1}{2}(D_2^2 / D_1)] u_{\nu z}^2 \end{aligned}$$

Director relaxes to zero

$$C_5^R = C_5 - \frac{1}{2}\frac{D_2^2}{D_1} = 0 \Rightarrow \text{Soft}$$

Harmonic Free energy with Frank part

$$F = F_u + F_n + F_{u-n}$$

$$F_u = \int d^3x[\tfrac{1}{2} C_1 u_{zz}^2 + C_2 u_{zz} u_{\nu\nu} + \tfrac{1}{2} C_3 u_{\alpha\alpha}^2 + C_4 u_{\nu\tau}^2 + C_5 u_{\nu z}^2]$$

$$F_n = \int d^3x[\tfrac{1}{2} K_1 (\partial_\nu n_\nu)^2 + \tfrac{1}{2} K_2 (\varepsilon_{\nu\tau} \partial_\nu n_\tau)^2 + \tfrac{1}{2} K_3 (\partial_z n_\nu)^2]$$

$$F_{u-n} = \int d^3x[\tfrac{1}{2} D_1 \tilde{n}_\nu^2 + D_2 \tilde{n}_\nu u_{\nu z}]$$

$$\tilde{n}_\nu = n_\nu - \tfrac{1}{2}(\partial_z u_\nu - \partial_\nu u_z)$$

NE: Relaxed elastic energy

Uniaxial solid when $C_5^R > 0$,
including Frank director energy

$$F_u^{\text{eff}} = \int d^3x [\tfrac{1}{2} C_1 u_{zz}^2 + C_2 u_{zz} u_{aa} + \tfrac{1}{2} C_3 u_{ii}^2 + C_4 u_{ab}^2 + C_5^R u_{az}^2 + \tfrac{1}{2} K_1^R (\partial_a^2 u_z)^2 + \tfrac{1}{2} K_3^R (\partial_z^2 u_a)^2]$$

$$C_5^R = C_5 - \frac{D_2^2}{2D_1}; \quad \text{Soft}: \; C_5^R = 0; \quad \text{Semi-Soft}: \; C_5^R \neq 0$$

$$K_1^R = \frac{1}{4}\left(1 + \frac{D_2}{D_1}\right)^2 K_1; \quad K_3^R = \frac{1}{4}\left(1 - \frac{D_2}{D_1}\right)^2 K_3$$

Slow Dynamics – General Approach

- Identify slow variables: φ Determine static thermodynamics: $F(\varphi)$
- Develop dynamics: Poisson-brackets plus dissipation
- Mode Counting (Martin,Pershan, Parodi 72):
  - One hydrodynamic mode for each conserved or broken-symmetry variable
  - Extra Modes for slow non-hydrodynamic
  - Friction and constraints may reduce number of hydrodynamics variables

Preliminaries

Harmonic Oscillator: seeds of complete formalism

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2; \qquad \{p,x\} = 1$$

$$\dot{p} = -\{p,x\}\frac{\partial H}{\partial x} - \Gamma\dot{x} = -kx - \Gamma v$$

$$\dot{x} = -\{x,p\}\frac{\partial H}{\partial p} = \frac{p}{m}$$

friction

$$p = mv$$

:

mechanical coupling between variables – time-reversal invariant.

:

not time-reversal invariant

Dissipative: time derivative of field (p) to its conjugate field (v)

Fluid Flow – Navier Stokes

Conserved densities:

$$F = \frac{1}{2}\int d^d x (g^2/\rho) + \int d^d x f[\rho]$$

mass: $\boldsymbol{\rho}$ Energy: ε

Momentum: $g_i = \boldsymbol{\rho} v_i$

$$\partial_t \rho + \partial_i g_i = 0$$

$$\partial_t g_i = \partial_i \sigma_{ij} = -\partial_i p + \eta \nabla^2 v_i$$

$$\partial_t \varepsilon + \partial_i j_i^\varepsilon = 0$$

$$\omega = \pm c q - \frac{2\eta}{3\rho} i q^2 \qquad (2 \text{ modes})$$

$$\omega = -i \frac{\eta}{\rho} q^2 \qquad (2 \text{ modes})$$

$$\omega = -i \frac{\kappa}{C_p} q^2 \qquad (1 \text{ mode})$$

Crystalline Solid I

Mass density is periodic

$$\rho \to \rho + \sum_{\mathbf{G}} \rho_{\mathbf{G}} e^{i\mathbf{G}\cdot(\mathbf{x}-\mathbf{u})}$$

