Quantifying the topology of loopy network architectures

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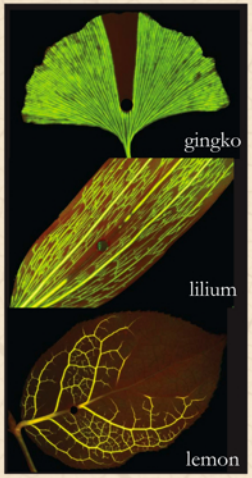
Motivation (why do we care?)

The architecture of distribution (and structural) networks is frequently dominated by *loops*.

Leaf veins are responsible for the transport and distribution of fluid along the leaf blade. They evolved from dichotomous branching networks to the modern complex and nested arrangements.

What drives leaf vein evolution?



Redundancy

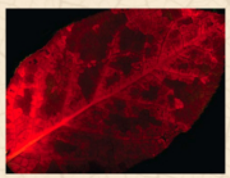
It ensures that the network will remain functional even after damage to the main veins.

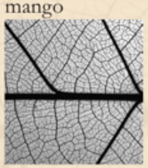
gingko: ~270myo

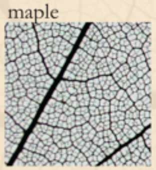
modern angiosperms: ~140myo

Stomatal patchiness

Photosynthetic activity (and transpiration rates) are not uniform on the leaf blade. *fluctuating load*







Leaf vein architecture exhibits spectacular **diversity** across species along with some consistent trends. How can we quantify the architecture?

Electrical circuit analogy

Given:

constitutive relation: $m_{kj} \sim C_{kj}^{\gamma}$

fixed cost: $\sum_{<kj>} m_{kj} \sim \sum_{<kj>} C_{kj}^{\gamma} = const$

source (net) currents: $\sum_{j \in \{<kj>\}} C_{kj}(V_k - V_j) = \mathcal{I}_k$

Find the network that minimizes:

dissipation, pressure drop, etc

$$F = \sum_{<kj>} C_{kj}(V_k - V_j)^2 \quad , \; F = -\frac{1}{N}\sum_k V_k$$

robustness: each bond has a probability to be cut. Average over all damaged networks and minimize <F>

fluctuations: the net currents fluctuate. Average over all realizations and minimize <F>

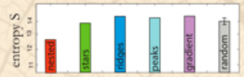
Robustness and fluctuations in the load induce *loops*. What are the elements of the characteristic architecture in each case?

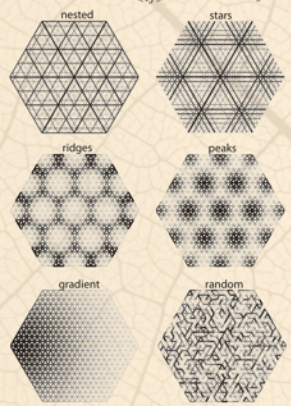
Entropy and node strength distribution

Network informational entropy function is a popular measure for the reliability of the architecture

Ang et al (2005)

weighted sum of Shannon entropies of incoming currents $S = -\sum_{<ij>} I_{ij}^{+} \log \frac{I_{ij}^{+}}{\sum_k I_{kj}^{+}}$

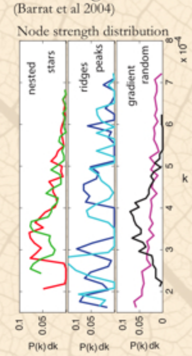




computer generated networks with identical weight distribution and connectivity

node strength: weighted vertex degree k

(Barrat et al 2004)

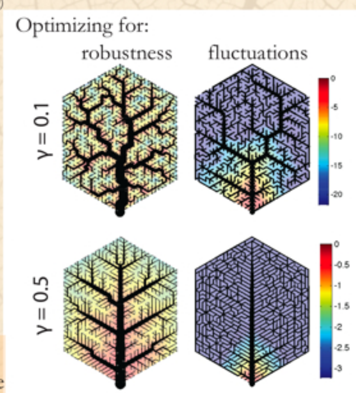


Traditional methods do not describe the architecture

Optimal transport networks
(with G.Szollosi)

What is the **optimal architecture for transport** given a fixed investment in vascular tissue?

Katifori et al, PRL (2010)



Hierarchical decomposition

Each edge deletion results in joining of two loops.

Each planar graph without subtrees can be represented as a binary tree. Each node of the binary tree represents an edge deletion.

Horton-Strahler stream ordering system for trees:

nodes with no children: ω=1

Subsequent nodes:

$\omega = \max(\omega_1, \omega_2) + \delta_{\omega_1, \omega_2}$

Bifurcation ratio

$$B_\omega = \frac{\#\text{ streams of order } \omega}{\#\text{ streams of order } \omega+1}$$

We can identify two building blocks of the graph:

nested and **ordered** loops

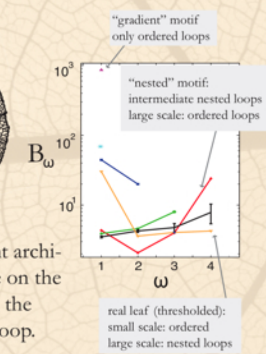
nested: low B_ω ordered: high B_ω

Comparing leaf vascular architecture

The bifurcation ratios of the tree representation of the loopy network encode the structure at different orders.

high max(ω), low Bω: nested

low max(ω), high Bω: ordered



Nodes in the binary tree with high ω represent architecturally significant loops. Their y-coordinate on the binary tree is proportional to the thickness of the deleted vein that led to the formation of the loop.

The architecture of the loopy network is decoded through incremental decomposition

connections to persistent homology

Future work

Study of other biological networks: corals, insect wings, retina vasculature

Leaf venation: classification scheme based on architecture. Connections to phylogeny or evolution? Universality classes?