Quantum Transport in Solids

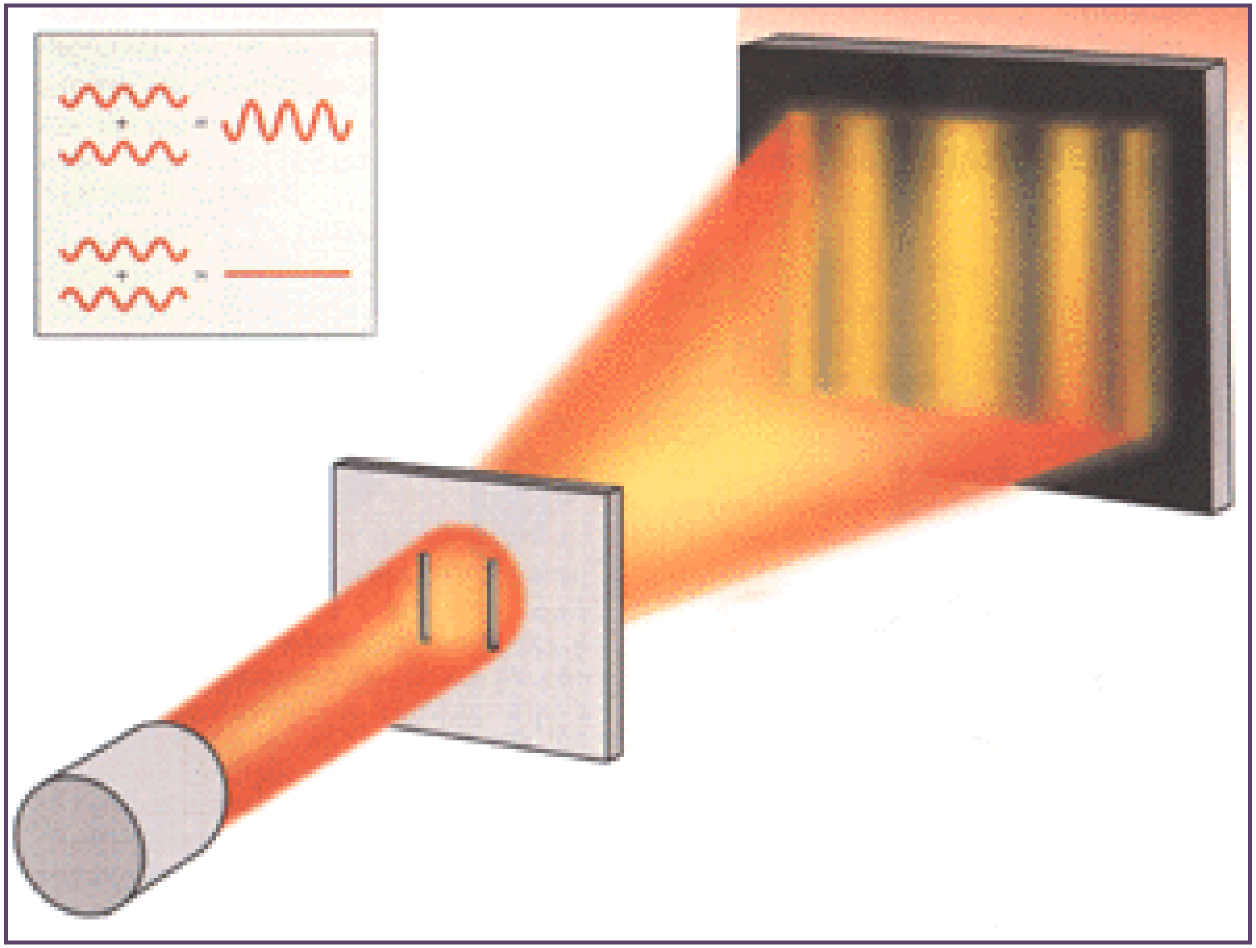
- Waves and particles in quantum mech.
- Quantization in atoms
- Insulators : Energy gap
- Emergent particles in a solid
- Landau quantization in a magnetic field and the quantum Hall effect