Conserved densities:

mass: $\boldsymbol{\rho}$ Energy: ε

Momentum: $g_i = \boldsymbol{\rho} v_i$

Broken-symmetry field:

Phase of mass-density field:

describes displacement of periodic part of density

Strain $$u_{ij} = (\partial_i u_j + \partial_j u_i)/2$$

Aside: Nonlinear strain is not the Green Saint-Venant tensor

Free energy

$$F = \int d^d x \left[\tfrac{1}{2}(g^2/\rho) + f[\rho] - \lambda \delta\rho u_{ii} + \tfrac{1}{2} K_{ijkl} u_{ij} u_{kl} \right]$$

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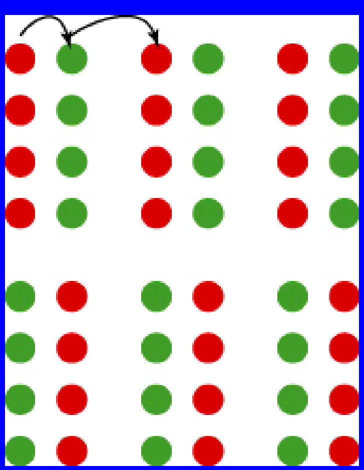
Crystalline Solid II

$$\partial_t \rho + \partial_i g_i = 0$$

$$\partial_t u_i = v_i - \frac{1}{\gamma}\frac{\delta F}{\delta u_i} \quad \text{permeation}$$

$$\partial_t g_i = -\partial_i p - \frac{\delta F}{\delta u_i} + \eta \nabla^2 v_i$$

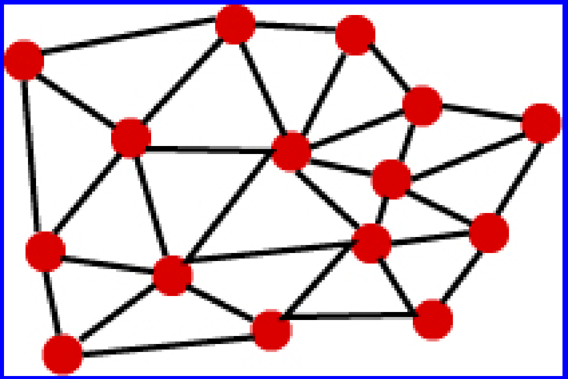


Modes:
Transverse phonon: 4
Long. Phonon: 2
Permeation (vacancy diffusion): 1
Thermal Diffusion: 1

Aside: full nonlinear theory requires more care

Permeation: independent motion of mass-density wave and mass: mass motion with static density wave

Tethered Solid



No permeation : $\partial_t \mathbf{u} = \mathbf{v}$

Density locked : $\delta\rho \,/\, \rho = -\nabla \cdot \mathbf{u}$

7 hydrodynamic variables: 1 density,3 momenta, 3 displacements, 1 energy + 1 constraint = 8-1=7

Classic equations of motion for a Lagrangian solid; use Cauchy-Saint-Venant Strain

Isotropic elastic free energy

$$F = \tfrac{1}{2}\int d^d x \left(\lambda u_{ii}^2 + 2\mu u_{ij}^2\right)$$

$$\rho \partial_t^2 u_i = -\frac{\delta F}{\delta u_i} + \eta \nabla^2 v_i$$

$$\omega_T = \pm\sqrt{\frac{\mu}{\rho}}q + i\frac{\eta}{2\rho}q^2 \qquad (4)$$

$$\omega_L = \pm\sqrt{\frac{\lambda+2\mu}{\rho}}q + i\frac{2\eta}{3\rho}q^2 \qquad (2)$$

+ energy mode (1)

Gel: Tethered Solid in a Fluid

$$\rho_s \partial_t^2 u_i = -\frac{\delta F}{\delta u_i} + \eta_s \nabla^2 \dot{u}_i - \Gamma(\dot{u}_i - v_i)$$

Friction only for relative motion- Galilean invariance

Frictional Coupling

$$\partial_t g_i = -\partial_i p + \eta \nabla^2 v_i - \Gamma(v_i - \dot{u}_i)$$

Total momentum conserved

$$\partial_t (g_i + \rho_s \dot{u}_i) = \partial_j \sigma_{ij}^T$$

Fast non-hydro mode: but not valid if there are time scales in Γ

$$\omega_F = -i\tau^{-1} = -i(\rho^{-1} + \rho_s^{-1})\Gamma$$

$\omega\tau \ll 1:$ Effective Tethered Hydro.

