# Optics of the Human Eye

## Goals of this lab

- **understand how the lens of the eye forms an image on the retina**
- **diagram image formation by single lenses and two lenses combined**
- **learn how the focal length of a lens is affected by the index of refraction of the medium**
- **Understand how a corrective lens helps the eye form images on the retina**

## Overview

The eye is much like a camera. The pupil is the muscle controlled variable aperture that regulates the amount of light entering the eye. The cornea on the front surface and the crystalline lens inside the eye combined together act as a lens that focuses light on the retina at the back of the eye. The retina is a membrane containing light-sensitive nerve cells known as rods and cones. The optic nerve connects to the retina, and there is a blind-spot at the point of attachment where there are no rod and cones. A good web-site to look at for an explanation of common vision defects is: http://hyperphysics.phy-astr.gsu.edu/hbase/vision/eyedef.html#c2

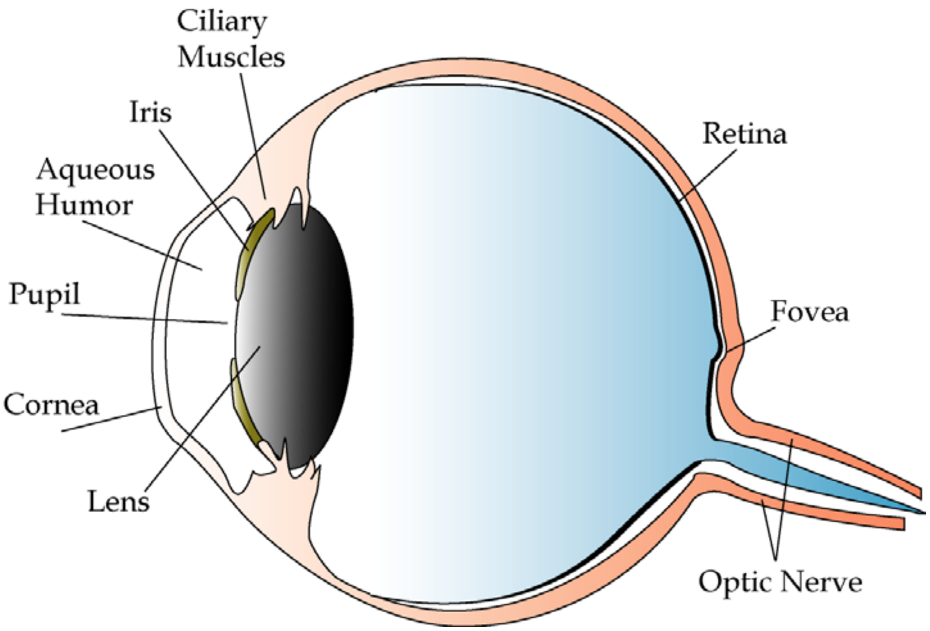


**Figure 1: The Human Eye in cross section. The large lightly shaded (blue) area represents the vitreous humor.**

The combined corneal lens and crystalline lens act like a single convergent or positive thick lens. Light entering the eye from an object is focused to form an inverted, real image on the retina. The eye can focus on objects at varying distances by accommodation, or changing the curvature of the crystalline lens. When viewing near objects, the muscles of the eye contract causing the crystalline lens to bulge in the center, shortening the focal length. The farthest distance at which the eye can accommodate is called the **far point**, and the closest distance is called the **near point**. The crystalline lens is simulated in our model with two positive lenses: +62 mm for near vision, and +120 for far vision which have to be switched in the model.

A lens bends the rays emerging from a point source of light (part of an extended object) to focus them at a point in space called the focal point. The geometry of the focal point of a thin lens is illustrated in **Figure 2**. The mid-plane of the lens is the vertical dotted line which is perpendicular to the optical axis. The top drawing shows how parallel light entering the lens perpendicular to the mid-plane focuses at the focal point. The bottom drawing shows how parallel light entering the lens off-axis, i.e., at an angle to the mid-plane, focuses at a point off-axis in the focal plane.

