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Web Resources

The book's Web site (<http://www.macmillanhighered.com/physicalmodels1e>) contains links to the following resources:

- The *Student's Guide* contains an introduction to some computer math systems, and some guided computer laboratory exercises.
- *Datasets* contains datasets that are used in the problems. In the text, these are cited like this: Dataset 1, with numbers keyed to the list on the Web site.
- *Media* gives links to external media (graphics, audio, and video). In the text, these are cited like this: Media 2, with numbers keyed to the list on the Web site.
- Finally, *Errata* is self-explanatory.