$$(\rho + \rho_s)\ddot{u}_i = -\frac{\delta F}{\delta u_i} + (\eta + \eta_s)\nabla^2 \dot{u}_i$$

Fluid and Solid move together

Nematic Hydrodynamics: Harvard I

$$F = \tfrac{1}{2}\int d^d x (g^2 / \rho) + \int d^d x f[\mathbf{n}, \rho]$$

g is the total momentum density:
determines angular momentum $\boldsymbol{\ell} = \mathbf{x} \times \boldsymbol{g}$

$$f[\mathbf{n},\rho] = \frac{1}{2}K_1(\rho)(\nabla\cdot\mathbf{n})^2 + \frac{1}{2}K_2(\rho)[\mathbf{n}\cdot(\nabla\times\mathbf{n})]^2 + \frac{1}{2}K_3(\rho)[\mathbf{n}\times(\nabla\times\mathbf{n})]^2$$

Frank free energy for a nematic

Nematic Hydrodynamics: Harvard II

$$\partial_t n_i = \lambda_{ijk}\partial_k v_j - \frac{1}{\gamma}\frac{\delta F}{\delta n_i}$$ permeation

$$\partial_t g_i = -\partial_i p + \partial_j\left(\lambda_{kij}\frac{\delta F}{\delta n_k}\right) + \partial_j \sigma'_{ij}$$

$$A_{ij} = \tfrac{1}{2}\left(\partial_i v_j + \partial_j v_i\right)$$

$$\omega_i = \tfrac{1}{2}\varepsilon_{ijk}\partial_j v_k$$

$$\sigma'_{ij} = \eta_{ijkl}A_{kl};$$

$$\lambda_{ijk} = \tfrac{1}{2}\left(\delta^T_{ij}n_k - \delta^T_{ik}n_j\right) + \tfrac{1}{2}\lambda\left(\delta^T_{ij}n_k + \delta^T_{ik}n_j\right)$$

ω – fluid vorticity not spin frequency of rods

$$\partial_t \mathbf{n} = \omega \times \mathbf{n} + \lambda \mathbf{n} : \mathbf{A} + \frac{1}{\gamma}\mathbf{h}$$

Symmetric strain rate rotates $\mathbf{n}$

Modes: 2 long sound, 2 “slow” director diffusion. 2 “fast” velocity diff.

Stress tensor can be made symmetric

NE: Director-displacement dynamics

Stenull, TCL, PRE 65, 058091 (2004)

Tethered anisotropic solid plus nematic

Note: all variables in target space

$$\partial_t n_i = \lambda_{ijk} \partial_k v_j - \frac{1}{\gamma} \frac{\delta F}{\delta n_i}$$

$$\dot{u}_i = \frac{\delta F}{\delta g_i} = \frac{1}{\rho} g_i$$

$$\partial_t g_i = \partial_j \left(\lambda_{kij} \frac{\delta F}{\delta n_k} \right) - \frac{\delta F}{\delta u_i} + \partial_j \sigma'_{ij}$$

Director relaxes in a microscopic time to the local shear – nonhydrodynamic mode

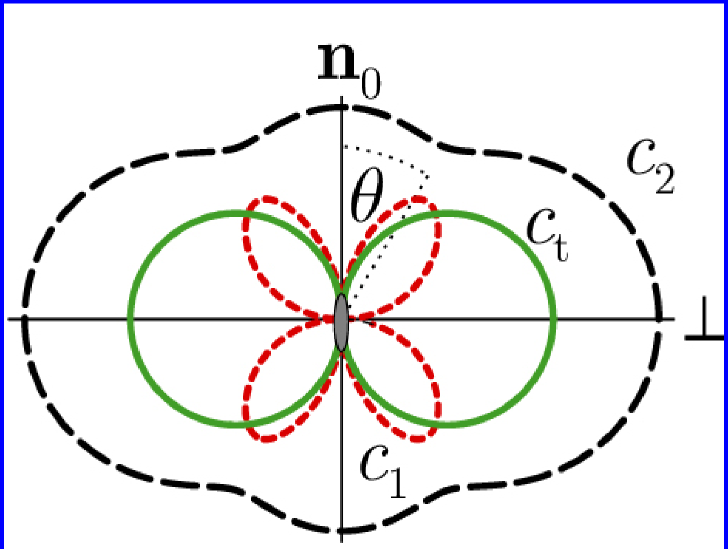
$$\omega_f = -i \frac{D_1}{\gamma} = -\frac{i}{\tau_1}$$