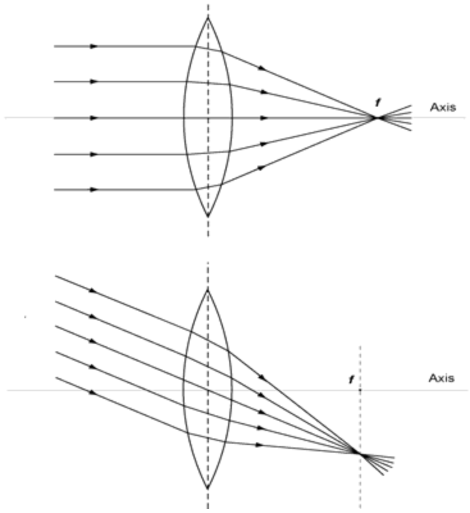


**Figure 2: The Focal point of the thin lens**

For a thin lens forming an image in air with the object and image also in air, the object and image distances, $S_o$ and $S_i$, respectively are related by the thin lens formula

$$\frac{1}{S_o}+\frac{1}{S_i}=\frac{1}{f} \quad (1)$$

We cannot apply equation (1) to a lens immersed in a medium other than air. The focal length of a lens in water is very different than it is for the same lens in air. This occurs because the wavelength of light is larger in water than air which changes the effective path length of the light.

The frequency of light does not change going from one medium to another, only velocity changes and as a result, the wavelength changes:

$f\lambda = c$ in air and $f\lambda_n = v = \frac{c}{n}$ in a medium with index of refraction $n$ .

As a result, $\lambda_n = \frac{\lambda_{air}}{n}$ , and similarly for distances light travels in the medium

To find a generalized form of equation (1), where media other than air may be present on either side of the lens, we must account for what occurs at each surface of the lens. Derived from Snell's

Law, the following equation relates $S_o$ and $S_i$ at a single lens surface. Here *n* is the index of refraction and is differentiated by means of subscripts for the lens or medium, and ***R*** is the radius of the surfaces of the lens

$$\frac{n_1}{S_o} + \frac{n_2}{S_i} = \frac{n_2 - n_1}{R} \tag{2}$$

Known as the Gaussian formula for a single spherical surface, equation (2) can be used to derive equations analogous to (1) for more complicated setups (i.e. multiple lenses, lens in mixed media) as well as the lens-makers formula. For our setup with the model eye, the two-lens mixed media system can be approximated to be a thick lens. Distances are measured to a point halfway between the lenses. $n_\alpha$ and $n_\Omega$ represent the indices of refraction of the media that light rays pass through before and after the lenses, respectively. This yields:

$$\frac{n_\alpha}{S_o} + \frac{n_\Omega}{S_i} = \frac{n_\Omega}{f} \; . \tag{3}$$

The lens-maker's Formula relates the focal length of a lens to the curvature(s) of its surfaces. This formula can be written for a thin double convex lens (which corresponds to the crystalline lens in the model) in a medium other than air. For this equation, both surfaces have the same radius:

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_{medium}} - 1\right)\frac{2}{R} \tag{4}$$

The focal length of a lens in water can be calculated from the focal length of the same lens in air, using equation (4). In the eye model, the corneal lens has a curved side which faces the object in air and a flat side in water which faces the retina screen. When the model is filled with water the focal length of the corneal lens is still the focal length in air. But the focal length of the crystalline lens is the focal length in water which can be calculated from equation (4). The index of refraction of the human crystalline lens is 1.40 at the center and 1.38 along the edge.

Pre Lab Question 1: Derive the relationship between $f_{water}$ and $f_{air}$. Show that the ratio of the focal length of the crystalline lens of the eye in water to the focal length in air is about 7.6 .

Far-sightedness or hyperopia is a condition where the eye-ball is shorter than normal, so that the retina is too close to the lens system. A near-sighted or myopic person, on the other hand, has a longer than normal eye-ball making the retina too far from the lens of the eye for the image of a distant object to form on the retina. These vision defects are simulated by moving the position of the retina in the model. (The corrective lenses focus light on the actual position of the retina as in the real eye.)

