Radioactive Decay

Goals of this lab

- understand the concepts of half-life and the decay constant
- measure the decay constant of a radioactive source

WARNING

The radioactive source used in this experiment is quite harmless when properly used; nonetheless, handle it as little as possible.

Although the low currents available from the high voltage supply are not dangerous, it is possible to obtain an uncomfortable shock from this supply. Avoid touching either the terminals of the supply or the wires that connect the voltage source to the voltmeter.

Overview

In this experiment we will study the radioactive decay of the artificial radioactive species Barium-137, ${}^{137}_{56}Ba$. This species is made by the beta decay of a neutron in Cesium-137, ${}^{137}_{55}Cs$. In beta decay a neutron decays into a proton, a high-energy electron, and an anti-neutrino:

$$ {}^{137}_{55}Cs = {}^{137m}_{56}Ba + e^{-} + \overline{\nu}_e \qquad (1) $$

However, the Barium-137 is created in an excited (or metastable) state, as indicated by the "m" in equation (1), and must emit energy in the form of a gamma ray:

$$ {}^{137m}_{56}Ba = {}^{137}_{56}Ba + \gamma \qquad (2) $$

The source you will be using in lab will be the excited Barium-137. An isotope generator of Cesium-137 naturally generates excited Barium-137, and your TA will use a solution to flush out the excited Barium-137 for your sample.

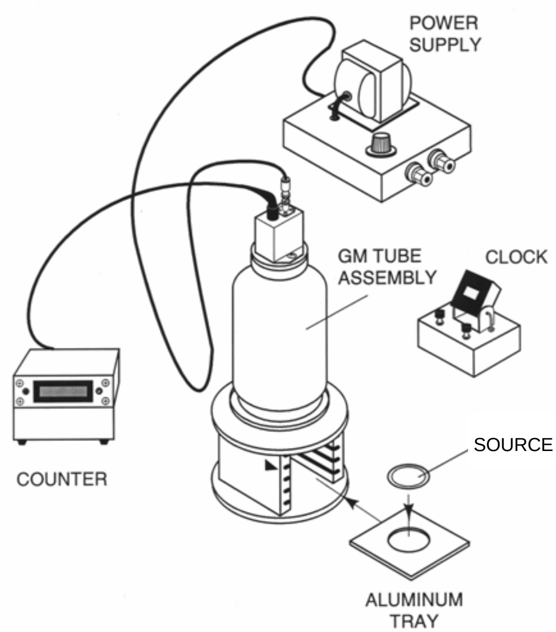

Figure 1: Lab Setup

The rate at which a particular radioactive material disintegrates in a given time interval is called the decay constant, λ. The decay constant is characteristic of the particular nuclear species and is independent of all external physical and chemical conditions. Radioactive decay is a statistical process, for one cannot predict when a particular nucleus will decay. The most that one can say is that if a nucleus is as of yet undecayed the probability that it will decay in the next small time interval dt is λdt.

When a large number of nuclei of a single species are studied, statistics can be used to make definite predictions about the average number of decays that occur in a small time interval. Assume at time t there are $N(t)$ undecayed nuclei in some sample. If one studied many identical samples, then on average the number of decays that occur in the interval dt is $\lambda N(t)dt$. Thus, we get

$$dN = -\lambda N(t)\,dt \qquad (3)$$

Equation (3) may be integrated to give

$$N(t) = N_0 e^{-\lambda t} \qquad (4)$$

where N_0 is the number of nuclei present at time $t = 0$.

A radioactive species is often characterized by its half-life, $T_{1/2}$, which is defined as the time it takes for one-half of the nuclei to decay. Using Equation (4) we see that $T_{1/2}$ satisfies

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda T_{1/2}} \qquad \text{or} \qquad \ln\frac{1}{2} = -\lambda T_{1/2}$$

which leads to the formula

$$T_{1/2} = \frac{0.693}{\lambda} \qquad (4)$$

> **Prelab Question 1**: In Part I of this lab, you'll be using dice to model radioactive decay. One side of six-sided dice has been painted black. When you roll the dice, if a die lands with the black side facing up, it means that die has "decayed." The decay constant of these dice is 1/6, since the decay constant is the fraction of the sample that decays in a time interval. Given the decay constant of the dice, what is the half-life of the dice?

