PHYSICS 101 QUIZ, MAR 29, 2000    NAME: SOLUTION

Given: \( L = mr_v = mr_\perp v, \quad L = I\omega, \quad \text{KE}= \frac{1}{2}I\omega^2, \quad \Gamma = Fr_\perp = F_\perp r. \)

A) (5 pts) At right is a 4 meter long bar initially at rest, suspended from a pivot. The moment of inertia of the bar for rotations about this pivot is 1.92 kg\cdot m^2. A 0.02 kg bullet, traveling horizontally at a speed of 200 m/s, hits the bar at its center (2 meters below the pivot) and then sticks to the bar. Find the angular velocity of the bar just after the collision.

SOLUTION: During the collision the top of the bar bumps against the pin, which prevents the top of the bar from moving. This impulsive force exerted by the pin on the bar is an \( F_{\text{ext}}\Delta t \) which can NOT be neglected. So linear momentum is NOT conserved. But angular momentum about the pin is conserved, because this impulsive force exerts zero torque. So

\[ mv r_\perp = I_f \omega_f, \]

where \( m \) is the mass of the bullet and \( r_\perp \) is the distance from the line of the velocity to the fulcrum (at the pin). So \( r_\perp = L/2 = 2 \) m. Thus \((0.02)(200)(2) = 8 = I_f \omega_f. \) But

\[ I = \sum m_i r_i^2 = I_{\text{bar about pivot}} + mr^2, \]

where \( r \) is the distance the bullet is from the pivot (2 m). So \( I_f = 1.92 + (0.02)(2)^2 = 2 \text{ kg}\cdot\text{m}^2. \) Then \( 8 = 2\omega_f, \) or \( \omega_f = 4 \text{rad}/\text{sec}. \)

B) (4 pts) At right is shown a meter stick mounted at its center on a fixed frictionless horizontal axle, A, and from whose left end is suspended a 300 Nt weight. A force \( \mathbf{F} \) is applied to the right end of the meter stick at an angle \( \phi \) with respect to the horizontal, as shown, with \( \sin \phi = 0.6, \cos \phi = 0.8. \) Determine the magnitude of \( \mathbf{F} \) so that the meter stick will not rotate.

SOLUTION: The forces on the bar are a) the tension downward in the left rope = \( M g = 300 = 300 \) Nt, b) the upward force, which I call \( \mathbf{S}, \) at the axle A which holds the bar on the axle, and c) the
applied force $\mathbf{F}$. Use $\Gamma = F_\perp r$ to get the torque, $\Gamma$, due to a force applied at a distance $r$ from the axle. The torque due to the force at the axle is zero because for this force, $r = 0$. The torque due to $Mg$ is $Mg(L/2)$ CCW. Here I used that $F_\perp = Mg$ because this force is perpendicular to the vector $\mathbf{r}$ (which goes from the axle to the point of application of the force). The perpendicular component of $F$ is $F \sin \phi$. (Note that the parallel component, $F \cos \phi$ does not give rise to a torque.) So the torque due to $\mathbf{F}$ is CW and is of magnitude $F_\perp r = F \sin \phi (L/2)$. Equating the magnitudes of these torques gives $Mg(L/2) = F \sin \phi (L/2)$ so that $F(0.6) = 300$, or $F = 300/(0.6) = 500$ Nt.

C) Determine the magnitude of the force, $\mathbf{S}$, the axle exerts on the massless meter stick.

The vector sum of the forces must be zero. Taking components gives

\begin{align*}
  x : & \quad 0 = F_x + S_x \quad (3) \\
  y : & \quad 0 = F_y + S_y - Mg \quad (4)
\end{align*}

where $+x$ is horizontally to the right and $+y$ is vertically up. So $S_x = -F_x = -F \cos \phi = -(0.8)(500) = -400$ Nt and $S_y = -F_y + Mg = -(500)(-0.6) + 300 = 600$ Nt. Note that $F_y$ is negative so that $-F_y$ is positive. The magnitude of $\mathbf{S}$ is thus $[F_x^2 + F_y^2]^{1/2} = [(-400)^2 + (600)^2]^{1/2}$, or $100\sqrt{16 + 36} \approx 720$ Nt.