## Graphical Analysis of Image Formation

The thin lens formula provides an analytic relationship between $S_o$ and $S_i$, but a graphical approach known as ray-tracing provides a visual picture of how the image is formed and can also give an estimate of where the image will form. Ray tracing relies on two principles:

1) Rays going through the center of the lens do not bend.
2) Rays coming from the focal point emerge from the lens perpendicular to the mid-plane (and vice versa).

Consider a lens with focal length $f$ and an object at distance $3f$ from the lens, as shown in **Figure 3**. The object is a distance $S_o = 3f$ to the left of the midplane. Light rays are bent at the curved surfaces of the lens and converge to form an image at a distance $S_i$ on the other side of the lens.

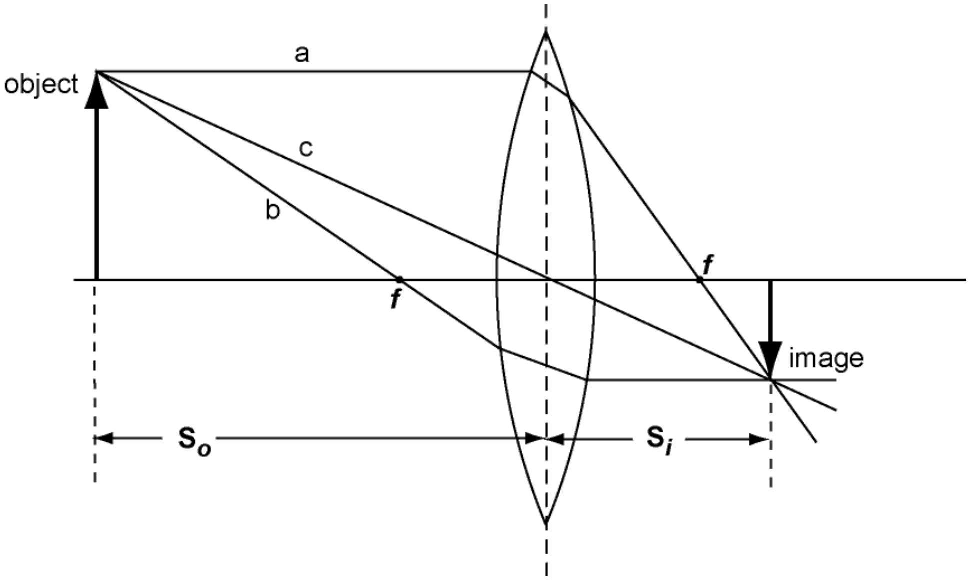


**Figure 3: A real image formed with object at $S_o = 3f$**

In Figure 3, ray c goes through the center of the lens and rays a and b pass through a focal point. (For a more detailed explanation of image formation, please refer to the optics section of your text book.)

The magnification of the image, m is defined as the ratio of the height of the image to the height of the object.

$$m = \frac{h_1}{h_2} = \frac{S_i}{S_o}$$

**Pre-Lab Question 2:** Use ray tracing as in Figure 3, to draw the image of an object placed 2 focal lengths in front of a thin lens. Construct the image of the same object placed one half a focal length in front of the same lens. Find the magnification in both cases.

When two lenses are combined, the image formed by the first lens becomes the object for the second lens. This image is called an intermediate image and may be located on the same side of the second lens as the object or on the other side of the second lens. Ray tracing with multiple lenses requires some tricks with the definitions of object and image distances that we cannot study here. In lab you will have a ray box and special (cylindrical) lenses that will allow you to create a picture of the way two lenses focus light.

The corneal lens used in this experiment is plano-convex, meaning that one face is flat and the other bulges outward in the middle. For this special case in air, the lens-maker's formula reduces to

$$f = \frac{R}{n_{lens} - 1} \tag{5}$$

where $R$ is the radius of curvature of the curved face of the lens.

