A de Gennes Legacy: Liquid Crystals as Inspiration for Fundamental Physics

40 Years of Liquid Crystal Physics

P.G. de Gennes, C.R. Acad. Sci. Paris. **266**, 15 (1968): Nematic Fluctuations and Light Scattering

Grand Synthesis: Liquid Crystals Are Ideal

1. **Equilibrium states of matter characterized by symmetry and conservation laws**
2. **Broken continuous symmetry**
   a. **Elasticity with** $\varepsilon \sim q^2$
   b. **Topological defects**
   c. **Goldstone hydrodynamics with** $\omega \rightarrow 0$ as $q \rightarrow 0$
3. **Fluctuations**
   a. **Modify MF critical behavior below** d_c
   b. **Destroy long-range order for** $d<d_L$
   c. **Modify elasticity and hydrodynamics in low** d
   d. **Unbinding of topological defects destroys elastic rigidity especially at** $d=d_L$
4. **Order produces Higgs bosons in gauge theories**

De Gennes Deux Chevaux, Rue Froidvaux -1969

My Wife

PGG's car – lent to me for summer

Les Houches – 1967 – Guitar

Orsay – 1969 – from magnetism to Liquid crystals

Superconductors and smectic liquid crystals – 1973

Polymers, branched polymers, gels, and percolation – 70's

Twist-grain boundary phase – 1989

Liquid Crystal Elastomers – 1980-89 and now

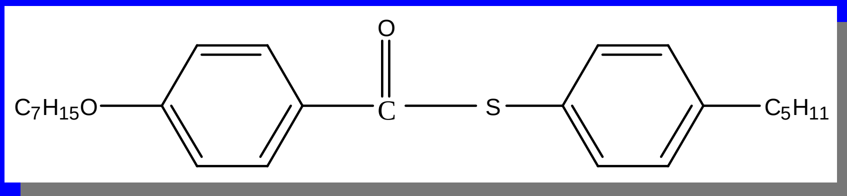
Outline

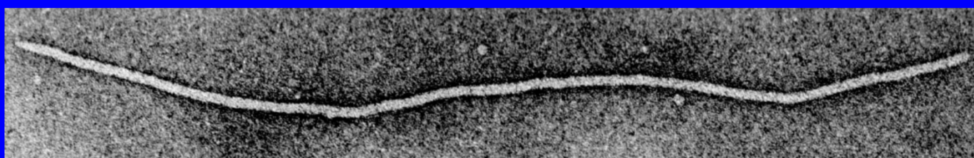
- Nematics
- Nematic Elatomers
- Smectic

From PGG's early contributions to modern questions

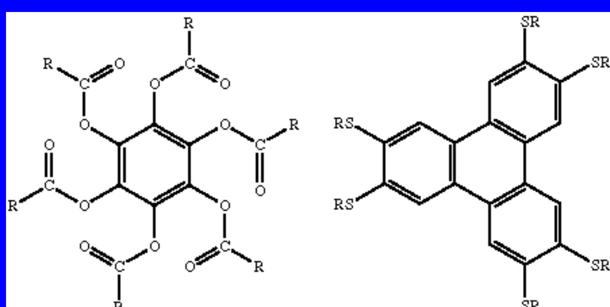
LC Mesogens

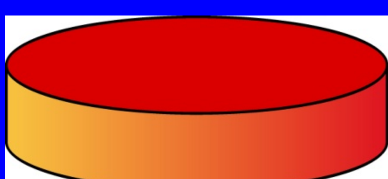
Chemistry is king





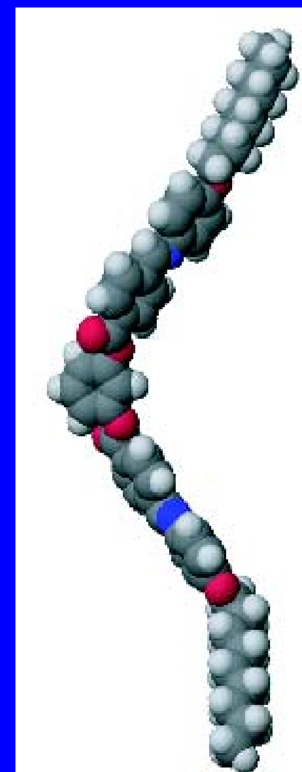
fd Virus



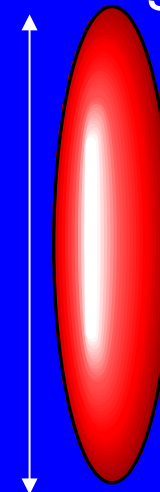


Disc-like:
discotic

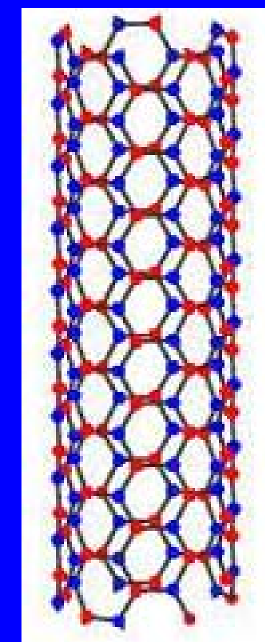
7S5 30A



Bent Core or
Banana NOBOW



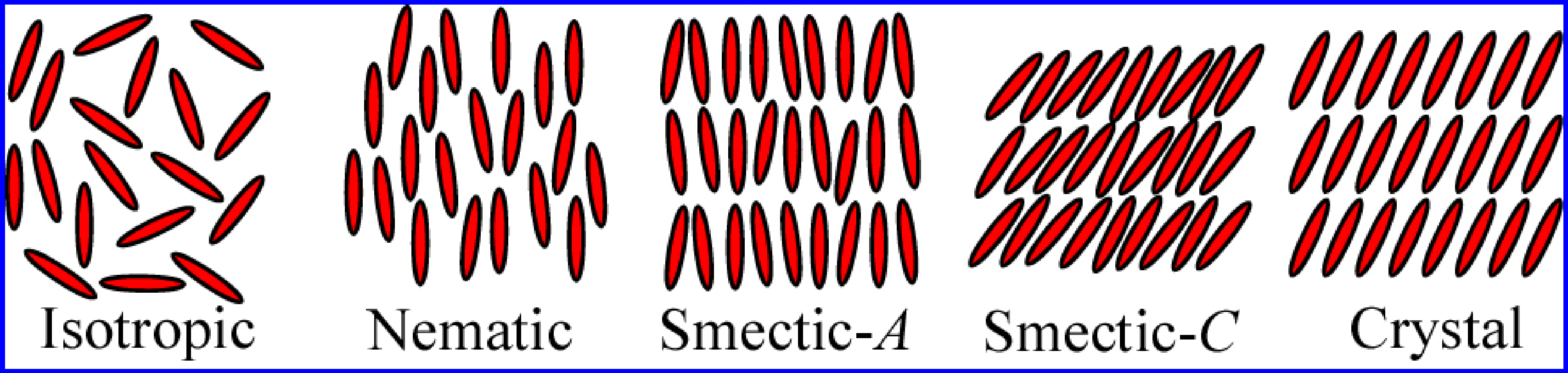
Rod-like:
calamatic



Carbon
Nanotube

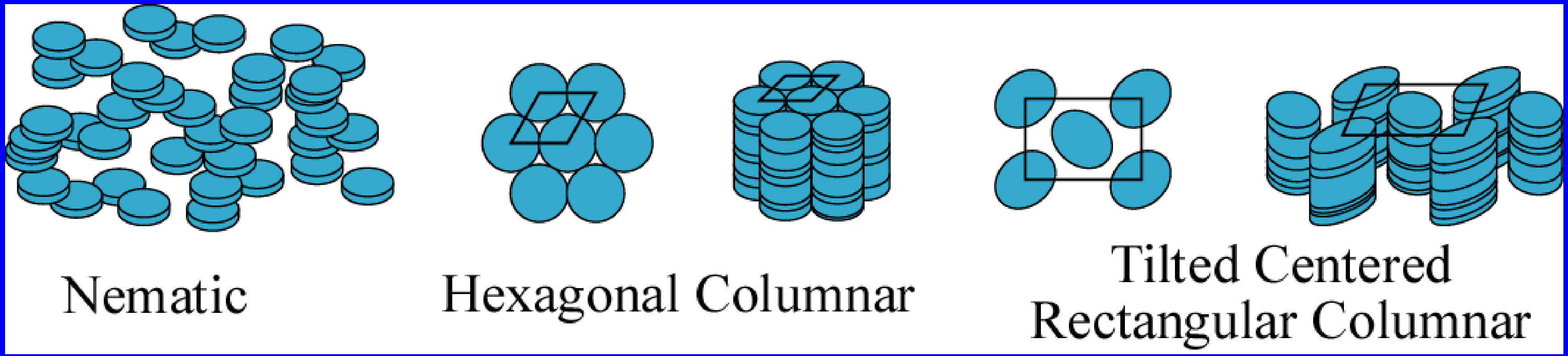
Liquid Crystal Phases

νεμωσ=Thread



Decreasing Symmetry

Σμεγμα = soap



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Chirality

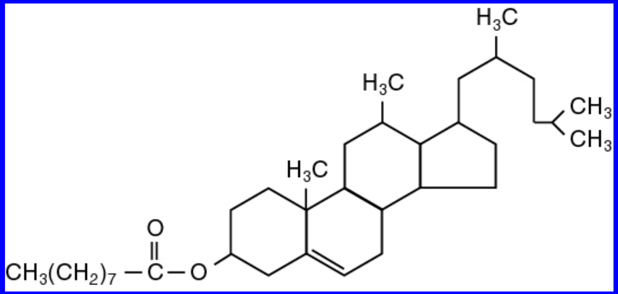
Chiral molecule: no combination of translations and rotations can superpose molecule on its mirror image