The average lifetime, T_a, of any single nucleus may be computed by multiplying $\lambda N(t)dt$, the number of nuclei disintegrating in time dt, by the time t for which the decaying nuclei exist, summing these products over all nuclei, and then dividing by N_0, the total number of nuclei at the start. We want to average over a large number of atoms, so the sum becomes an integral. The average lifetime of a single nucleus can be found with the following expression:

$$T_a = \frac{1}{N_0}\int_0^\infty t\lambda N_0 e^{-\lambda t}dt = \frac{1}{\lambda} \qquad (5)$$

When studying radioactivity, one observes the number of decays over some time interval and not the number of surviving nuclei. The number of decays that one observes in some time interval is the number of decays that occur in that interval multiplied by a factor, *f*, characterizing the efficiency of detection. Thus, in a small, finite, time interval Δt the number of "counts" in the detector is $N_C = f\lambda N(t)\Delta t$.

Substituting for *N(t)* and taking the natural logarithm of this equation yields

$$\ln N_C = \ln(f\lambda N_0\Delta t) - \lambda t \qquad (6)$$

which shows that the logarithm of the number of counts in a fixed time interval is a linear function of the time at the center, say, of that interval.

The formulas given above refer to the average of identical observations hypothetically performed on a large number of identical systems. Any single measurement can differ greatly from the expected result. Statistical arguments show that the order of magnitude of the fluctuations is $\sqrt{N_C}$ where N_C is the expected value. Thus, the uncertainty associated with a counting measurement is the square root of the number of counts observed.

> **Prelab Question 2**: You are given the equation $N(t) = N_0 e^{-\lambda t}$. If you graph the number of counts on the y-axis and the time on the x-axis, you will get the graph of a decaying exponential. If you wanted to get a graph of a line, what would you want to plot on the x-axis and on the y-axis? See the **Linearizing Data** section of the lab manual for help. What would be the slope of the line? What would be the intercept of the line?

The Geiger Counter

We detect the products of radioactivity using a Geiger-Mueller tube, which is part of a Geiger counter. The tube consists of a wire mounted coaxially within a metal cylinder but electrically insulated from it. The cylinder is filled with a combination of gases at very low pressure. One end of the metal cylinder has a very thin mica window that allows photons and electrons to enter.

In operation a high voltage (approximately 1 kV) is applied between the thin wire and the metal cylinder. The wire is the anode, positive with respect to the cylinder.

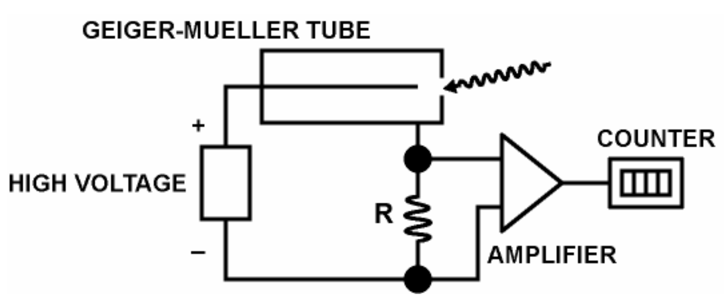

Figure 2: Diagram of a Geiger-Mueller Counter

When an electron or gamma ray enters the tube it can ionize an atom of the gas. The electron produced is rapidly accelerated toward the anode by the strong electric field. On its way it gains enough energy to ionize additional atoms along its path --- the process cascades until a large number of electrons and positive ions are produced. The effect of the cascade is to produce a large electron current flowing onto the anode and through the circuit where it triggers the external circuit: a "count" is recorded. The current also produces a voltage drop across the resistor, which decreases the voltage across the tube itself, and causes the cascade to stop.

For the first part of this lab you will be using dice to model radioactive decay. You will be performing the function of the Geiger counter and checking for "decays." Each six-sided die represents a radioactive nucleus. One side of the die is painted black, which represents the die's "decaying." When you roll the dice, some will land with the black side facing up. When you remove those dice from the sample, you act like the Geiger counter reporting the number of dice which "decayed" in that time interval.

The decay constant is the fraction of the sample that "decays" in each time interval. You can calculate the decay constant of the dice as $\lambda = \frac{\Delta N / N}{\Delta t}$, where ΔN represents the number of "decays" counted, N represents the number of dice rolled, and Δt represents the time interval (the roll). Since only one side of the die is painted black, this makes the decay constant for 1 die 1/6, or 0.17. Using Equation (4), we find that the dice have a half-life of about 4.2 rolls.