Light is a Wave

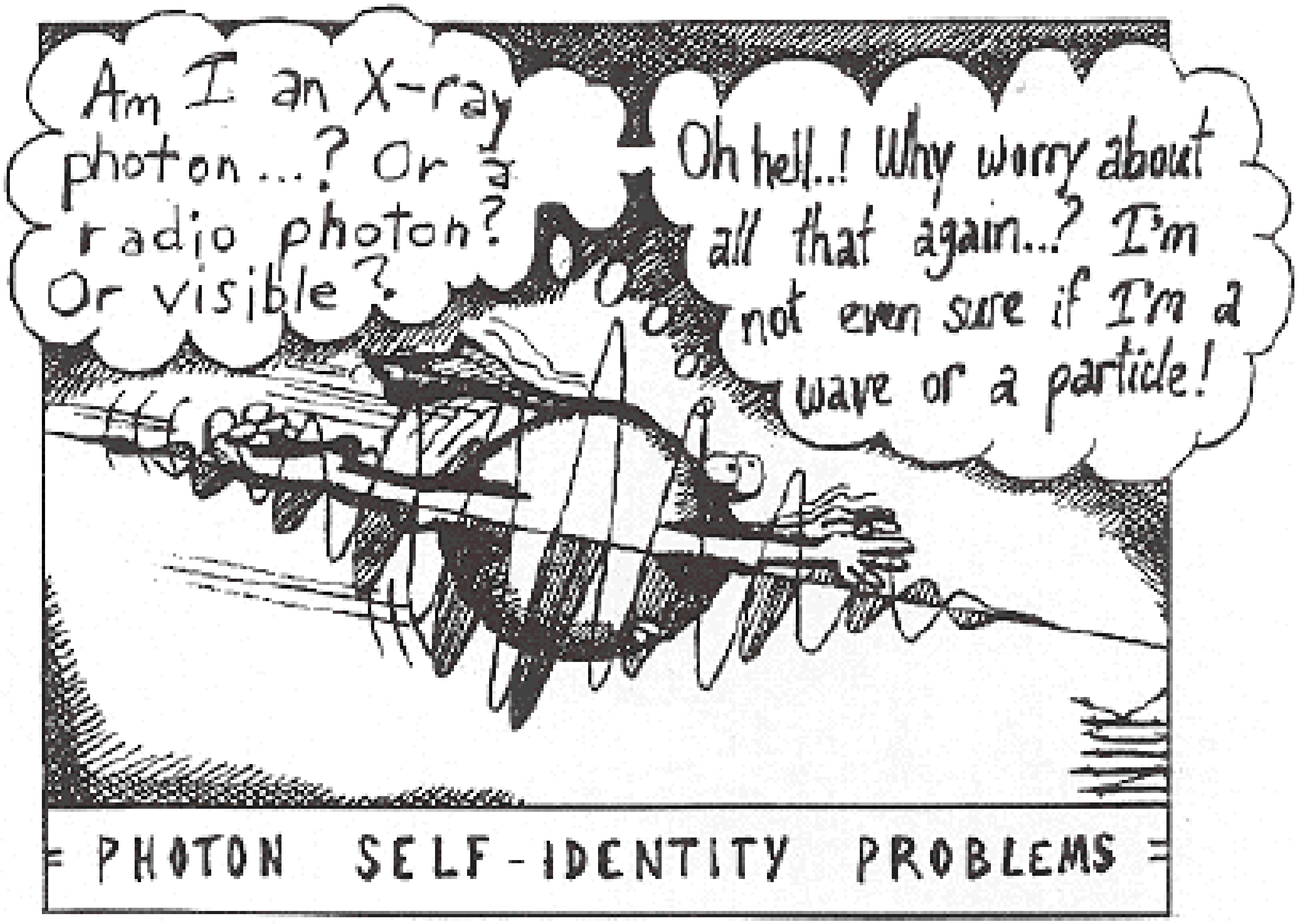
Hallmark of a wave : Interference

constructive

destructive

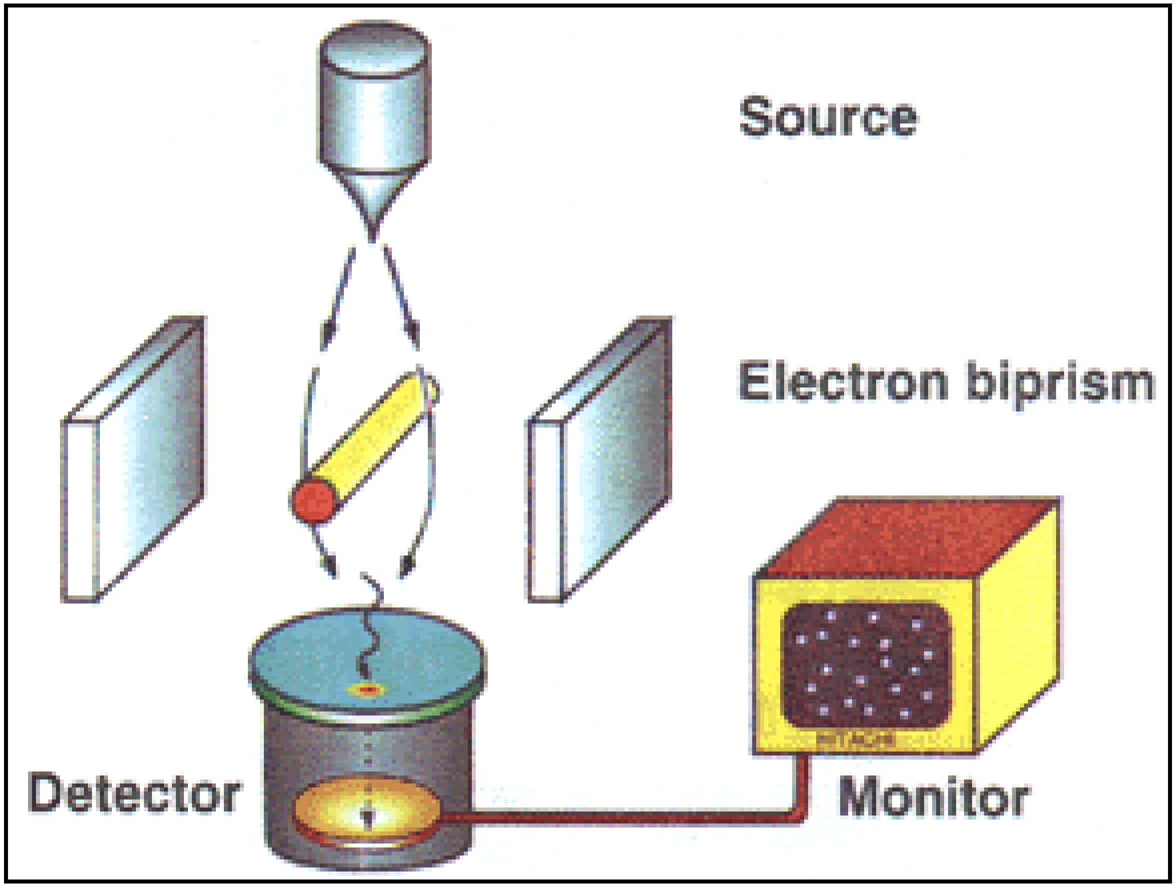


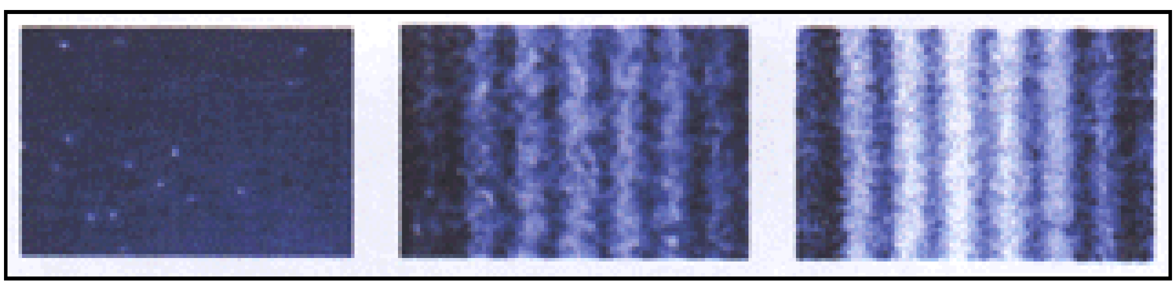
Wave Particle Duality



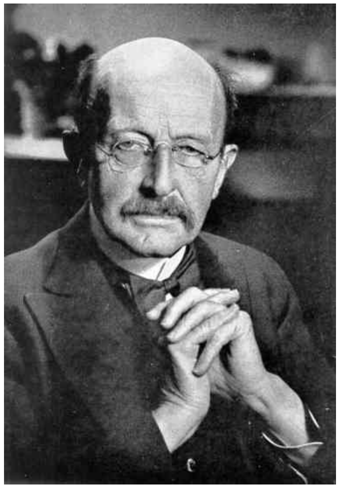
A photon is a particle too

