(5A) Higher-dimensional propagators

Consider the Euclidean Green’s function in $d$ Euclidean spacetime dimensions:

$$S^{(d)}(x) = \frac{1}{(2\pi)^d} \int d^d k \frac{\exp(-ik \cdot x)}{k \cdot k + \mu^2}.$$ 

Note that the sign of $\mu^2$ is plus because we switched to the mostly-plus signature. Incidentally, the analytical continuation of $S^{(d)}$ to imaginary values of one coordinate is related to the Minkowski Green’s functions. Answer the following questions:

- Show that $S^{(d)}$ only depends on $x \cdot x$ i.e. its rotational invariance.
- For a fixed but arbitrary $d, \mu > 0$, compute the asymptotic behaviors of $S^{(d)}$ for $r = |x| \to 0$ and $r \to \infty$.
- Compute $S^{(d)}(x)$ for $d = 1$ and $d = 3$ exactly. For $d = 3$, find the relation of $S$ and the Yukawa potential.

(5B) Two to two scattering at the tree level

Consider a theory with three real scalar fields $\phi_{1,2,3}$ and two types of interactions:

$$\mathcal{L} = \frac{3}{2} \sum_{i=1}^{3} (\partial_\mu \phi_i \partial^\mu \phi_i - m_i^2 \phi_i^2) - g (\phi_1)^2 \phi_2 - h \phi_1 \phi_2 \phi_3.$$ 

Here, $g, h$ are the coupling constants and the particles have three different kinds of masses. Compute all possible matrix elements of the S-matrix between two-particle initial states and two-particle final states up to the second order in the coupling constants. Note that all the matrix elements will only differ in one detail or two: draw the corresponding Feynman diagram but do the calculation at the level of Wick’s theorem if you’re not certain.