Achiral mirror images

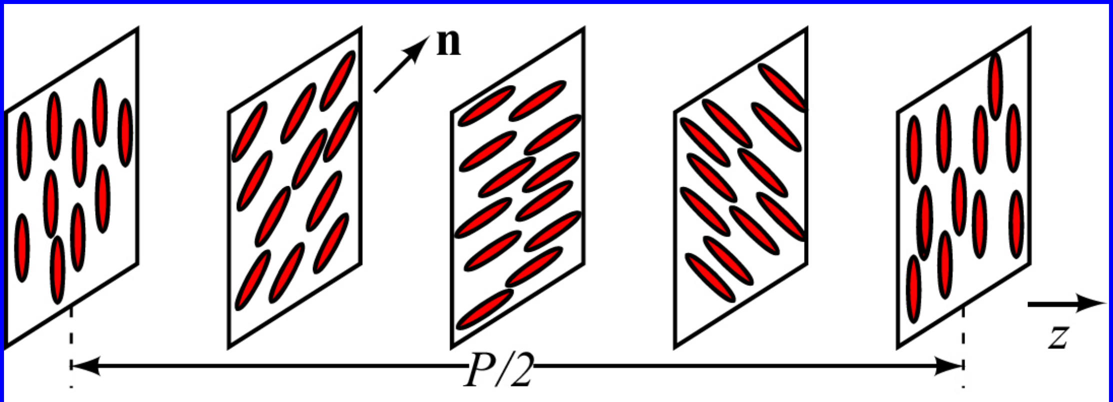
Chiral mirror images

Chirality favors twist

Chiral LC Phases

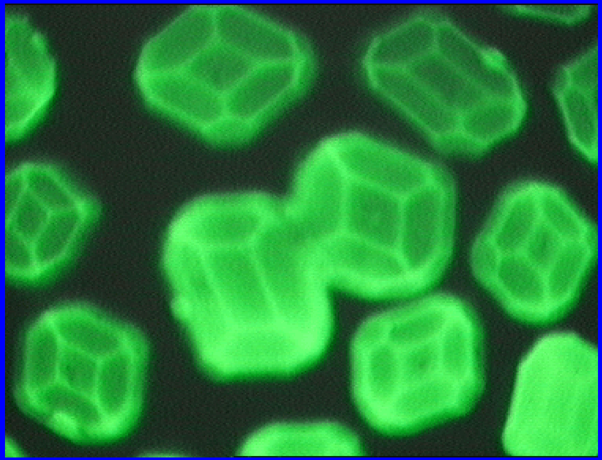


Cholesteric

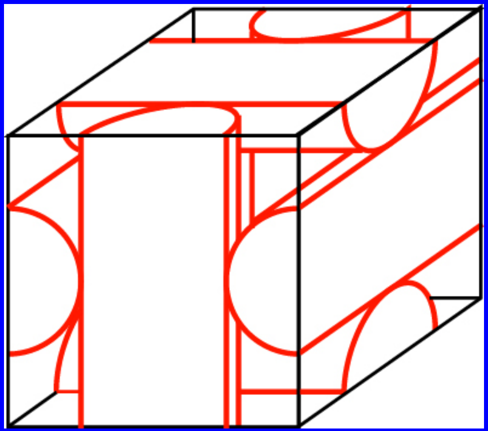


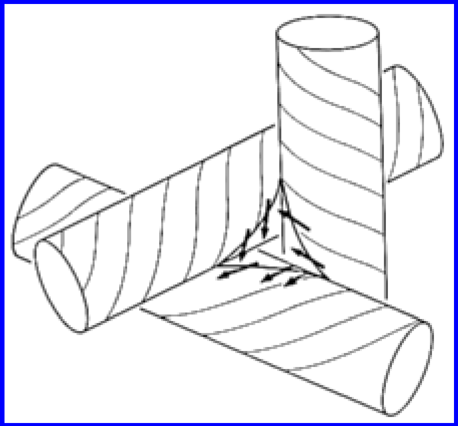
New length:
P~500nm>>d

Fig: B. Pansu



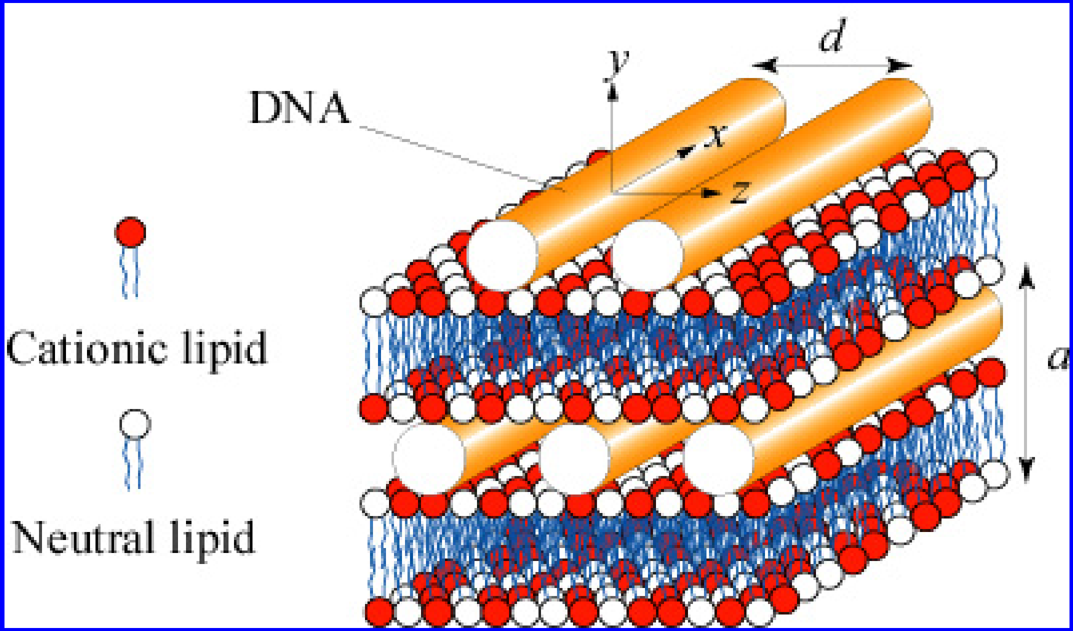
Blue Phase





Lyotropic – Mixed Rods and Layers

Sliding Columnar Phases

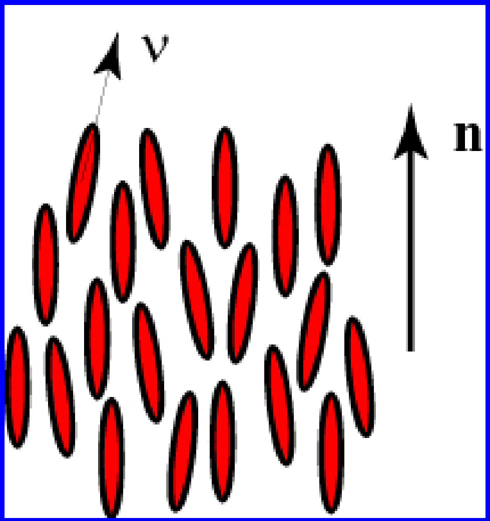


DNA-Lipid Complexes: Gene Therapy

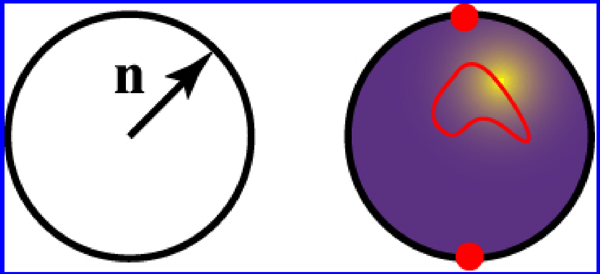
O'Hern, C.S., and Lubensky, T.C., Phys. Rev. Lett. 80, 4345-4348 (1998).

L. Golubovic and M. Golubovic, Phys. Rev. Lett. 80, 4341 (1998).

Nematic I



n = director



- Homogenous but anisotropic
- All origins are equivalent but not all directions
- Order parameter (deGennes-Maier-Saupe):

P.G. de Gennes, Physics Lett. **30A**, 454 (1969)

$$Q_{ij} = S\left(n_i n_j - \tfrac{1}{3}\delta_{ij}\right) = \langle \nu_i \nu_j - \tfrac{1}{3}\delta_{ij} \rangle$$

- Rotations about **n** leave phase unchanged
- Rotations perp. to **n** take nematic from one state to another with equal energy on the ground state-manifold
- Slowly Varying, spatially non-uniform rotations cost elastic energy with $\varepsilon = Kq^2$; $q = 2\pi/\lambda$; $\lambda =$ *length.*
- Two new hydrodynamic modes with
- $\omega = -i(K/\eta)q^2$ = frequency – but decays

P.G. de Gennes, C.R. Acad. Sci. Paris. **266**, 15 (1968)
Groupe d'Etudes des Cristaux Liquides (Orsay), J. Chem. Phys. 51, 816 (1969)

de Gennes-Landau Energy

Volume 30A, number 8 PHYSICS LETTERS 15 December 1969

PHENOMENOLOGY OF SHORT-RANGE-ORDER EFFECTS IN THE ISOTROPIC PHASE OF NEMATIC MATERIALS

P. G. DE GENNES
Physique des Solides, Faculté des Sciences, 91 Orsay, France

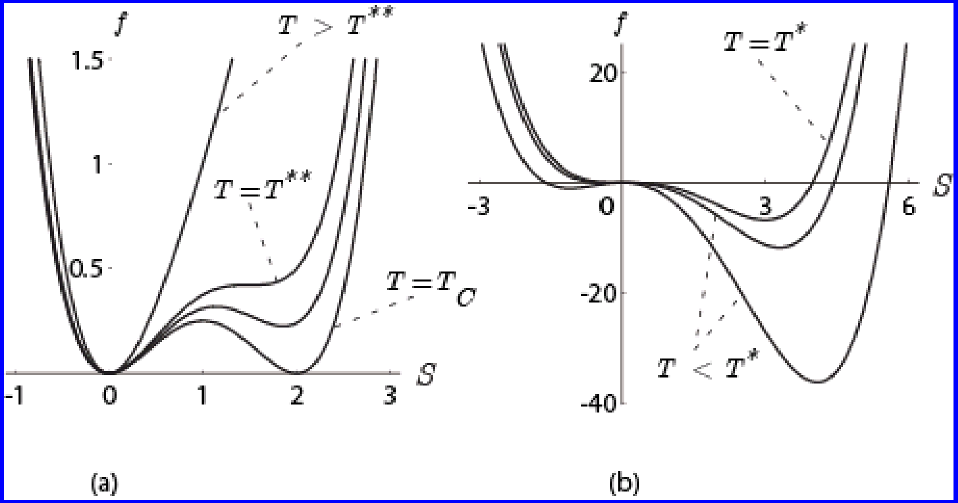
Received 6 November 1969

The magnetic birefringence, the intensity and the width in frequency of the Rayleigh scattering, and the flow birefringence, are discussed in terms of a small number of phenomenological parameters. A qualitative discussion of the nuclear spin-lattice relaxation times is also given.

First appearance of my name in a Physics Journal:

We have greatly benefited from discussions with D. Litster, T. Lubensky, M. Papoular and P. Pincus.

$$F = F_0 + \tfrac{1}{2} A\, Q_{\alpha\beta} Q_{\beta\alpha} + \tfrac{1}{3} B Q_{\alpha\beta} Q_{\beta\gamma} Q_{\gamma\alpha} + O(Q^4) + - \tfrac{1}{2} G\, Q_{\alpha\beta} H_\alpha H_\beta + + \tfrac{1}{2} L_1 \partial_\alpha Q_{\beta\gamma} \partial_\alpha Q_{\beta\gamma} + \tfrac{1}{2} L_2 \partial_\alpha Q_{\alpha\gamma} \partial_\beta Q_{\beta\gamma} \quad (2)$$



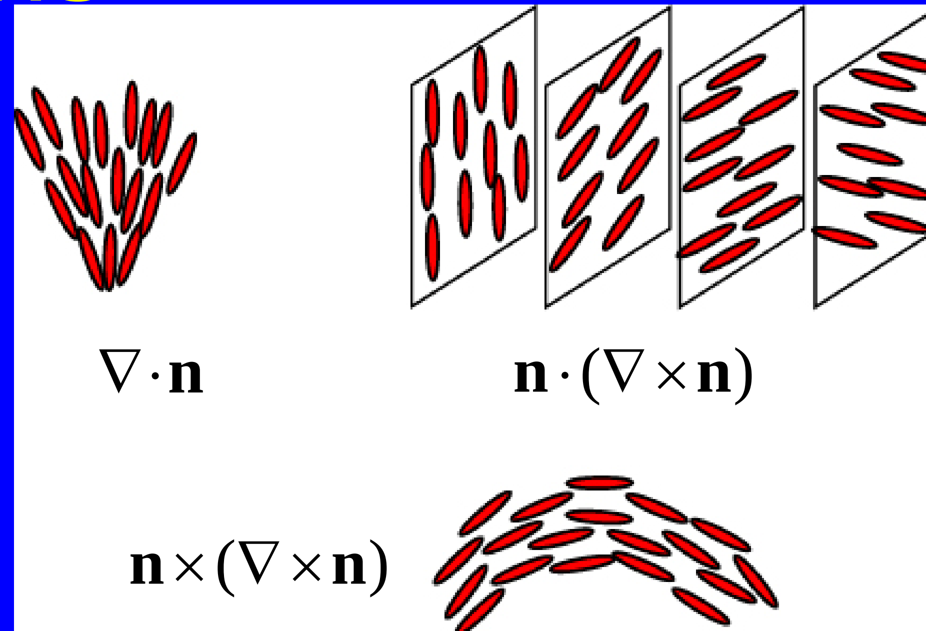
First-order phase transition

Short Range Order Effects in the Isotropic Phase of Nematics and Cholesterics, Mol. Cryst. Liq. Cryst.12, 193 (1971)

