PHYSICS 101 QUIZ, APR 5, 2000

NAME: SOLUTION

\[ F = -kx, \quad x = A \cos \omega t, \quad v = -A \omega \sin \omega t, \quad PE = \frac{1}{2}kx^2. \]

A) (4 pts) At right is shown a 4 kg block constrained to move frictionlessly along a horizontal track and attached to the end of a spring. It is found that it requires a 200 Nt force to hold the mass on the end of the spring motionless at point P which is 2 meters to the left of its equilibrium point. Give the period \( T \) of the ensuing motion when the mass is released from point P.

SOLUTION:

\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{m/k}. \]  
(1)

To get \( k \) note that \( F = kx \). But from the given data: \( 200 = k2 \), so that \( k = 100 \text{ Nt/m} \). So \( T = 2\pi \sqrt{4/100} = 2\pi/5 \approx 1.25 \text{ sec.} \)

(1 pt) After the mass is released from point P it will take a time (circle the correct answer) \( (T/4), \quad (T/3), \quad (T/2), \quad (T), \quad (2T) \) for the spring to first attain its longest length during the motion.

ANS: to go from left to right involves a half cycle, so the answer is \( T/2 \).

B) (4 pts) Now the spring used in part A is replaced by another spring whose spring constant is 400 Nt/m. The 4 kg block is set into oscillatory motion about its equilibrium point. At exactly 1PM it is observed that the block is 0.3 meters to the right of its equilibrium point and is moving to the right with speed 4 m/s. Find the amplitude of this simple harmonic motion.

SOLUTION: Use the conservation of energy:

\[ \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{TE}. \]  
(2)

The TE is the same at all stages of oscillation. The total energy when the mass is at the amplitude is \( \frac{1}{2}kA^2 \), so

\[ \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2. \]  
(3)

We can evaluate the left-hand side of this equation for the initial point, so that
\[
\frac{1}{2}(4)(4)^2 + \frac{1}{2}(400)(0.3)^2 = \frac{1}{2}(400)A^2, \tag{4}
\]
so that \(A^2 = (32 + 18)/200 = 1/4\), so that \(A = 0.5\) m.

C) (3 pts) Continuing with part B; how long after 1PM will it be when the block first reaches its maximum displacement to the right of the equilibrium position?

The block is to the right and moving to the right. We want the time it takes to go from \(x = 0.3\) m to the amplitude (0.5 m). This is the same as the time \(t_0\) to go from the amplitude to \(x = 0.3\) m. To find \(t_0\) use

\[
0.3 = A \cos(\omega t_0), \tag{5}
\]
so \(0.3 = 0.5 \cos(\omega t_0)\). So \(\omega t_0\) is the angle in radians whose cosine is 0.6. This angle is \(53^\circ = 53^\circ (\pi \text{rads}/180^\circ) = 0.925 \text{ rads}\). (Of course you get this with one punch of a button, if you do it right.) So \(\omega t_0 = 0.925\) or \(t_0 = 0.925/\omega = 0.925/\sqrt{k/m} = 0.925/\sqrt{400/4} = 0.0925\) sec.