PHYSICS 101 QUIZ, MAR 1, 2000

A small cart of mass 0.5 kg, which you are to treat as being a point particle, is set into motion on a frictionless loop-the-loop of radius $R$. The cart has speed 14 m/s when it passes the 3 o’clock position, as shown in the top panel at right. When the cart reaches its lowest point, as shown in the bottom panel at right, its speed is 16 m/s. Determine

a) the radius $R$ of the loop-the-loop.

SOLUTION: Apply the conservation of energy between the two situations shown.

$$KE_f + PE_f = KE_i + PE_i.$$  \hspace{1cm} (1)

where ”f” is the top diagram and ”i” the bottom one. So

$$\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_i^2 + mgh_i.$$  \hspace{1cm} (2)

Cancel out the $m$’s. Then $\frac{1}{2}(14)^2 + (10)R = \frac{1}{2}(16)^2$, or $98 + 10R = 128$, so that $R = 3$ m.

b) Give the magnitude AND direction of the acceleration of the cart when it is at its lowest point.

SOLUTION: The acceleration is radially in, i.e. UPWARD. Its magnitude is $v^2/R = 16^2/3 = 85.3$ m/s².

c) Give the magnitude AND direction of the force the track exerts on the cart when the cart is at its lowest point.

SOLUTION: Apply $F = ma$. The forces on the cart (when it is at the bottom) are a) the force of gravity, $mg$, downward and b) the force exerted by the track on the cart, which is $N$, upward. The net upward force equals $m$ times the upward acceleration, so

$$N - mg = mv^2/R,$$  \hspace{1cm} (3)

so $N = mg + ma = 0.5(10 + 85.3) = 47.6$ Nt.