Focusing and Images: the Thin Lens

Goals of this lab

- **understand how lenses focus light**
- **learn how to calculate the focal length of a lens with the Lens-maker's formula and by focusing images**

For more information on lenses, refer to the Powerpoint file at www.physics.upenn.edu/~uglabs

Overview

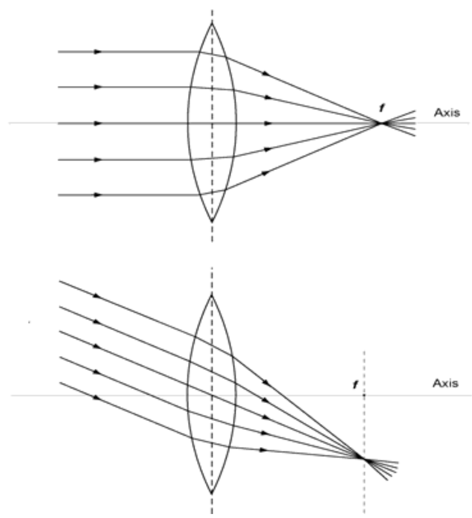

Figure 1: The Focal point of the thin lens

A lens is an optical device that takes the rays emerging from a point source of light (typically part of an extended object) and bends each ray differently to bring them together, or focus them, at a point in space called the focal point.

The geometry of a thin lens is illustrated in **Figure 1**. The mid-plane of the lens is the vertical dotted line which is perpendicular to the optical axis and passes through the optical center. The top drawing shows how parallel light entering the lens perpendicular to the mid-plane focuses at the focal point. The bottom drawing shows how parallel light entering the lens off-axis, i.e., at an angle to the mid-plane, focuses at a point off-axis in the focal plane.

For a thin lens the object and image distances (S_O and S_i respectively) are related to the focal length f by the thin lens formula

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \qquad (1)$$

where f is the focal length of the lens. This can also be written (using a little algebra)

$$S_i = f\left(1 + \frac{S_i}{S_O}\right) \qquad (1a)$$

This expression is the equation for a straight line in standard slope-intercept form. We will use S_i as the y-axis variable and the ratio $\frac{S_i}{S_o}$ as the x-axis variable. (The focal length f is both the slope and the intercept.)

Prelab Question 1:
Derive equation (1a) from equation (1).

Graphical Analysis of Image Formation

The thin lens formula provides an analytic relationship between S_o and S_i, but ray-tracing often provides better understanding and can give an estimate of where the image will form. Consider a lens with focal length f and an object at distance $3f$ from the lens, as shown in **Figure 2**. The object from which the light emerges is a distance S_o to the left of the mid-plane. Light rays are bent at the curved surfaces of the lens and converge to form an image at a distance S_i on the other side of the lens.