Penn University of Pennsylvania

Nematic Distortions

State invariant under **n** to **–n.**
Energy invariant under uniform rotations of **n** – depends only on gradients of **n** ~ $(\partial \mathbf{n})^2 \sim \mathbf{n}^2/R^2$



Frank Energy

$$F = \frac{1}{2} \int d^3x \left\{ K_1 (\nabla \cdot \mathbf{n})^2 + K_2 [\mathbf{n} \cdot (\nabla \times \mathbf{n})]^2 + K_3 [\mathbf{n} \times (\nabla \times \mathbf{n})]^2 \right\}$$

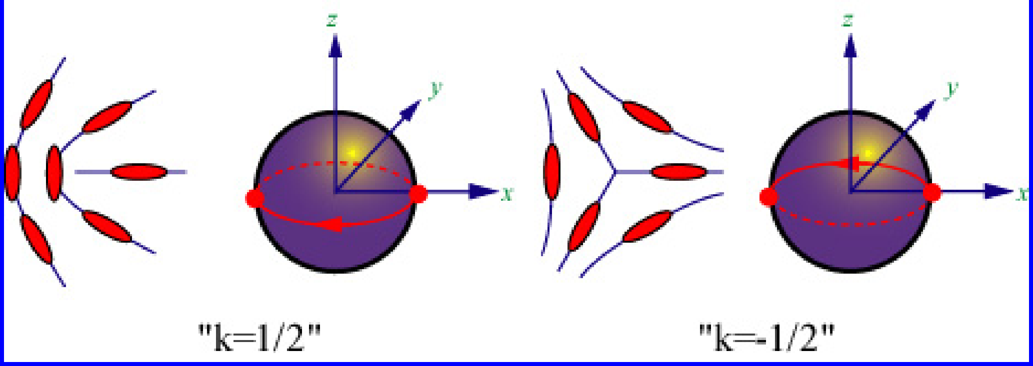
Fluctuations:

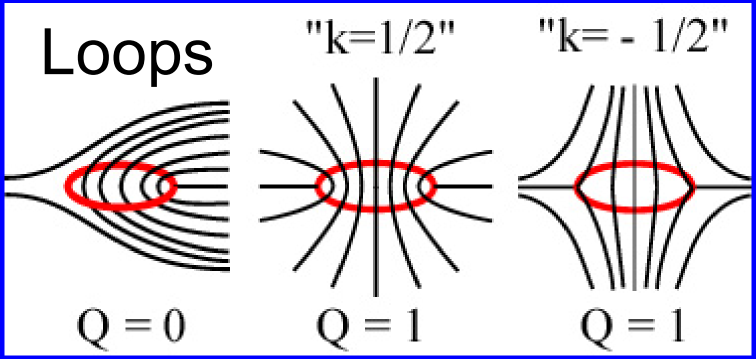
$$\left\langle |n(\mathbf{q})|^2 \right\rangle \sim \frac{k_B T}{K q^2}$$

P.G. de Gennes, C.R. Acad. Sci. Paris. **266**, 15 (1968)

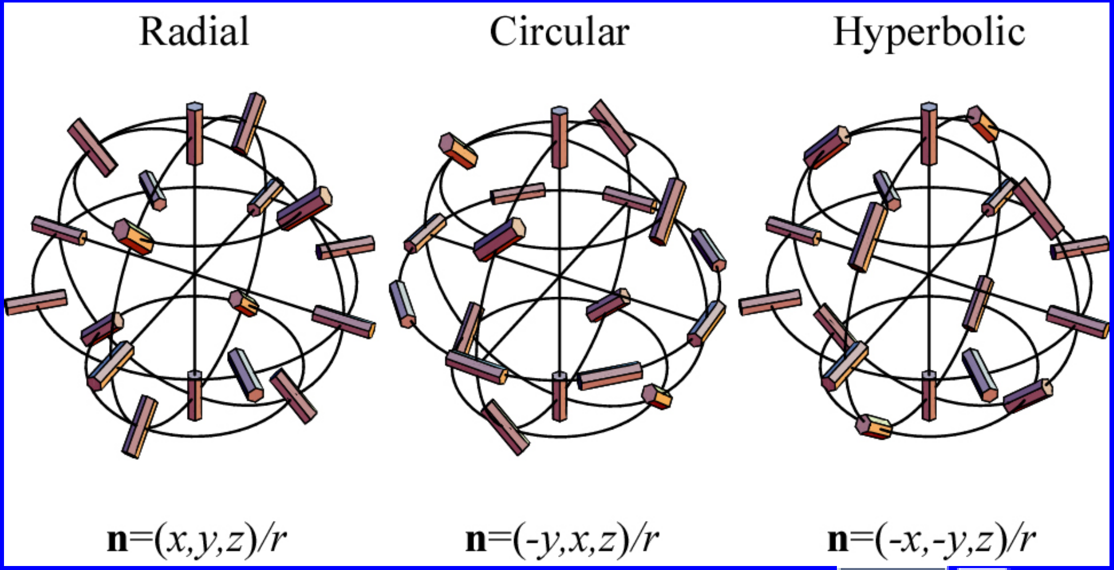
Nematic Defects

- Disclinations: Mapping from a closed loop in nematic from any point to its antipode in the ground-state manifold





- Hedgehogs: Mappings from surface enclosing defect to 2D surface of the ground-state manifold



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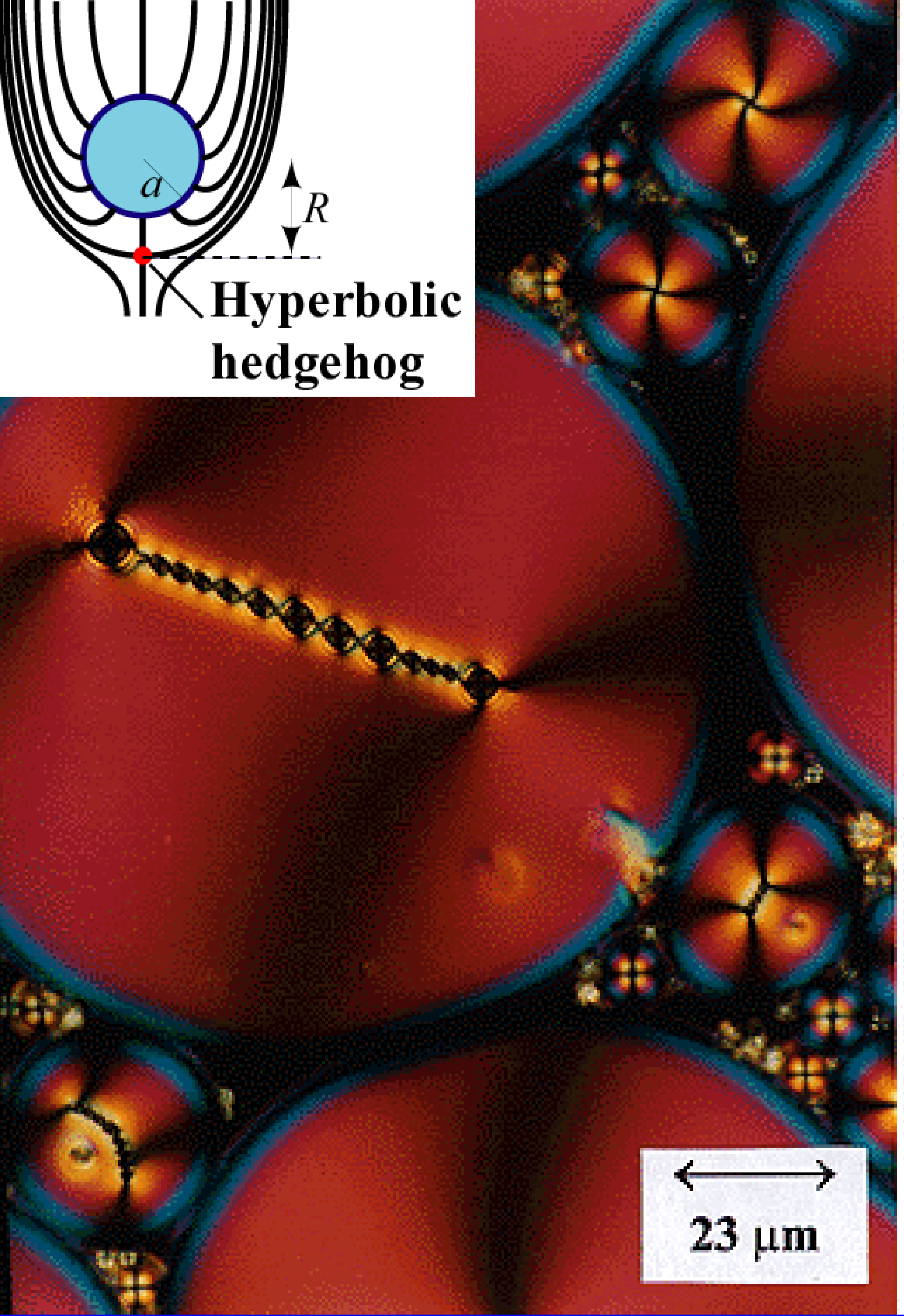
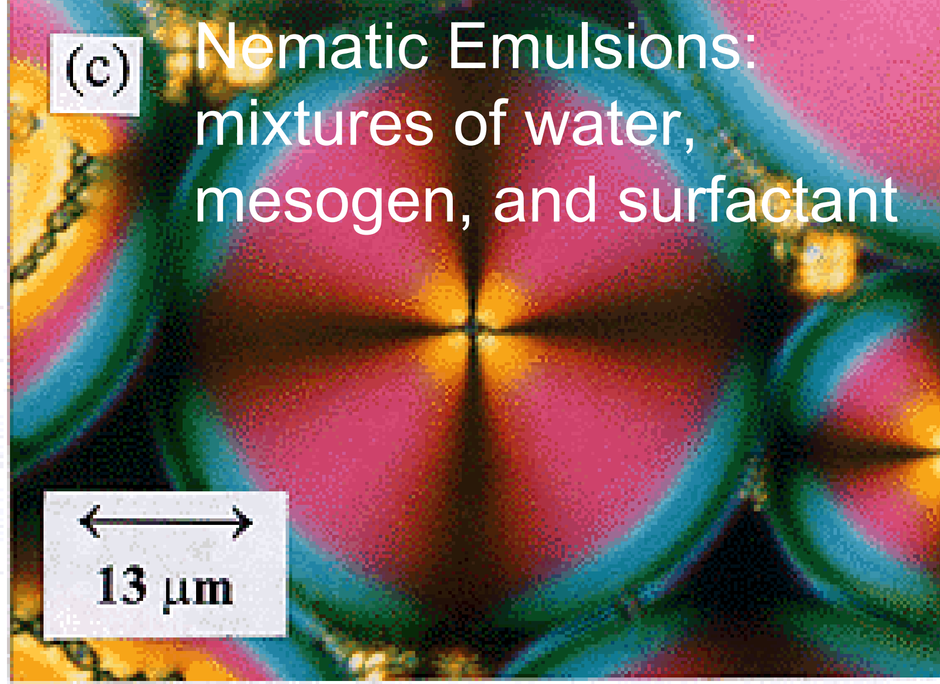
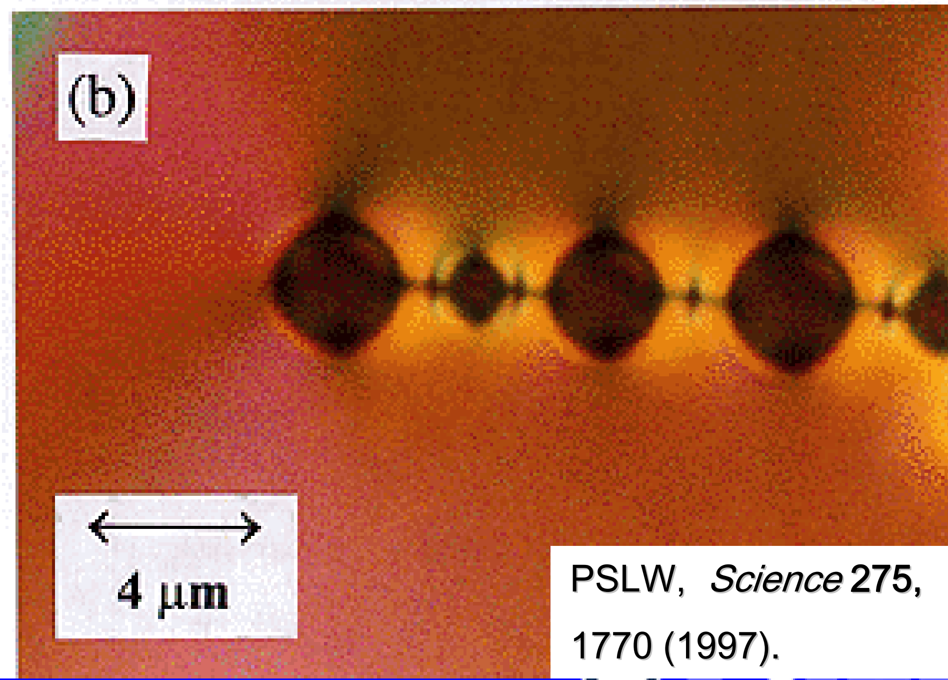
Nematic Emulsions: mixtures of water, mesogen, and surfactant