Electrons are Waves





Energy and Momentum



Max Planck 1858-1947

Planck : Energy ~ Frequency

$$E = hf = \hbar\omega \qquad \omega = 2\pi f$$

$$\hbar = h/2\pi = 1.05\times10^{-34}\,Js$$

Louis de Broglie 1892-1987

de Broglie: Momentum ~ (Wavelength)$^{-1}$

$$p = \frac{h}{\lambda} = \hbar k \qquad k = 2\pi/\lambda$$

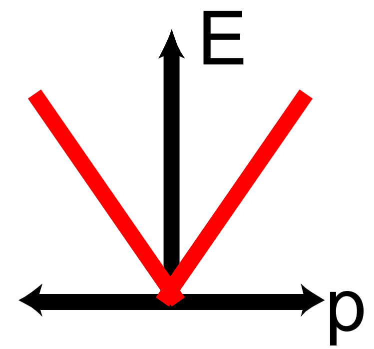
Dispersion Relation

Relation between

- Frequency and Wavelength
- Energy and Momentum

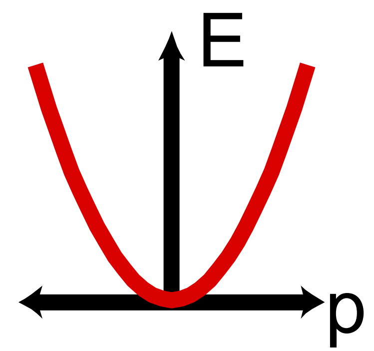
For a photon:

$$f = \frac{c}{\lambda} \quad \Rightarrow \quad \omega = ck \quad \Rightarrow \quad E = cp$$