**Prelab Question 3:** A jello "lens" is made by pouring and cooling the jello in a spherical bowl which has a radius of 10 cm. If the index of refraction of the jello is 1.40, find the focal length of this jello "lens"

### Measurements of the images

We will explore the relationship between the sizes of an object and its image as well as their distances from the corneal and "crystalline" lenses in the eye model. The image and object distances are measured from the outer rim of the black container as shown in Figure 4. The rim is located about midway between the corneal and crystalline lenses so that it can be used as the center of the combination of the two lenses to find the effective focal length.

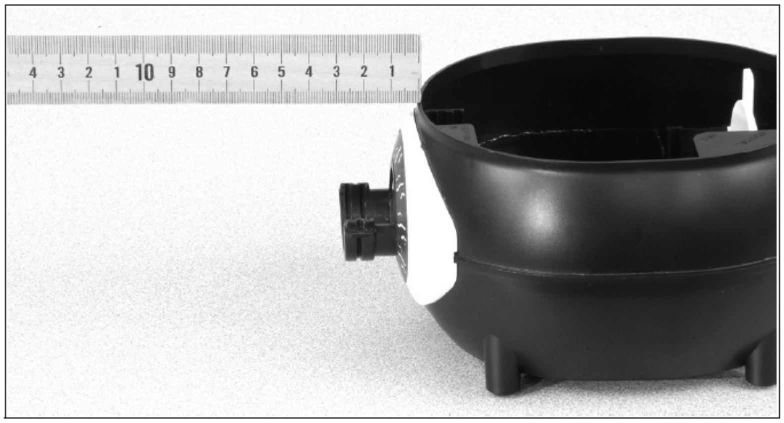
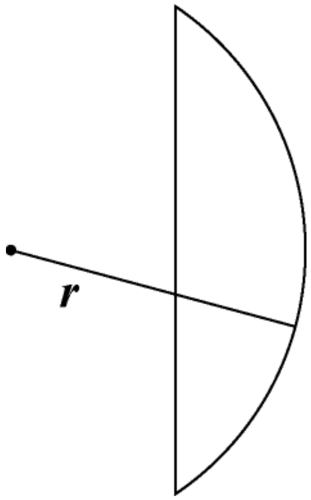


**Figure 4: Measuring the object distance with the eye model**

**Questions:**

1. Why can we treat the two lens system in the eye model like a "thick" lens?

2. *Answer this before you do the lab procedure:* The lens below has an index of refraction of 1.5 and a radius, r of 0.5 meters. Draw the path of a ray of light through the lens (starting from the left side of the page). Use Snell's law to find the focal length in air and in water (n= 1.33, water). Be sure to use small angle approximations, meaning that tan can be substituted in for sin of the small angle:



**Question 2 Figure- find the focal length using Snell's Law**

3. When the eye model is used to simulate far vision, why do objects located over a large range of distances all focus on the retina?

| Useful Data for the Eye Model | | |
|---|---|---|
| | Focal length | Index of refraction |
| Corneal lens | *f*= 142 mm; R= 71 mm, | 1.52 |
| Crystalline lens | *f*=+62mm (near accommodation) | 1.58 |
| Crystalline lens | *f*=+120mm (far accommodation) | 1.58 |
| Water | ------------------------------------ | 1.33 |

**Procedure Part I: Images Formed in the Eye**

1. Find the focal point of the semi-circular lens using the ray box and Snell's law.

2. Find the focal point of the positive double convex (DCX) lens and the negative (concave) lens using the ray box .

3. Place the DCX lens in front of the ray box. Place a second positive lens close to the first lens and adjust the position of it until the light is focused. Repeat with the negative lens as the second lens.

4. Sketch the object and lenses, the (intermediate) image formed by the first lens and the final image for the positive-positive and positive–negative lens combinations in the previous step.

5. Use the eye model with no water in it for this part. Put the retina screen in the middle slot, marked NORMAL and a +62 mm lens in the slot marked SEPTUM.

6. Focus the "object" and sketch the image formed on the retina. The light box with a pattern works best for this step. Move the "object" from side to side and describe what happens. Measure the object distance and the corresponding image distance on the retina screen.