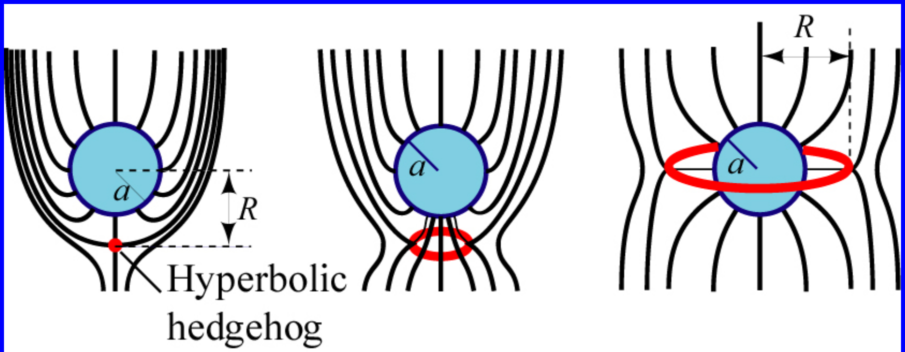




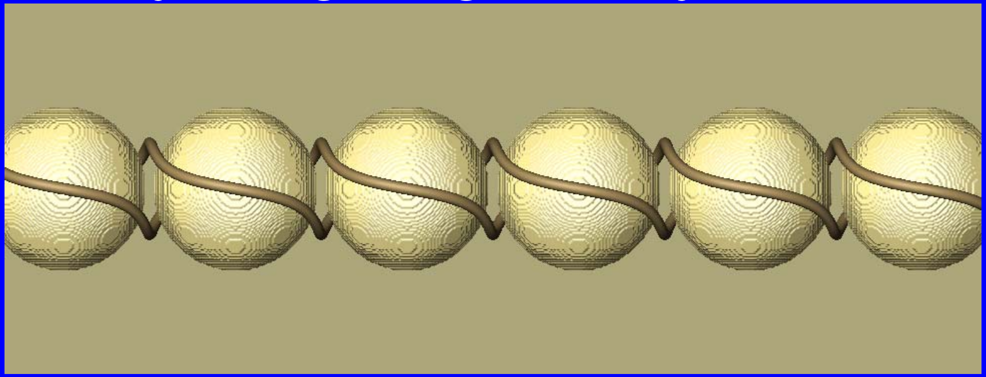


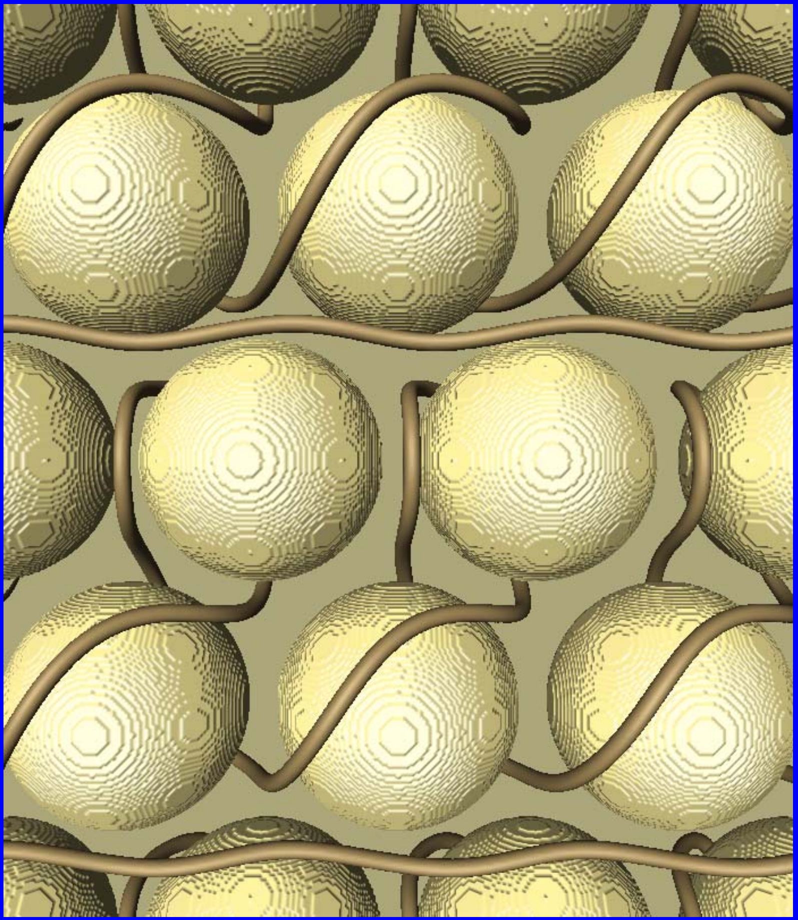
PSLW, *Science* **275**, 1770 (1997).

Disclinations and colloids



Each sphere produces one hedgehog: disclination lines carry hedgehog density





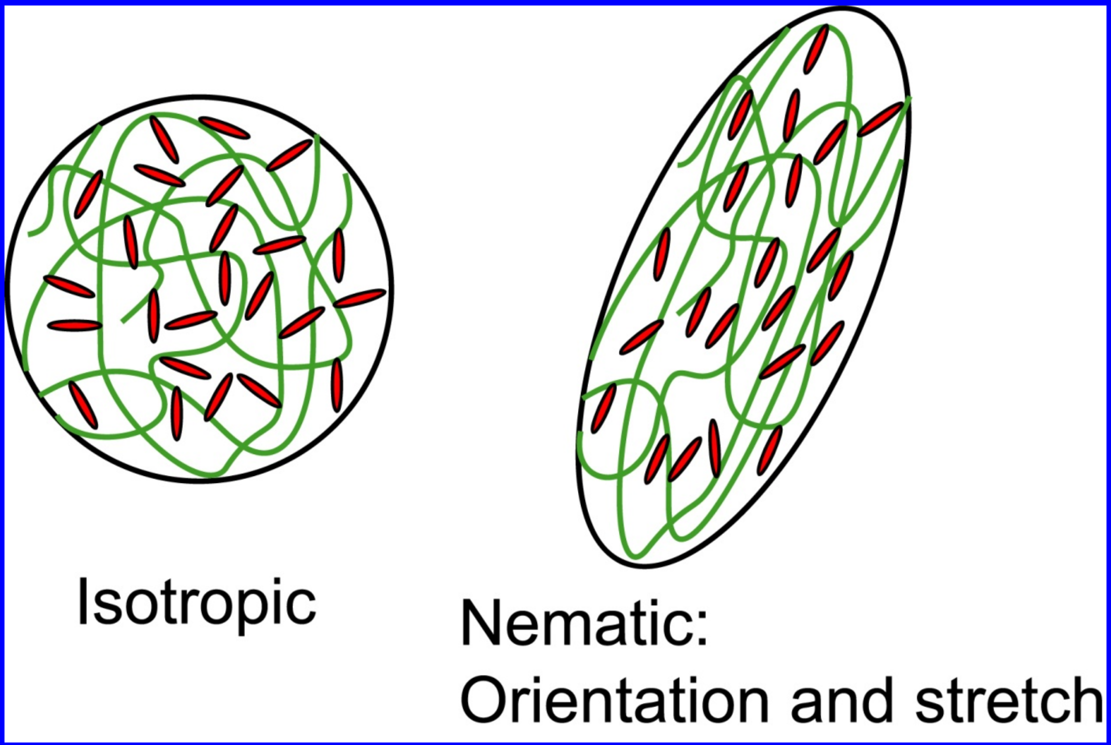
Musevic, I; Skarabot, M; Tkalec, U, et al., SCIENCE 313 954 (2006)

Ravnik, M; Skarabot, M; Zumer, S, et al., Phys Rev Lett 99, 247801 (2007)

Nematic Elastomers

Nematogens in a crosslinked network

Spontaneous rotational symmetry breaking in an isotropic elastic medium: nematic order drives stretch



M. Warner and E.M. Terentjev, *Liquid Crystal Elastomers* (Oxford University Press, New York, 2003)

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Nematic Elastomers

Volume 28A, number 11 PHYSICS LETTERS 10 March 1969

POSSIBILITES OFFERTES PAR LA RETICULATION DE POLYMERES EN PRESENCE D'UN CRISTAL LIQUIDE

P. G. DE GENNES
Service de Physique des Solides *
Faculté des Sciences, 91 - Orsay, France

Reçu le 3 février 1969

Un cristal liquide utilisé comme diluant lors de la réticulation permettrait en principe d'engendrer des matériaux plastiques anisotropes et éventuellement dotés de propriétés optiques remarquables.

