

Homework 12 (files hwc*.*) – due Tuesday, Dec 19th: optional

(11A) Feynman diagrams with fermions

Consider the Lagrangian with a real scalar field ϕ and a Dirac field Ψ . The Lagrangian is

$$\mathcal{L} = \bar{\Psi}(p^\mu \gamma_\mu - m)\Psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}\mu^2 \phi^2 - y\bar{\Psi}\Psi\phi$$

where y is the Yukawa coupling constant.

1. Show what the Feynman rules, propagators, external lines, and vertices, are. Be sloppy about overall signs but careful about relative signs (and factors of i that can change the signs of observable quantities). If you need different lines, use wiggly lines for the bosons.
2. For $\mu > 2m$, compute the total decay rate of ϕ particle at the leading order in y . Note: you must sum over the electron and positron polarizations. Always work in the center-of-mass frame.
3. Call the Ψ quanta electrons, and compute the differential cross section of $e^- + e^- \rightarrow e^- + e^-$ (elastic scattering) at the tree level, with the exchange of ϕ , as a function of θ , the angle in center-of-mass frame. Compute only the total cross section summed over the spin polarizations of the final electrons, and averaged over the polarizations of the initial electrons. Only the cross sections can be summed over different initial or final states, not the amplitudes! Amplitudes are only summed if the initial and final states are identical. Don't forget that there are two graphs and a relative sign if any.
4. Compute the self-energy correction to ϕ from an electron-positron loop, following the conventions we had for scalars. Don't forget signs and traces if any associated with the loops and fermions.
5. Write down all renormalizable terms for these fields Ψ, ϕ – the most general renormalizable Lagrangian for these degrees of freedom. Hint: you can always shift ϕ so that the linear terms vanish so that only two purely ϕ polynomial self-interactions survive. Also, there will be a new interaction between ϕ and Ψ because you are supposed to allow parity violation. What is it?
6. Compute the same elastic scattering of two electrons, as before, in the general theory at the tree level. Note that only some new term(s) will matter. Did the P violation cause a forward-backward asymmetry, i.e. asymmetry between θ and $\pi - \theta$. If you scatter two positrons instead of two electrons, will the cross section be an identical function of θ or will it be forward-backward reverted, $\theta \leftrightarrow \pi - \theta$?
7. Draw all one-loop 1PI diagrams that generate divergences – that require counterterms to be cancelled – but don't evaluate them. Include tadpoles and vacuum energy graphs.