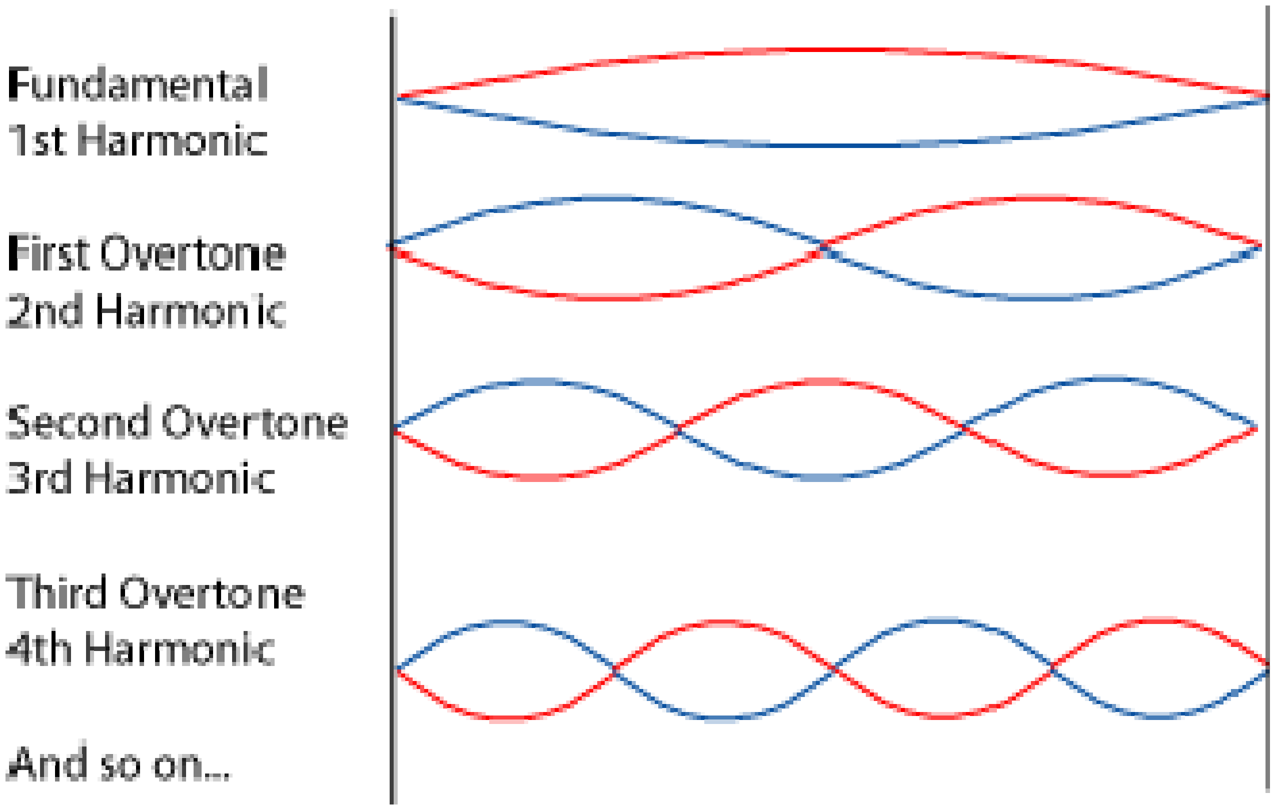
For an electron in vacuum:

$$E = \frac{1}{2}m\mathrm{v}^2 = \frac{p^2}{2m} \quad \Rightarrow \quad \hbar\omega = \frac{\hbar^2 k^2}{2m}$$



Quantization

Waves in a confined geometry have discrete modes



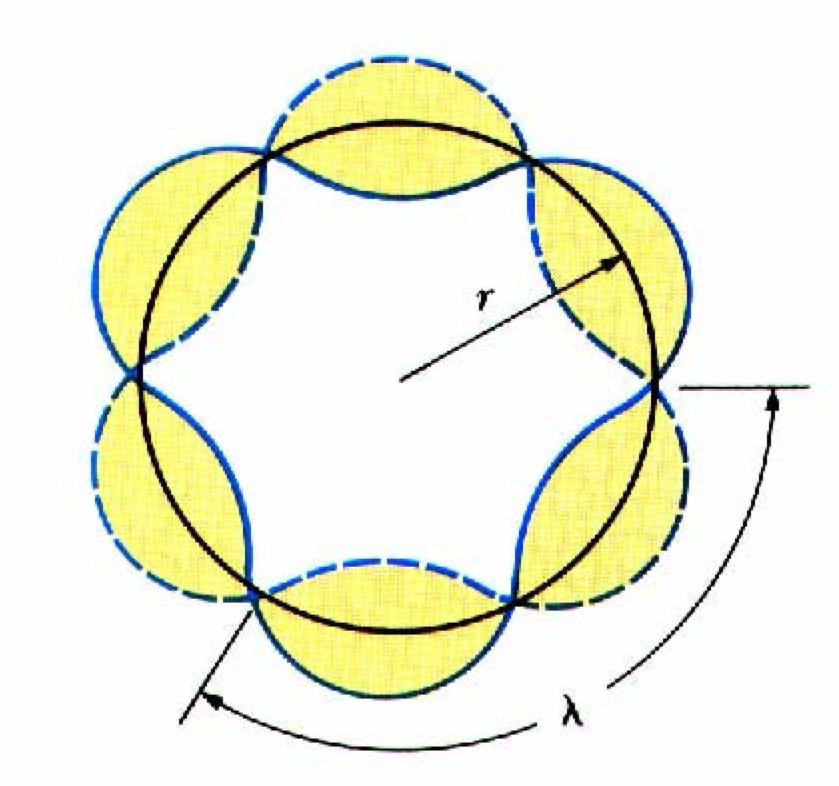
$$f_1 = \frac{v}{2L}$$

$$f_2 = 2\frac{v}{2L}$$

$$f_3 = 3\frac{v}{2L}$$

⋮

Quantization of circular orbits



Circular orbits have quantized angular momentum.