Considered effects crosslinking in LC solvents: Smectics and Cholesterics freeze in anisotropy

Other Elastomer publications:

"Réflexions sur un type de polymères nématiques," C.R. Acad. Sc. Paris B t. 281, 101 (1975)

In Liquid Crystals of One- and two-dimensional order (Springer, Berlin, 1980)

In Polymer Liquid Crystals (Academic Press, New York, 1982)

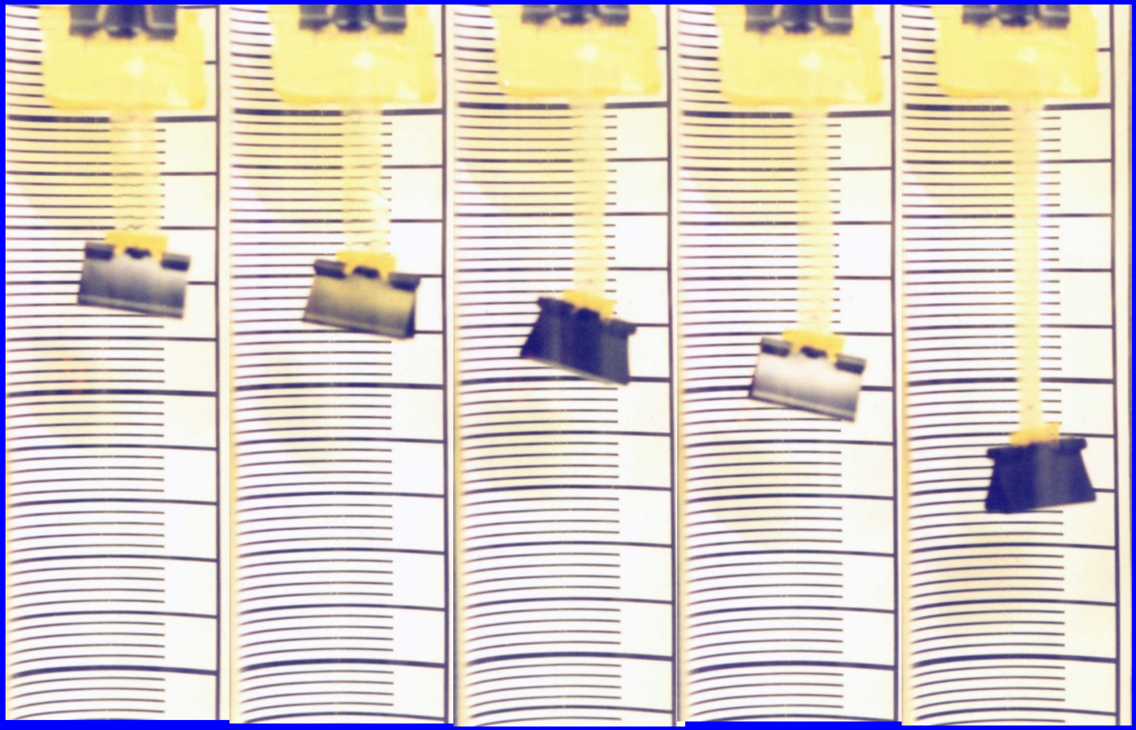
P.G. de Gennes, M. Hebert, et al. "Artificial muscle based on nematic gels," Macromolecular Symposia 113, 39 (1997).

M. Hebert, R. Kant, et al. "Dynamics and thermodynamics of artificial muscles based on nematic gels," J. de Physique 7, 909 (1997).

P.G. de Gennes and K. Okumura, "Phase transitions in nematic rubbers," Europhysics Letters 63, 76 (2003).

Thermoelastic Effect

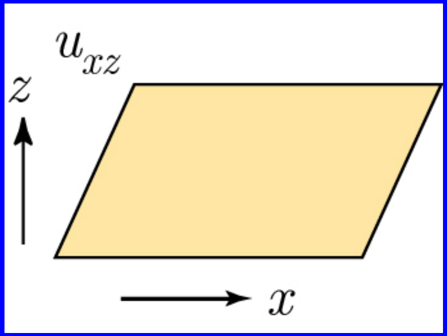
- Large thermally induced strains - artificial muscles



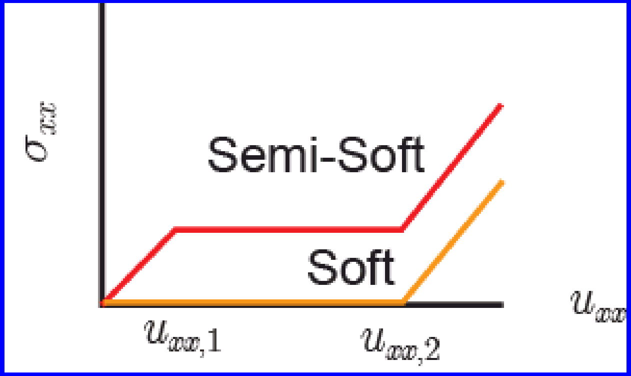
Courtesy of Eugene Terentjev

300% strain

Soft and Semi-soft Response

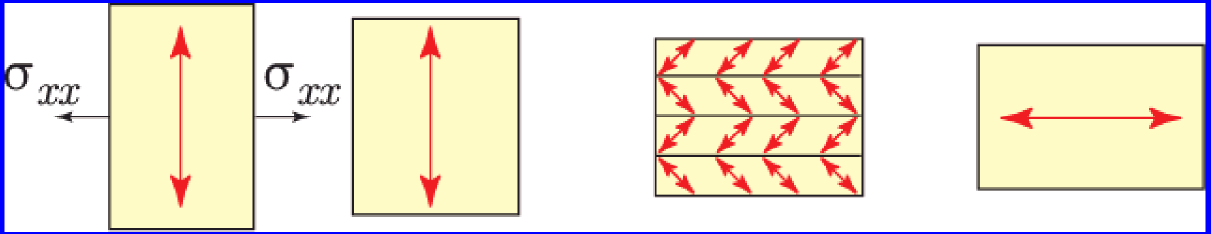


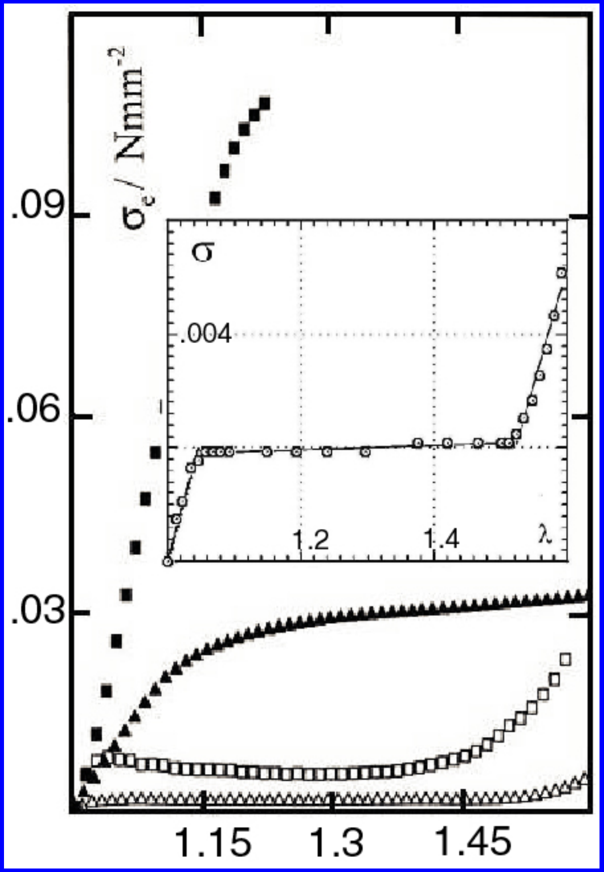
Vanishing xz shear modulus



Soft: spontaneseous symmetry breaking

Semi-soft: frozen-in nematic order with second crosslinking





Soft or semi-soft stress-strain for stress perpendicular to order

Finkelmann, et al., J. Phys. II **7**, 1059 (1997); Warner, J. Mech. Phys. Solids **47**, 1355 (1999)

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Elasticity of Nematic Elastomers

$$f = \tfrac{1}{2} B u_{ii}^2 + \mu \mathrm{Tr}\underline{\tilde{u}}^2 - C \mathrm{Tr}\underline{\tilde{u}}^3 + D\left(\mathrm{Tr}\underline{\tilde{u}}^2\right)^2$$

$$\tilde{u}_{ij} = u_{ij} - \tfrac{1}{3}\delta_{ij} u_{ii}^2$$

P.G. de Gennes, C.R. Acad. Sc. Paris B t. 281, 101 (1975)

$$u_{ij} = (\partial_i u_j + \partial_j u_i + \partial_i u_k \partial_j u_k)/2$$

nonlinear strain

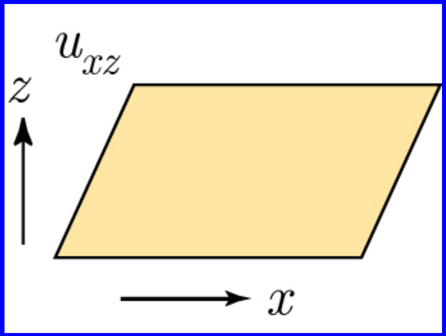
$\mu < 0$: Transition to stretched nematic phase with $\tilde{u}_{ij} \neq 0$

$$f_{\mathrm{el}} = \tfrac{1}{2} C_1 u_{zz}'^2 + C_2 u_{zz}' u_{aa}' + \tfrac{1}{2} C_3 u_{aa}' u_{aa}' + C_4 u_{ab}' u_{ab}' + C_5 u_{za}' u_{za}' + K_1^R (\partial_\perp^2 u_z)^2 + K_3^R (\partial_a^2 u_z)^2$$

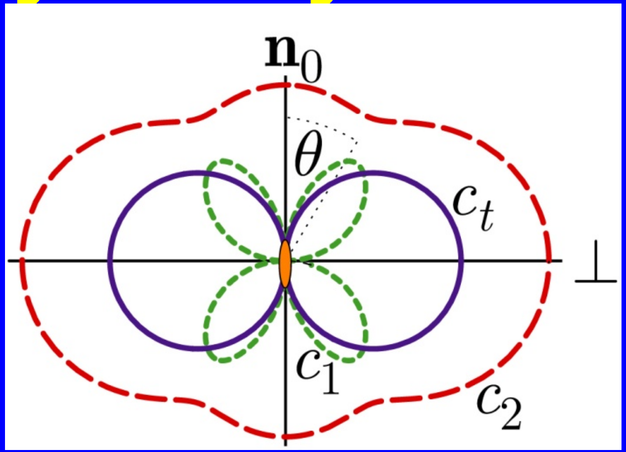
$a, b = x, y$

Broken continuous symmetry: soft or Goldstone mode, vanishing C_5

Golubovic, TCL, PRL 63, 1082 (1989); Olmsted, J. Phys. II 4, 2215 (1994)



Hydrodynamics, Nonlinearities



Three hydrodynamic sound modes with some velocities vanishing in symmetry directions

Stenull, **TCL**, PRE **69** ,051801(2004)

Nonlinearities lead to renormalization of elastic constants

G. Grinstein and R. Pelcovits, Phys. Rev. Lett. **47, 856** (1981); Phys. Rev. A **26,** 915 (1982)

$d<3$

$$C_4 = s_4 \,|\, q_\perp \,|^{\eta}\; g(q_d \,/\, |\, q_\perp \,|^{2+\omega})$$

$$K \sim |\, q_\perp \,|^{-\eta_K}\; g_K(q_d \,/\, |\, q_\perp \,|^{2+\omega})$$

$d=3$

$$K \sim \left[1 + a \ln |\, q_\perp \,|^{-1}\right]^{38/59}$$

$$C_4 \sim \left[1 + a \ln |\, q_\perp \,|^{-1}\right]^{-4/59}$$

$$\eta = \frac{4}{59}(3-d)$$

$$\eta_K = \frac{38}{59}(3-d)$$

$$\omega = -\frac{42}{59}(3-d)$$

Stenull, **TCL**, Europhys. Lett. 61, 776 (2003); PRE 69, 02180 (2004); Xing Radzihovsky, Europhys. Lett. 61