Prelab Question 3: The concept of decay applies to many things besides radioactive materials. The following table lists the value of Homer Simpson's stock portfolio for the last 5 years. The problem is simplified because there is no uncertainty in the news from Homer's broker, and there is no "background" to subtract because Homer has no other money. Find the decay constant of Homer's money by finding the average yearly fractional decay; refer to the analysis using averages mentioned in Part I. Use the decay constant to calculate the half-life of the Simpson Estate.

Year	Dollar Value
0	10,000.00
1	9,000.00
2	8,100.00
3	7,290.00
4	6,561.00
5	5,904.90

Questions Part I: Dice "decay"

1. Which analysis method, using averages or statistics, produced the most accurate result?

2. What would you change to make the statistical method give a more accurate result?

Procedure Part I: Dice "decay"

1) Count the total number of dice.

2) Roll the dice, making sure that no dice are resting on each other or on edge.

3) Count total number of dice with the black side facing up; that's your "decay" total for the roll.

4) Remove “decayed” dice from the sample.

5) Repeat steps 2) – 3) until all of the sample is gone.

Part I Analysis using averages

1) Calculate the fractional “decay” for each roll by dividing the number of “decays” per roll by the number of dice rolled. Watch out--the total number of dice you roll each time is not constant!

2) Average the fractional “decays” to get your decay constant λ.

3) Calculate the half-life using $t_{half-life} = \frac{0.693}{\lambda}$.

Part I Analysis using statistical method

1. Take the natural log of the “decay” counts. You may notice that some of your calculations involve dividing by zero --- that’s no good! To correct this, adjust your data by adding 1 to the “decay” counts and then take the natural log of the adjusted data.

2. Graph the adjusted natural log of the “decay” counts versus the rolls.

3. Find the best fit line to your data and get the slope and intercept of that line. Also find the r^2 value for the line.

Questions Part II: Barium-137 decay

1. Did your value for the decay constant come within error of the accepted value?

2. Why can’t you use the analysis method using averages for the Barium-137? What information don’t we have about the radioactive substance?

3. If you record 80 counts in 2 minutes, with a background count of 20 counts per minute, what is

   (a) the corrected number of counts per minute?

   (b) the uncertainty in the corrected number of counts per minute?

   (c) the uncertainty in the total number of counts per minute?

Procedure Part II: Barium-137 decay

1) The counter is turned on when the red display numbers on the scale are lit up. Make sure the switch on the back is set to CTR.

2) Set the voltage to 950 volts (as marked on metal chassis).

3) Even with no radioactive sample near the Geiger tube, counts are recorded. This is known as "background count" and is caused by cosmic rays or radioactive sources around the laboratory. Any "measurement" of a radioactive sample includes this background, which must be subtracted from the measurement to give a true value of the radioactivity of the sample. Determine this background by taking counts for four minutes. Make this determination before and after you take your measurements, and use the average divided by four as the background count for each one-minute interval.

4) Take the Barium-137 supplied by your instructor and place it under the Geiger tube. Record the readings of the counter for one-minute intervals, waiting one minute between counting intervals. Record the count by flicking the counter's CNT switch to the off position. Thus, you will start counting at t = 0, 2, 4, 6 ... etc. minutes, and stop counting at 1, 3, 5, 7 ... etc. minutes. Use the small clock to keep track of the total time elapsed from the time you begin taking counts. Continue this process for 10 to 15 minutes.

Analysis Part II: Barium-137 decay

1) Correct all of your counts for background radiation.

2) Plot the corrected counts per minute against time. Find two points on your curve where one point corresponds to twice the number of corrected counts of the other point. Draw perpendicular lines from these points to each axis to determine how long it took for the number of counts to decrease by a factor of 2. This is your observed half-life of Barium 137, $T_{1/2}$.

3) Find at least two other sets of points and do the same thing you did in step 2). Average these numbers to get an average value for your observed half-life.

4) Plot the natural logarithm of the corrected counts per minute against time. Indicate the uncertainties in your value of the counts per minute by error bars; remember, the uncertainty in any count, N, is $\sqrt{N}$. The uncertainty in the natural log may be estimated by differentiating:

$$\Delta(\ln N) = \frac{d(\ln N)}{dN}\Delta N = \frac{\Delta N}{N} \qquad (7)$$

5) Plot your data. Draw the straight-line that is the best fit to your points. From the slope of this line compute the decay constant λ. Estimate the uncertainty in λ.

6) Compute the half-life $T_{1/2}$ from the slope of the best-fit line, λ. Compare this value to your observed half-life.