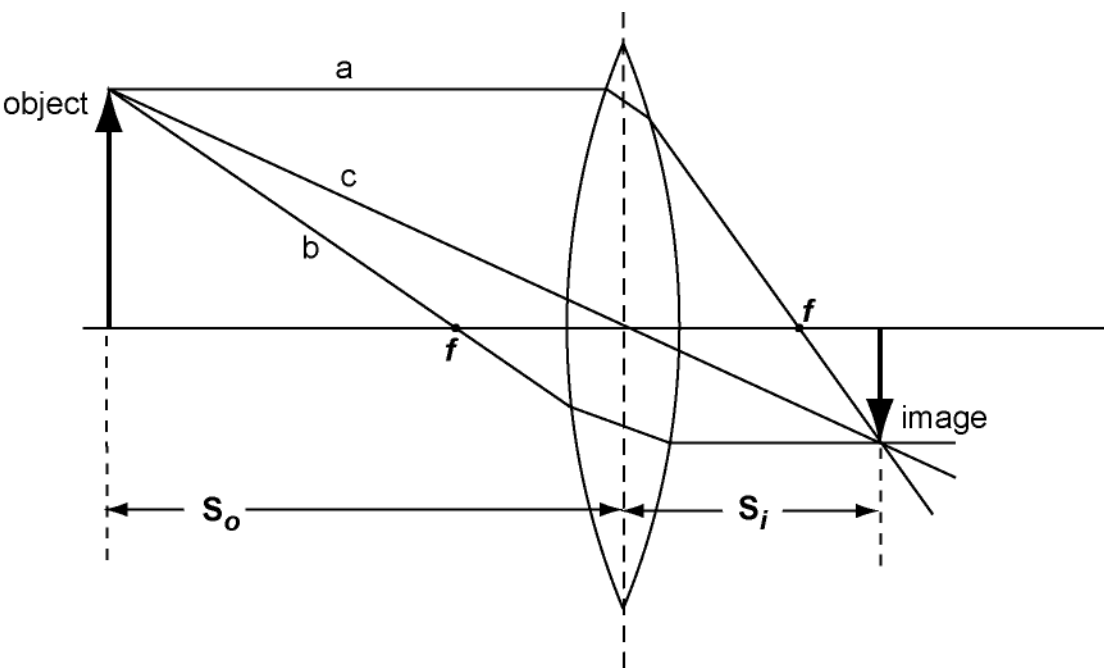

Figure 2: A real image formed with *S* = 3*f*

Ray tracing relies on these principles to construct an image:

a) A ray parallel to the axis (a), passes through the far focal point.

b) A ray through the near focal point (b), bends and exits parallel to the axis.

c) A ray through the center of the lens (c), is unchanged in direction.

As a result, an object placed at the focal point will not form an image because the outgoing rays are all parallel, and do not converge. Rays coming from a very distant object can be considered parallel and will converge at the focal point.

In Figure 2 ray c goes through the centre of the lens and rays a and b pass through a focal point. The image has to be located where all three rays intersect. (For a more detailed explanation of image formation, please refer to the optics section of your text book.)

The magnification, m, is defined as the ratio of the height of the image to the height of the object.

$$m = \frac{h_1}{h_2} = \frac{S_i}{S_o} \qquad (2)$$

Equation (1a) can be simplified by replacing the ratio $\frac{S_i}{S_o}$ with *m*.

> **Prelab Question 2:** Refer to the example in Figure 2 for this question.
>
> **a)** If the focal length of the lens is 0.3 meters, find the image distance.
> **b)** Find the magnification.

Index of Refraction

The index of refraction is defined as

n = physical depth / optical depth= d/ d'

The physical depth of the fish in Figure 3 is the distance from the surface of the water to the fish. The optical depth of the fish is the <u>apparent</u> distance from the surface of the water to the fish. Light bends as it passes from air into water, making the apparent distance shorter than the actual distance.

In this experiment we measure the index of refraction by looking through the plastic lens and comparing the position of the mark we focus on to the actual position. The top of the lens in this measurement is equivalent to the water surface in Figure 3.

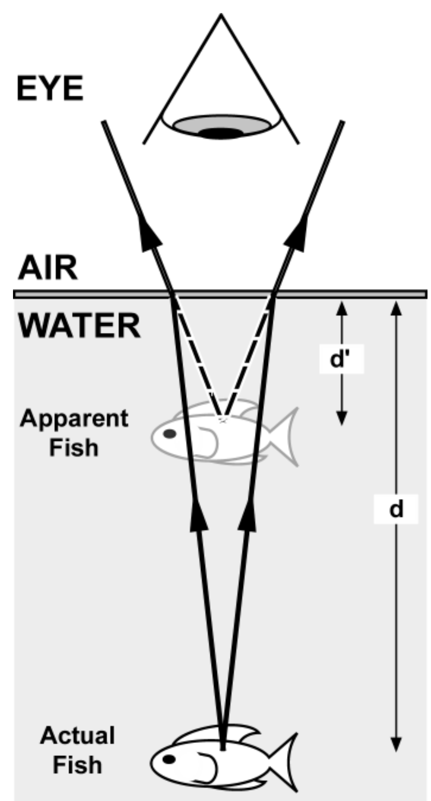

Figure 3: Apparent Depth

Prelab Question 3: Graph the Voltage-Current data in Table 1 below by hand. Plot the Voltage on the y-axis and Current on the x-axis. Label each axis, include proper units and show error bars. The uncertainty in a single voltage measurement is ±3% and the uncertainty in a current measurement is ±1%. Describe the relationship shown by the graph. Find the slope and uncertainty in the slope.