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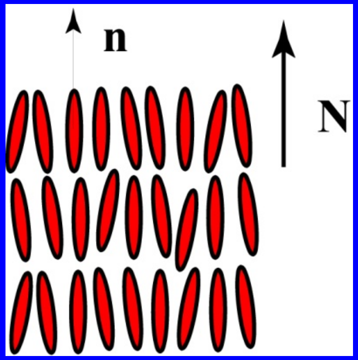
Smectics I



Smectic-*A*

- Layered structure: periodic in one dimension
- Order parameter: $\rho(\mathbf{x}) = \rho_0 + \psi e^{iq_0\mathbf{n}\cdot\mathbf{x}} + c.c.$
  $\psi(\mathrm{x}) = |\psi_0| e^{-iq_0 u}$
- Uniform increments of u translate layers and leave energy unchanged
- Directions of molecules and layer normals locked
- Ground-state manifold: line with $u = u + nd$

Smectics II

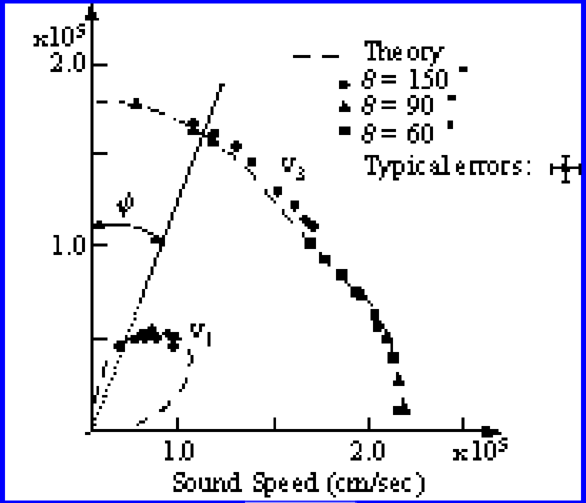
Invariance w.r.t. simultaneous rotation of layers and director: Elastic Energy locks *u* to **n**; The **Higg's** mechanism (D not 0) leaves only one elastic variable

$$F_{\mathrm{sm}} = \tfrac{1}{2}\int d^3x \left[B\left(\nabla_{||}u\right)^2 + D\left(\nabla_{\perp}u - \delta\mathbf{n}\right)^2 \right] + F_{\mathrm{n}}$$

$$F_{\mathrm{sm}} = \tfrac{1}{2}\int d^3x \left[B u_{zz}^2 + K_1\left(\nabla_{\perp}^2 u\right)^2 \right]$$

$$u_{zz} = \partial_z u - \tfrac{1}{2}\left(\nabla u\right)^2$$

Liao, Clark, Pershan, *PRL* 30, 639 (1973)



Fluctuations

F. Brochard, J. Physique 34, 411 (1973)

$$\left\langle \left|\delta n(q)\right|^2 \right\rangle = \frac{k_B T}{D + K q^2}$$

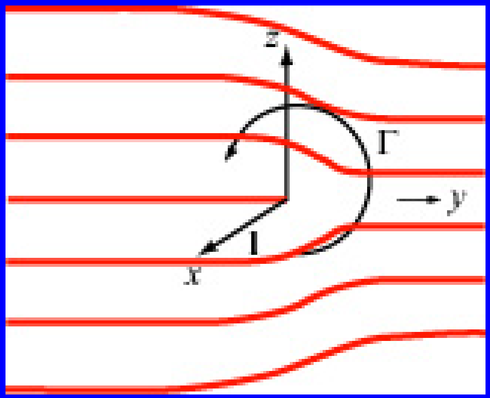
Hydrodynamics

PGG, J. Physique (Paris) Colloq 30, C4 65 (1969)

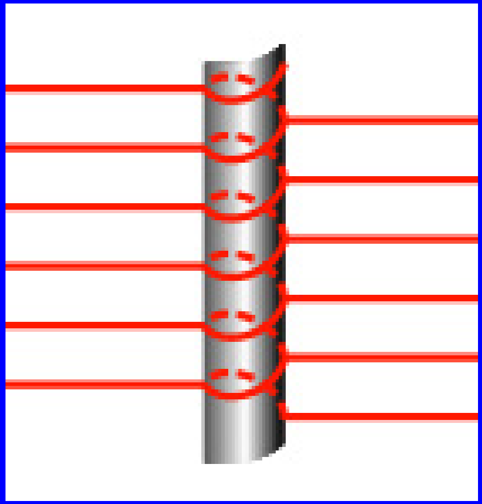
$$\frac{\partial u}{\partial t} = v_z - \zeta \frac{\delta F}{\delta u}$$

$$\omega^2 = \frac{B q_{\perp}^2 q_{||}^2}{\rho q^2}$$

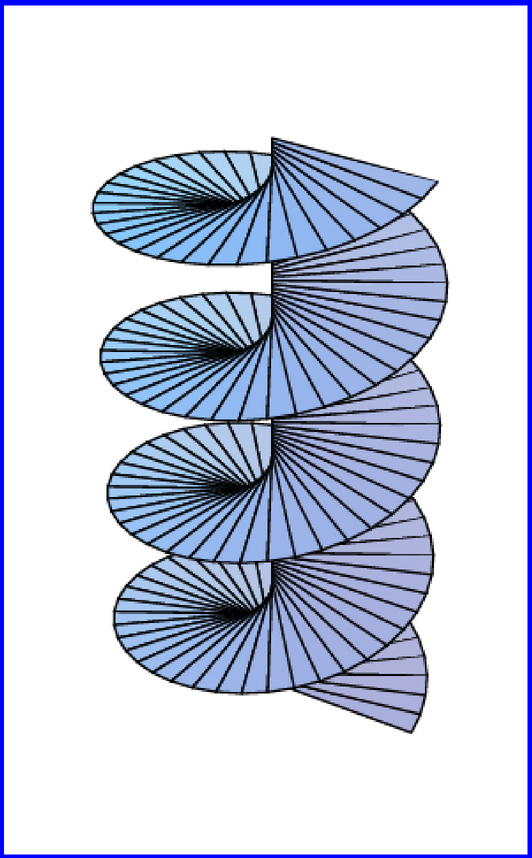
Smectic Topological Defects



Edge Dislocation



Screw Dislocation



Screw dislocation: helicoid or Renaissance staircase

Smectics III

Fluctuation destruction of long-range order:

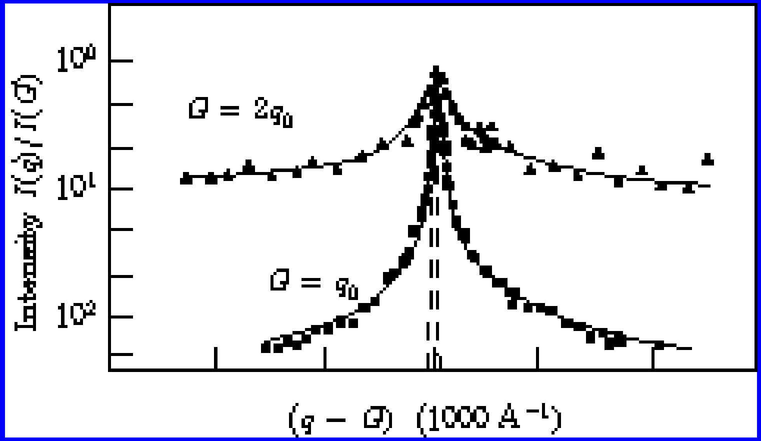
A. Caillé, *C. R. Acad. Sci*. Ser. B **274**, 891 (1972).

$$\langle u^2 \rangle \sim \ln L \to \infty \qquad \langle \psi \rangle \to 0$$

Als-Nielsen et al., *PR B* **22**. 312 (1980)

$$I(q) \sim \begin{cases} (q_{||} - nq_0)^{-2+n^2\eta} & q_\perp = 0 \\ q_\perp^{-4+2n^2\eta} & q_{||} = 0 \end{cases}$$

Safinya, *et al. PRL* **57**, 2718 (1986)



Non-linearities modify elasticity

$$B(\mathrm{q}) \sim \left[\ln\left(\mathrm{q}_{||}^2 + \lambda^2 q_\perp^4\right)^{-1}\right]^{-4/5}$$

$$K_1(\mathrm{q}) \sim \left[\ln\left(\mathrm{q}_{||}^2 + \lambda_2 q_\perp^4\right)^{-1}\right]^{2/5}$$

Grinstein & Pelcovits, *PRL* **47**, 856 (1981)

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Smectics and Superconductors

LC

$$F = F_\psi + F_n + F^*$$

$$F_\psi = \int d^3x [r|\psi|^2 + C_{||}|\nabla_{||}\psi|^2$$
$$C_\perp |(\nabla_\perp - iq_0\delta\mathbf{n})\psi|^2 + \tfrac{1}{2}g|\psi|^4]$$

de Gennes, *Solid State Commun.* 10, 753 (1972).

$$F_n = \int d^3x\{K_1(\nabla\cdot\mathbf{n})^2 + K_2(\mathbf{n}\cdot\nabla\times\mathbf{n})^2 + K_3[\mathbf{n}\times(\nabla\times\mathbf{n})]^2\}$$

$$F^* = h\int d^3x\, \mathbf{n}\cdot\nabla\times\mathbf{n}$$ Chiral

SC

$$F = F_\psi + F_A + F_H$$

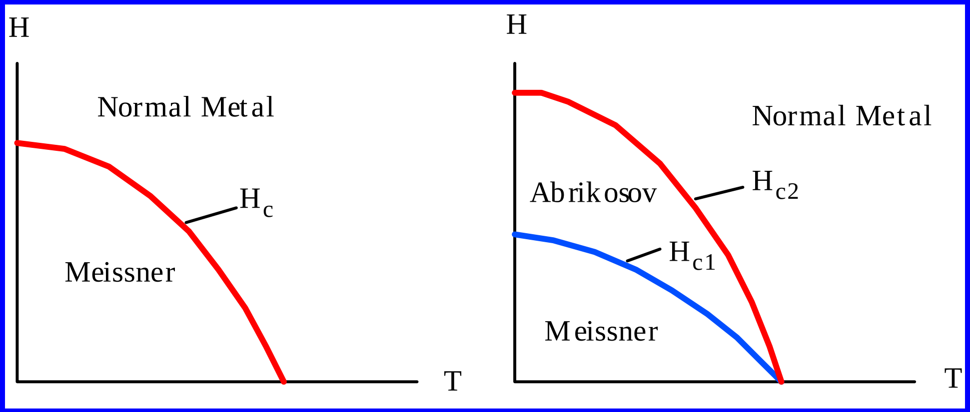
Landau-Ginzburg free energy

$$F_\psi = \int d^3x[r|\psi|^2 + C|(\nabla - i2(e/\hbar c)\mathbf{A})\psi|^2 + \tfrac{1}{2}g|\psi|^4]$$

$$F_A = \frac{1}{8\pi\mu_0}\int d^3x(\nabla\times\mathbf{A})^2$$

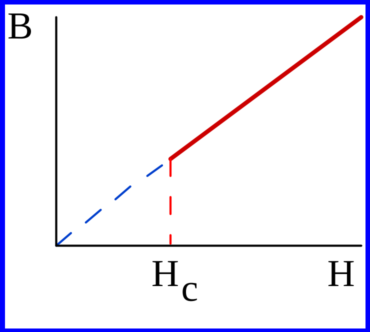
$$F_H = -\frac{1}{4\pi}\int d^3x\,\mathbf{H}\cdot\nabla\times\mathbf{A}$$