Modifications if γ depends on frequency

Semi-soft: Hydrodynamic modes same as a uniaxial solid: 3 pairs of sound modes

Soft Elastomer Hydrodynamics

$$\rho \ddot{u}_i = -\frac{\delta F_u^{\text{eff}}}{\delta u_i} + \eta_{ijkl} \partial_j \partial_l v_k$$



Same mode structure as a discotic liquid crystal: 2 "longitudinal" sound, 2 columnar modes with zero velocity along $\mathbf{n}$, 2 smectic modes with zero velocity along both symmetry directions

Slow and fast diffusive modes along symmetry directions

$$\omega_s = -i \frac{2K}{\eta_5} q^2$$

$$\omega_f = -i \frac{2\eta_5}{\rho} q^2$$

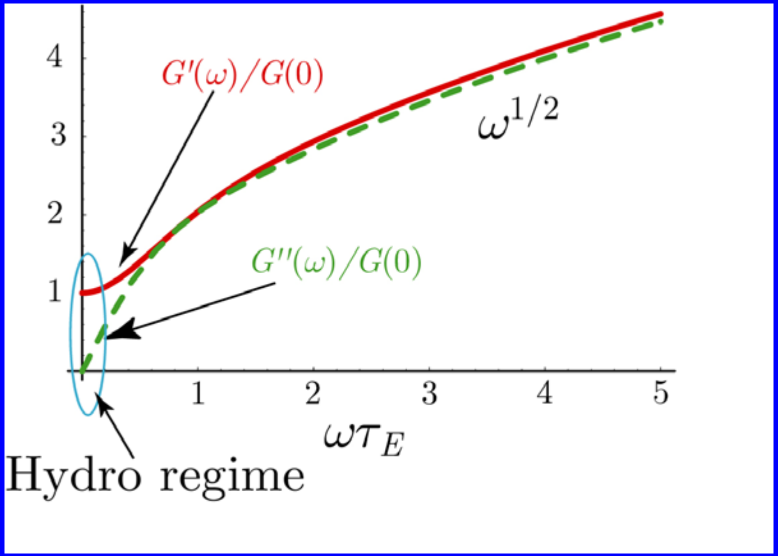
Beyond Hydrodynamics: 'Rouse' Modes

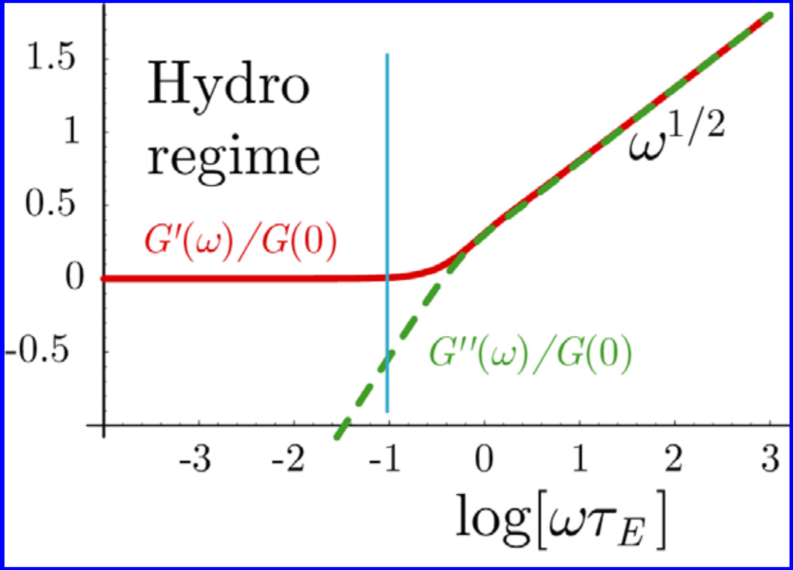
$$G(\omega) = \mu - i\omega\eta(\omega\tau_E)$$

$$\eta(\omega) = \eta f(-i\omega\tau_E)$$

$$f(x) = \frac{\sum_1^N (p^2 + x)^{-1}}{\sum_1^N p^{-2}} \rightarrow \begin{cases} 1, & x \rightarrow 0 \\ 3/(2\pi\sqrt{x}), & x \gg \infty \end{cases}$$

Standard hydrodynamics for $\omega\tau_E \ll 1$; nonanalytic $\omega\tau_E \gg 1$