Table 1: Voltage vs. Current

Voltage (Volts)	Current (amps)
0.50	1.00
1.00	2.30
1.50	3.50
2.00	4.70
2.50	5.80

Procedure

Part I: Find the Index of Refraction

(If microscopes are in use, go on to Part II and come back.)

1) Place the plastic plate with cross-lines on the microscope stage. Focus on the cross-lines and zero the dial caliper.
2) Place the lens on top of the plastic plate, flat side facing up. Re-focus the microscope on the cross-lines viewed through the lens and record the position.
3) Make a mark on the flat side of the lens and focus the microscope on the mark. (Please turn off the dial caliper display when you're done.)

Part II: Finding the focal length using image formation

1) The object will be a pattern inserted on the front of the "cube light".
2) Measure three or more different combinations of object and image distances for which you can find a clear focused image on the screen.
3) Take your lens out into the hallway, and find the location of the image of the lights on the board at the end of the hallway.

4) Measure one S_i with the curved surface of the lens facing the light-board.
5) Repeat the measurement with the flat surface of the lens facing the lights.
6) Place the lens 25cm from the object. Where is the image?
7) Set the lens about 2 meters from the object with the flat surface toward the object. Find the image. Now place an aperture in front of the lens to restrict rays to the central portion of the lens, and note the image again. Explain the changes in the definition of the image.
8) Place the other aperture in front of the lens which covers the center part of the lens and explain what you observe.

Questions to be answered in your report

1) How does reversing the lens with the flat side or curved side facing the distant object affect the location of the image? By doing this, can you estimate where the center of the lens is located?

2) How do the images differ when using the different apertures in front of the lens ?
3) How much error is introduced by assuming $s_o = \infty$, if the "distant object" is located at a distance of 100 times the focal length of the lens.

<u>Appendix: "Quick Notes" on Graphs</u> (See the IntroLab Manual section 1: Making Graphs)

General Guidelines for Graphs:

Do you have enough points to characterize your curve shape?

For example, if you have 2 points, you can <u>always</u> draw a line between them, similarly 3 points define a parabola, 4 a cubic function, and so forth. Therefore, to make a meaningful fit, you need the number of data points to *exceed* the minimum number of points necessary to define the shape. In general, the more data points you collect, the better the fit becomes.

Graph Presentation:

- Titles should tell the reader exactly what is graphed.
- Remove any stray lines, points, and any other unintended additions by the computer that do not add to your graph. (For example, many programs will add meaningless piecewise lines connecting your data points.)
- Remember the error bars! They give your fit meaning.

Figure 6: Example using Excel-A linear fit of a data set with error in both x and y. The error in x is constant, the error in y is a percentage of the value and hence varies. Note: This table represents a <u>good</u> format for recording and displaying data. Your TA will be very happy to see this.

Figure 6: **Example of a linear fit**

X (units)	X error (±)	Y	Y error (±%)
0	0.02	0.7	5
0.1	0.02	1.3	5
0.2	0.02	2.66	5
0.3	0.02	3.4	5
0.4	0.02	4.26	5
0.5	0.02	5.2	5
0.6	0.02	6.34	5
0.7	0.02	7.04	5
0.8	0.02	8.62	5
0.9	0.02	9.52	5