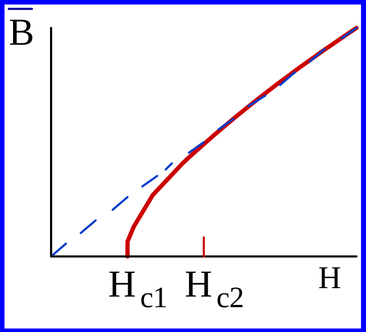
SC in a Field



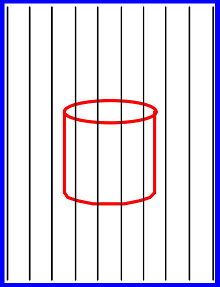
Type I: $\lambda<\xi$

Type II: $\lambda>\xi$

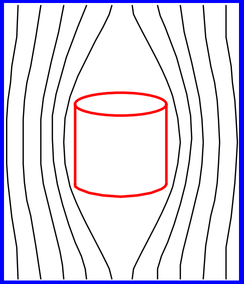




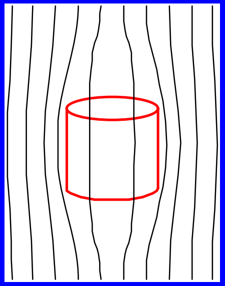
Normal Metal



Meissner: Type I



Abrikosov: Type II



H

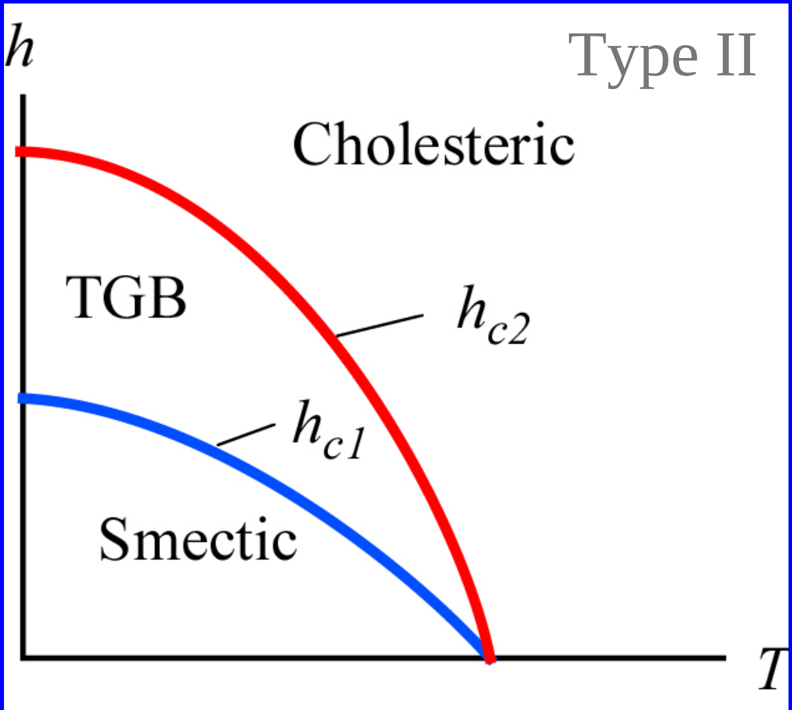
SC's and LC's

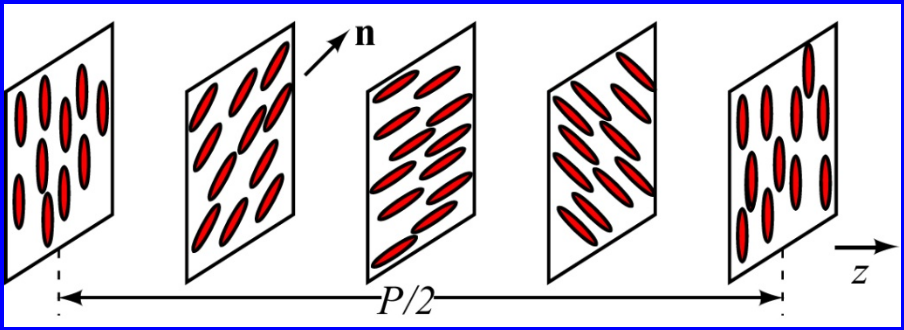
Superconductor	Liquid Crystals
$\psi=\lvert\psi\rvert e^{i\phi}$ **Cooper-Pair**	$\psi=\lvert\psi\rvert e^{i\phi}$ = **Mass-density-wave**
A= Vector potential	**n** = **nematic director**
H= magnetic intensity	h = **molecular chirality**
B=$\nabla\times$A= microscopic field	$k_0 = \mathbf{n}\cdot(\nabla\times\mathbf{n})$ = **twist**
$\overline{\mathrm{B}}$= Maxwell field	$\overline{k_0} = \overline{\mathbf{n}\cdot(\nabla\times\mathbf{n})}$ = **average twist**
normal metal	**nematic**
normal metal in a field	**cholesteric phase**
Meissner Phase	**smectic-A phase**
Meissner effect	**twist expulsion**
London Penetration depth	**twist penetration depth**
coherence length	**smectic coherence length**
Vortex	**screw dislocation**
Abrikosov Flux lattice	

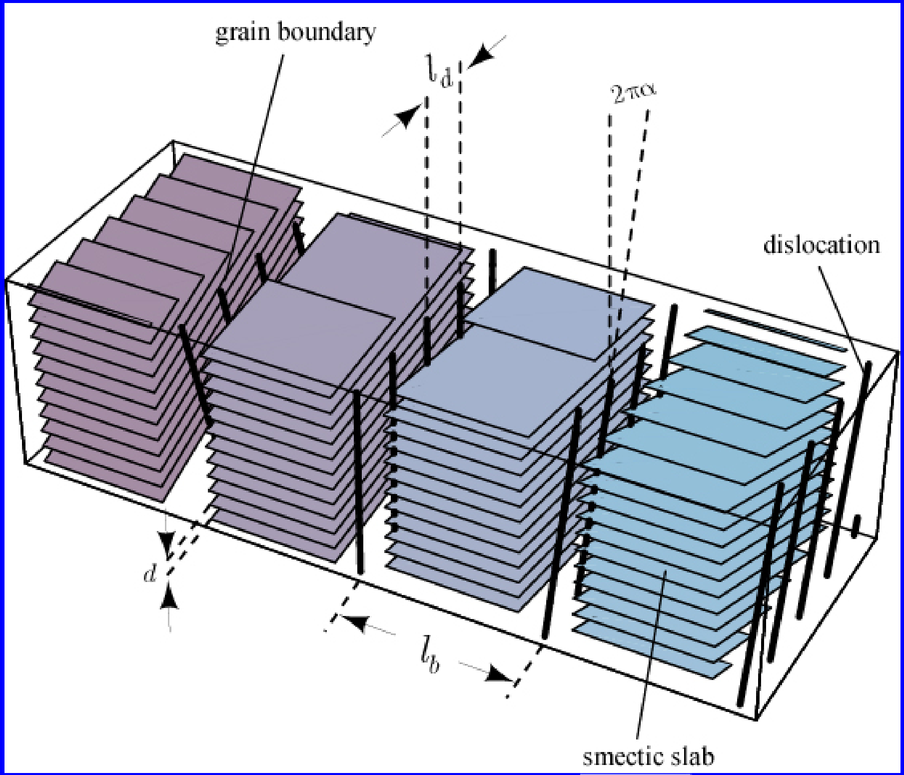
TGB Phase Diagram

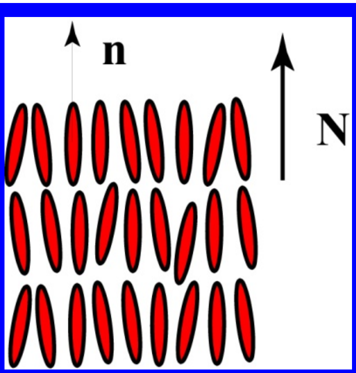
Renn-TCL, *PR A* **38**, 2132 (1988)
Goodby *et al.*, *Nature* **337**, 449 (1989)

Direct analogy with SC



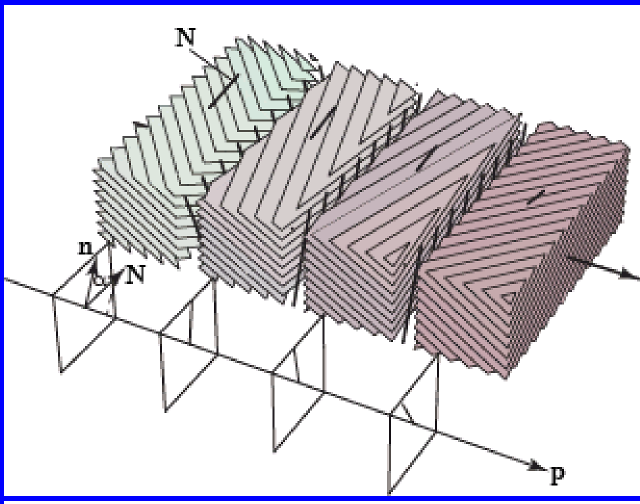






More TGB

TGBC phase

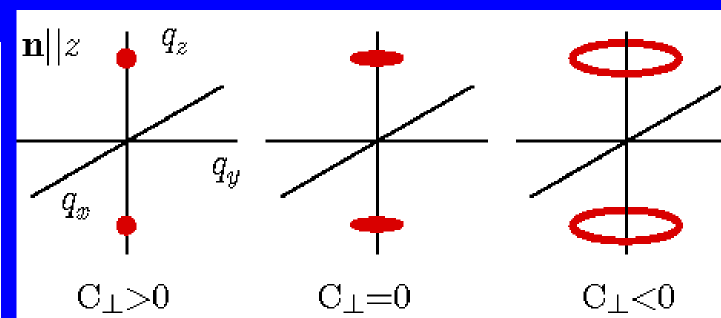


T. C. Lubensky and S. R. Renn, Mol. Cryst. Liq. Crys. **209**, 349-355 (1991).

H.T. Nguyen et al. J. Phys. II (France) **2** 1889 (1992).
[10] L. Navailles, P. Barois, and H.T. Nguyen, Phys. Rev. Lett **71**, 545 (1993); L. Navailles, P. Barois, and H.T. Nguyen, Phys. Rev. Lett. **72**, 1300 (1994).

$$F_H = \int d^3x[\tilde{r}|\psi|^2 + D_{||}|[\nabla_{||}^2(\mathbf{x}) + q_0^2]\psi|^2$$
$$+D_{\perp}|[\nabla_{\perp}^2(\mathbf{x}) + q_{0\perp}^2]\psi|$$
$$+D_{||\perp}[\nabla_{||}(\mathbf{x})^2 + q_0^2]\psi^*[\nabla_{\perp}(\mathbf{x})^2 + q_{0\perp}^2]\psi + \mathrm{c.c}$$

$$\nabla_{||}^2(\mathbf{x}) \equiv (\mathbf{n}(\mathbf{x}) \cdot \boldsymbol{\nabla})^2 \qquad \nabla_{\perp}^2(\mathbf{x}) = \nabla^2 - \nabla_{||}^2(\mathbf{x})$$

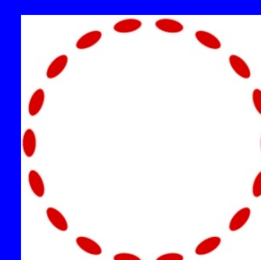


Arindam Kundagrami and T.C. Lubensky, *Phys. Rev. E 68,* 060703 (2004).