7. Fill the eye model with water to within 1 or 2 cm of the top. Move the eye model until it forms an image on the retina in NORMAL position. Measure the object distance and the image distance.

8. Find the image of the light-board at the end of the hallway with the +120 mm lens in the Septum slot. Measure the object distance and the image distance.

9. Calculate the effective focal length of the corneal-crystalline lens system from your measured image and object distances for the near vision and far vision cases.

## Procedure Part II: Corrective lenses

1. Start with the retina screen in the NORMAL position. Looking at a nearby light source focus the object on the retina screen.

2. Move the retina screen to the back slot labeled NEAR. Find a lens that brings the image into focus when you place it in front of the eye in slot 1 (farther from the rim of the model). Observe what happens when you move the eye far from the light source. Record the focal length of the corrective lens.

3. Move the retina screen to the front slot, labeled FAR. Find a lens that brings the image into focus when you place it in front of the eye in slot 1. Observe what happens when you move the eye model far away from the light source. Record the focal length of the corrective lens.

4. Find the effect of the cat's eye pupil and the round pupil on the retina image.

5. Using measured object and image distances, calculate the effective focal length of the eye model with the corrective lenses for the NEAR and FAR retina screen positions (near and far sighted simulations).

## Procedure Part III: Astigmatism

1. Astigmatism is a common vision defect that causes light rays propagating in two perpendicular planes to have different focal points (see astigmatism chart). This condition can be simulated in our eye model. First set the model up for near vision (62 mm lens in the septum slot) and focus a nearby light source (preferably LED lights) on the retina screen.

2. Now place the -128 mm cylindrical lens in slot A, keeping the side of the lens marked with the focal length toward the light source. This will give the eye model astigmatism. Describe the image now formed on the retina screen.

3. Rotate the cylindrical lens and describe what happens to the image. This shows that the astigmatism can have several orientations depending on they eye lens system's defects.

4. To simulate how astigmatism can be corrected with eyeglasses, place the +307 mm cylindrical lens in slot 1. Be sure to keep the side of the lens marked with the focal distance toward the light source. Rotate the corrective lens and describe what happens to the image. What is the angle between the two cylindrical lenses when the image is sharpest

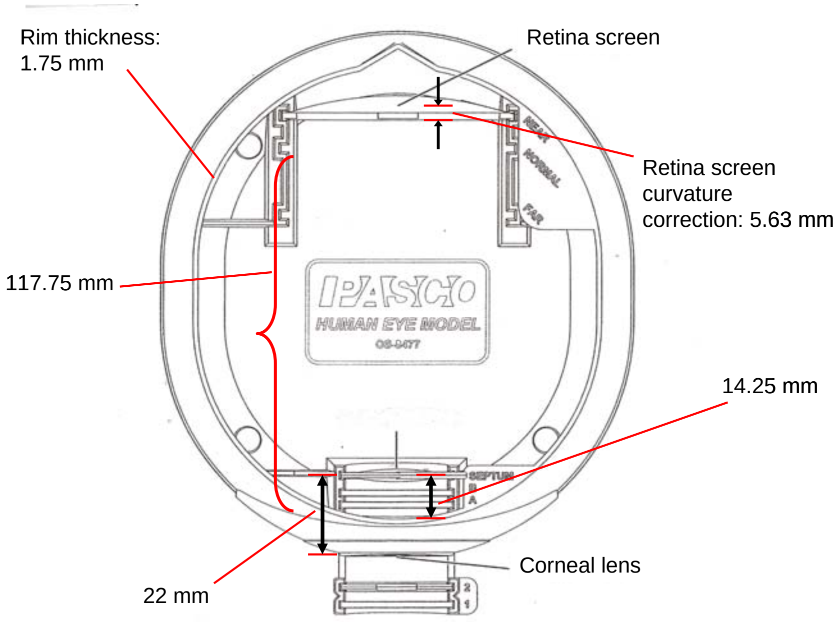


**Figure 5: Diagram of the eye model, with distance measurements**