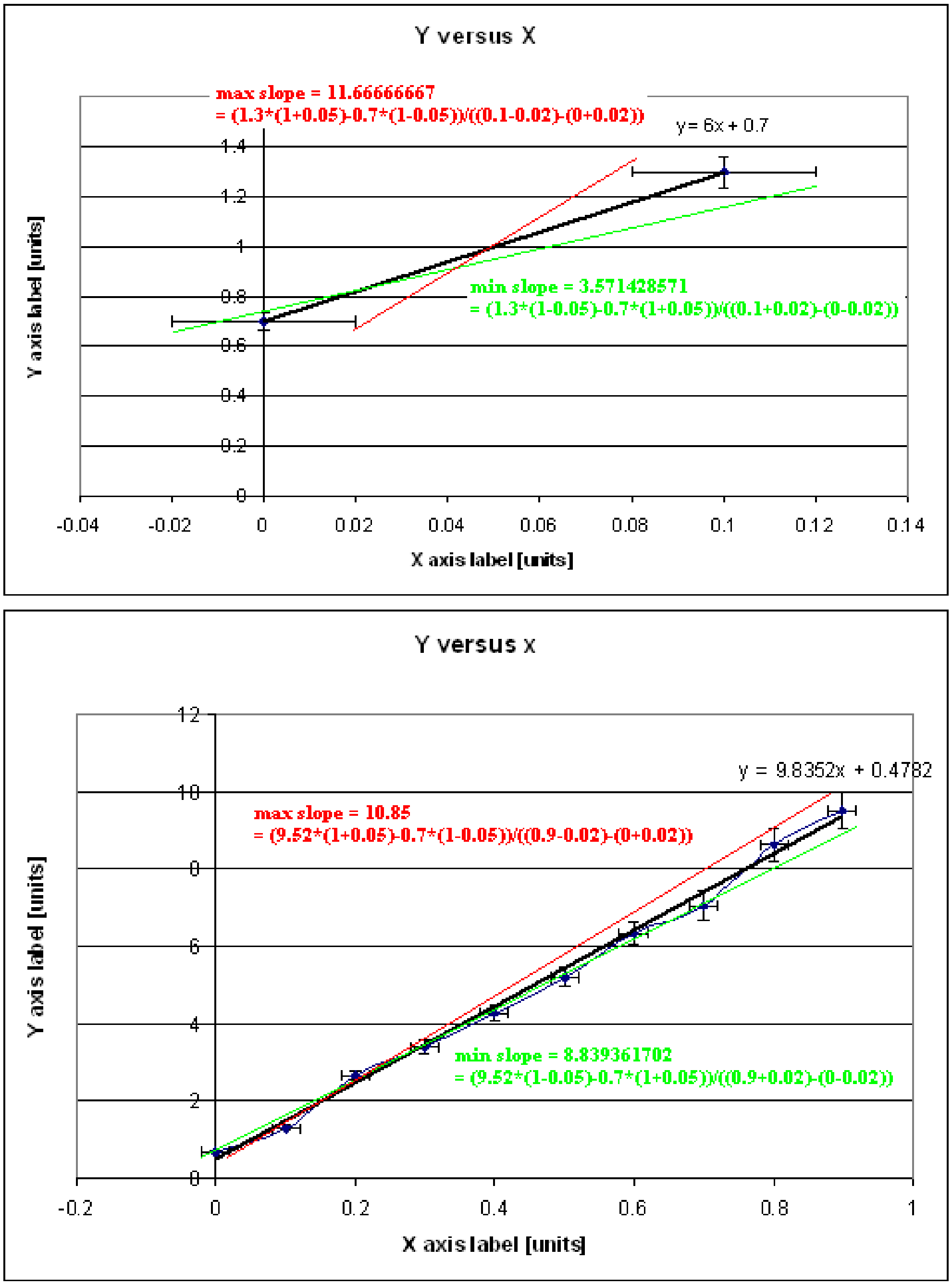

Figure 6

To determine the error in the slope use the max and min slopes, which are the most extreme lines that you can fit to the data in your judgement.

The top graph plots only the first 2 points of the data. The max slope is 11.67 and min slope is 3.57. So the slope is (11.67+3.57)/2 ± (11.67-3.57)/2 = 7.62 ± 4.05 (range/2 method). The lower graph plots all the data. Similarly the slope is (10.85+8.84)/2 ± (10.85-8.84)/2 = 9.84 ± 1.00.