I. Luk'yanchuk, Phys. Rev. E **57**, 574(1998).

Quasicrystalline TGBC phase

L. Naivailles, P. Barois, H.T. Nguyen, PRL **71**, 545 (1993)



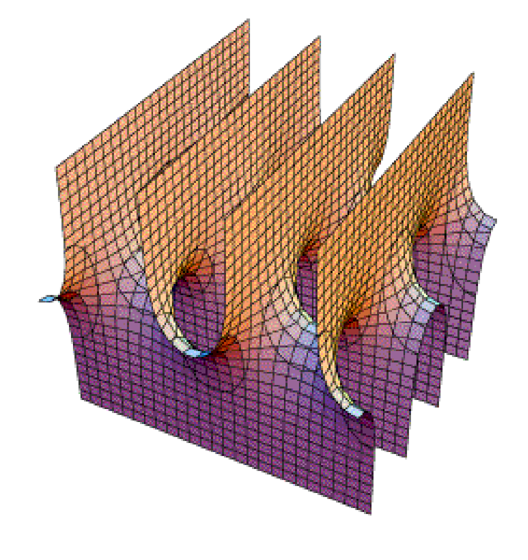
More TGB II

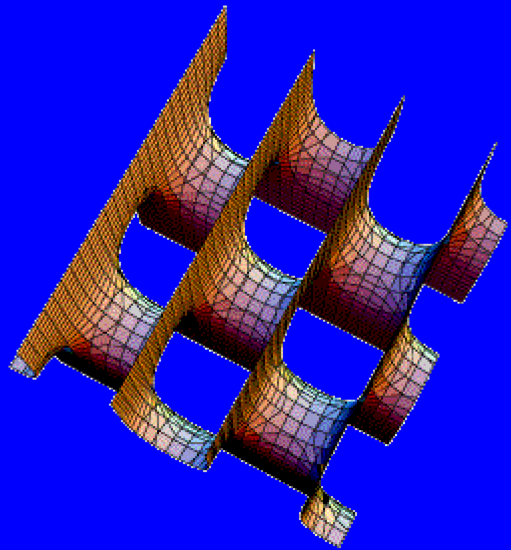
Nonlinearities important

$$F = \frac{1}{2}\int d^3x\left[Bu_{zz}^2 + KH^2\right]$$

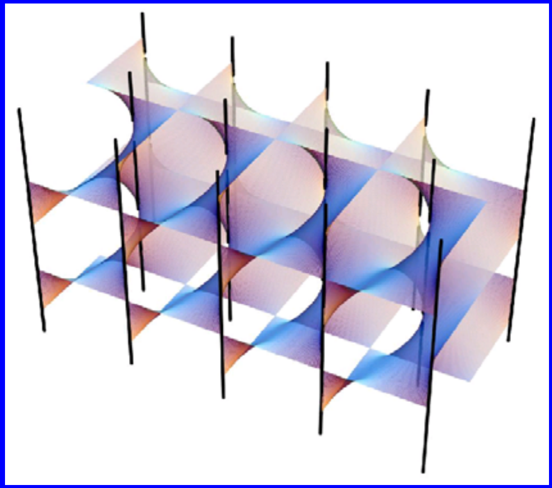
$$u_{zz} = \partial_z u - \left(\nabla u\right)^2; \quad H = \frac{1}{R_1} + \frac{1}{R_2}$$

Scherk's surface





Triply Periodic Smectic

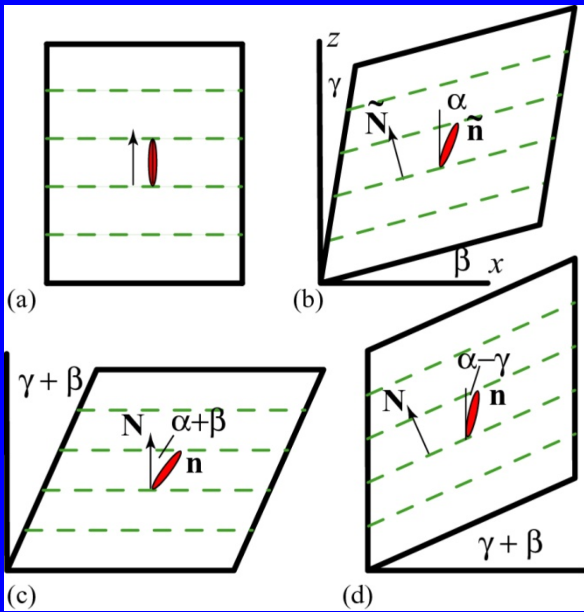


E.L. Thomas, D.M. Anderson, C.S. Henkee, and D. Hoffman, Nature 334, 598 (1988); S.P. Gido, J. Gunther,E.L. Thomas, and D. Hoffman, Macromolecules 26, 4506 (1993).

R.D. Kamien and T.C. Lubensky. *Phys. Rev. Lett. 82 (1999) 2892*.

C.D. Santangelo and R.D. Kamien, *Phys. Rev. E* **75** (2007) 011702.

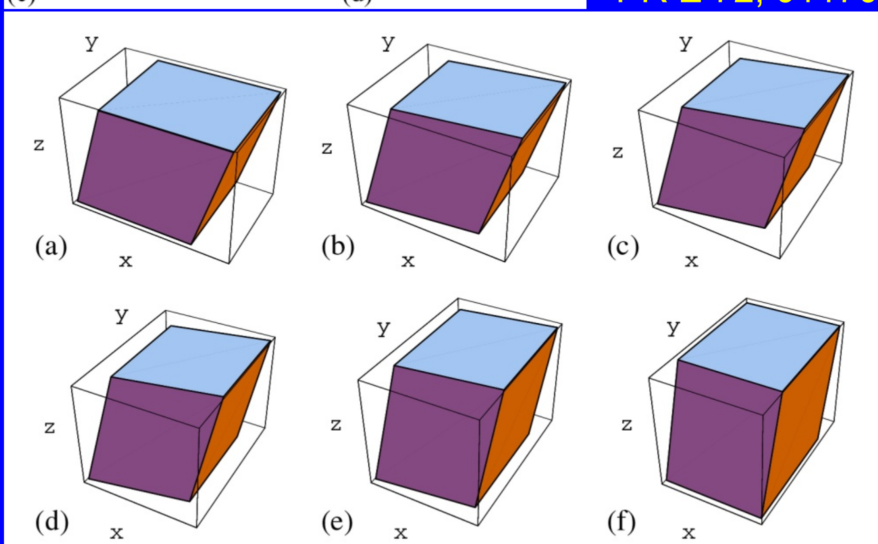
Soft SmC Elastomer

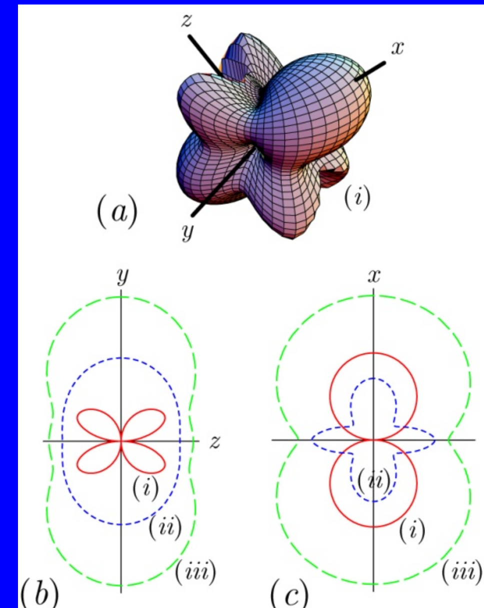


Director-strain coupling induces stain in Sm*C* phase

Olaf Stenull and T.C. Lubensky, PR E 74, 051709 (2006).

J. M. Adams and M. Warner, PR E 71, 021708 PR E 72, 011703 (2005).





Sound velocities of Sm*C* elastomer

Olaf Stenull and T.C. Lubensky, Phys. Rev. E 75, 031711 (2007).

Tilt in xz-plane; stress along y; Soft rotation of tilt direction to yz plane; then “hard” response

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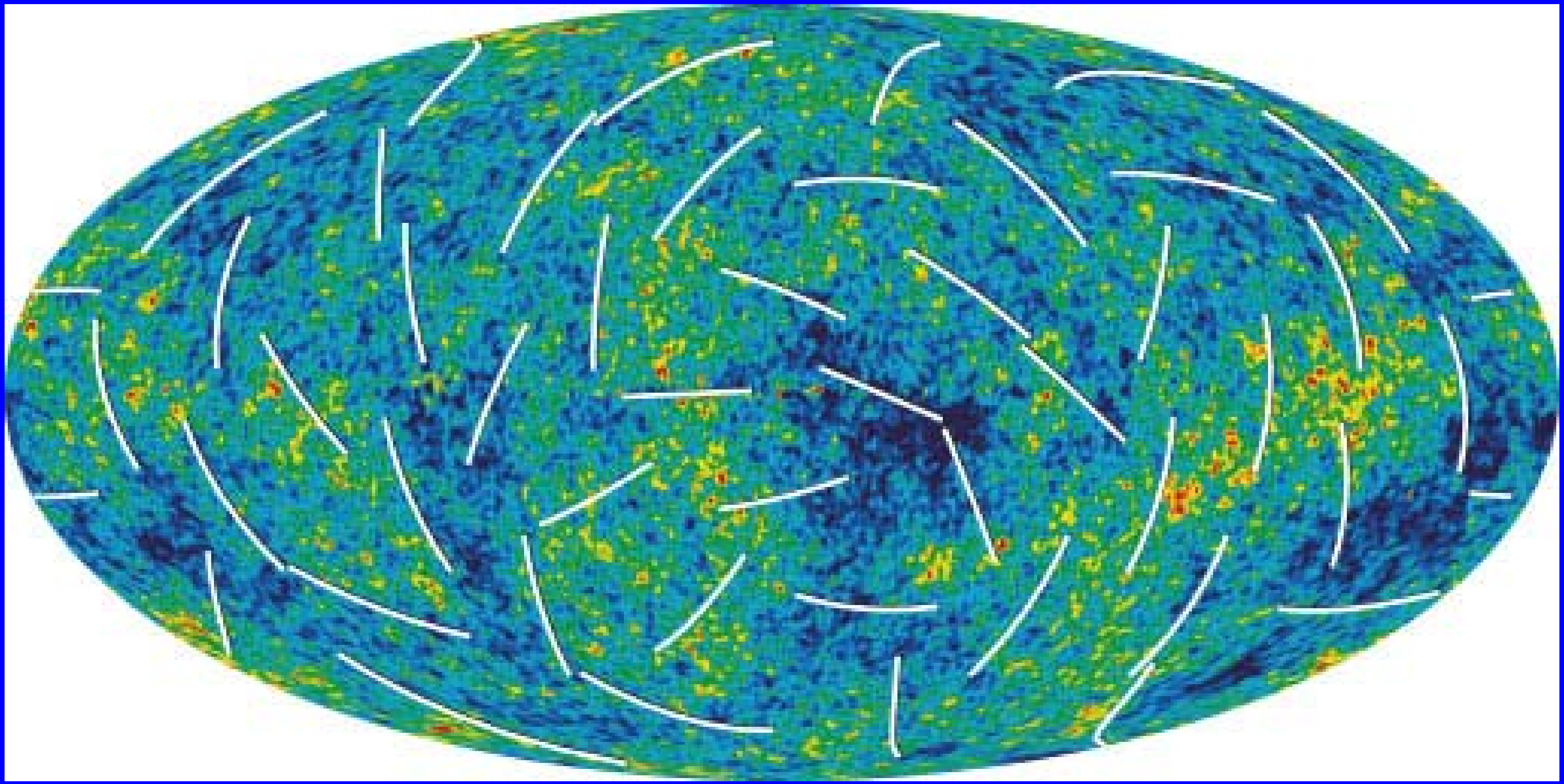
Cosmic Microwave Background

2D Nematic liquid crystal



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Polarization of the CMB