Rouse in NEs

References: Martinoty, Pleiner, et al.; Stenull & TL; Warner & Terentjev, EPJ **14**, (2005)

$$G(\omega) = C_5 - \frac{D_2^2}{2D_1} - i\omega[\nu_5(\omega) + (\lambda^2/2)\gamma(\omega)] - \frac{D_2^2}{2D_1}\left\{\frac{[1-i\omega\tau_2(\omega)]^2}{1-i\omega\tau_n(\omega)} - 1\right\}$$

$$\tau_n(\omega) = \gamma(\omega)/D_1;$$
$$\tau_2(\omega) = -\lambda(D_1/D_2)\tau_n(\omega)$$
$$\tau_n(\omega) = \tau_n f(-i\omega\tau_E);$$
$$\nu_5(\omega) = \nu_5 f(-i\omega\tau_E)$$

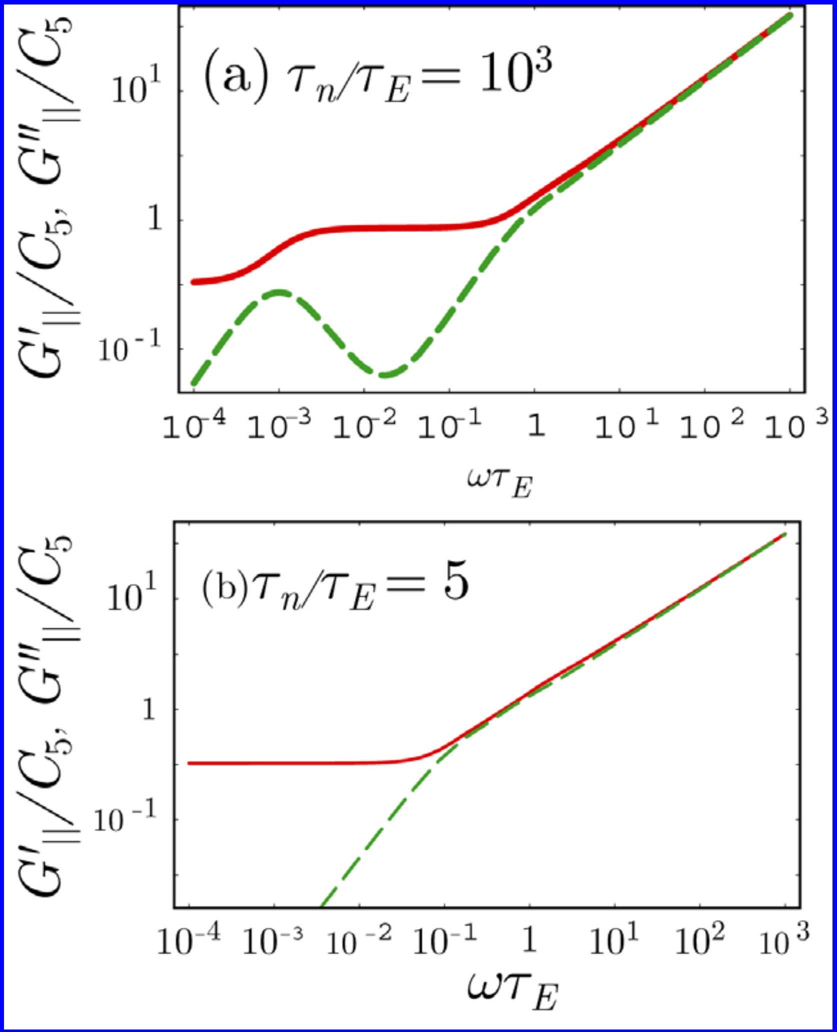
$\tau_n \gg \tau_E$ Second plateau in G'

$$G'(\omega) = C_5^R \qquad \omega\tau_n \ll 1$$

$$G'(\omega) = C_5^R + \frac{1}{2}|\lambda| D_2\left(1-|\lambda|\frac{D_1}{D_2}\right); \qquad \omega\tau_E \ll 1 \ll \omega\tau_n$$

$\tau_n \sim \tau_E$ or $\tau_n < \tau_E$ "Rouse" Behavior before plateau

Rheology



Conclusion: Linear rheology is not a good probe of semi-softness

Summary and Prospectives

- Ideal nematic elastomers can exhibit soft elasticity.
- Semi-soft elasticity is manifested in nonlinear phenomena.
- Linearized hydrodynamics of soft NE is same as that of columnar phase, that of a semi-soft NE is the same as that of a uniaxial solid.
- At high frequencies, NE’s will exhibit polymer modes; semisoft can exhibit plateaus for appropriate relaxation times.
- Randomness will affect analysis: random transverse stress, random elastic constants will complicate damping and high-frequency behavior.

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