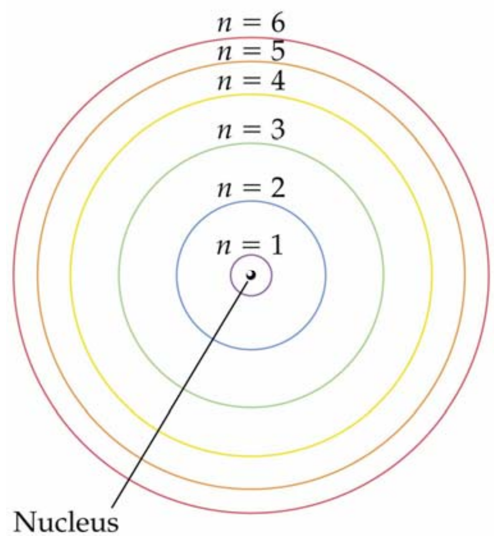
$$n\lambda = 2\pi r$$

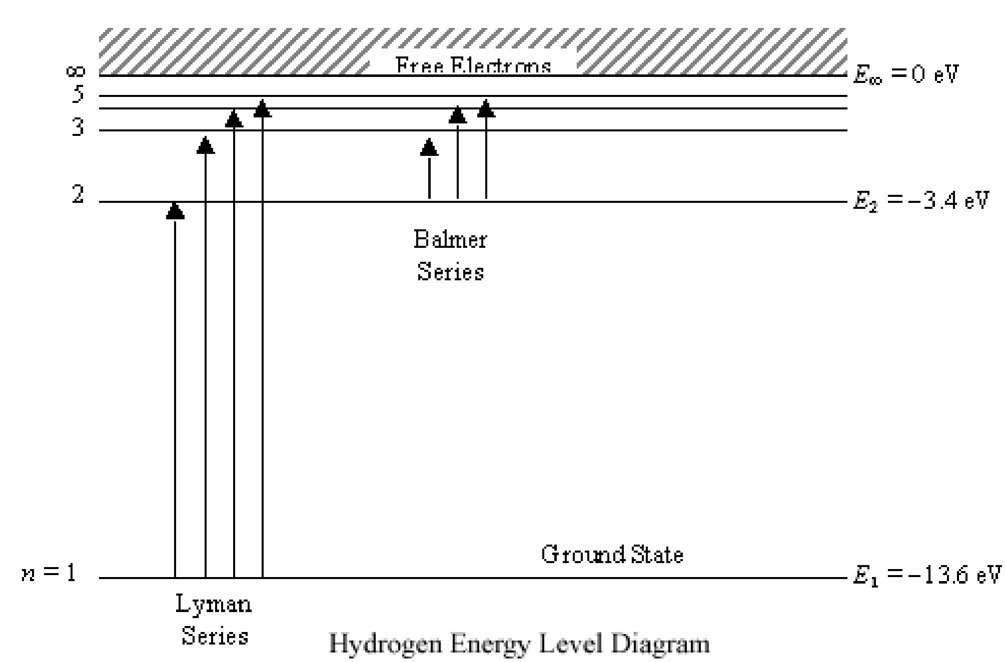
$$p = \frac{2\pi\hbar}{\lambda} = \frac{n\hbar}{r}$$

$$L = rp = n\hbar$$

Bohr Model

Niels Bohr 1885-1962





Explains line spectra of atoms

- Two equations :

$$L = mvr = n\hbar$$

$$F = k\frac{e^2}{r^2} = \frac{mv^2}{r}$$

- Solve for r_n and $E_n(r_n,v_n)$

$$E_n = -Ry/n^2$$

$$r_n = n^2 a_0$$

- Rydberg energy:

$$Ry = \frac{mk^2e^4}{2\hbar^2} = 13.6 \text{ eV}$$

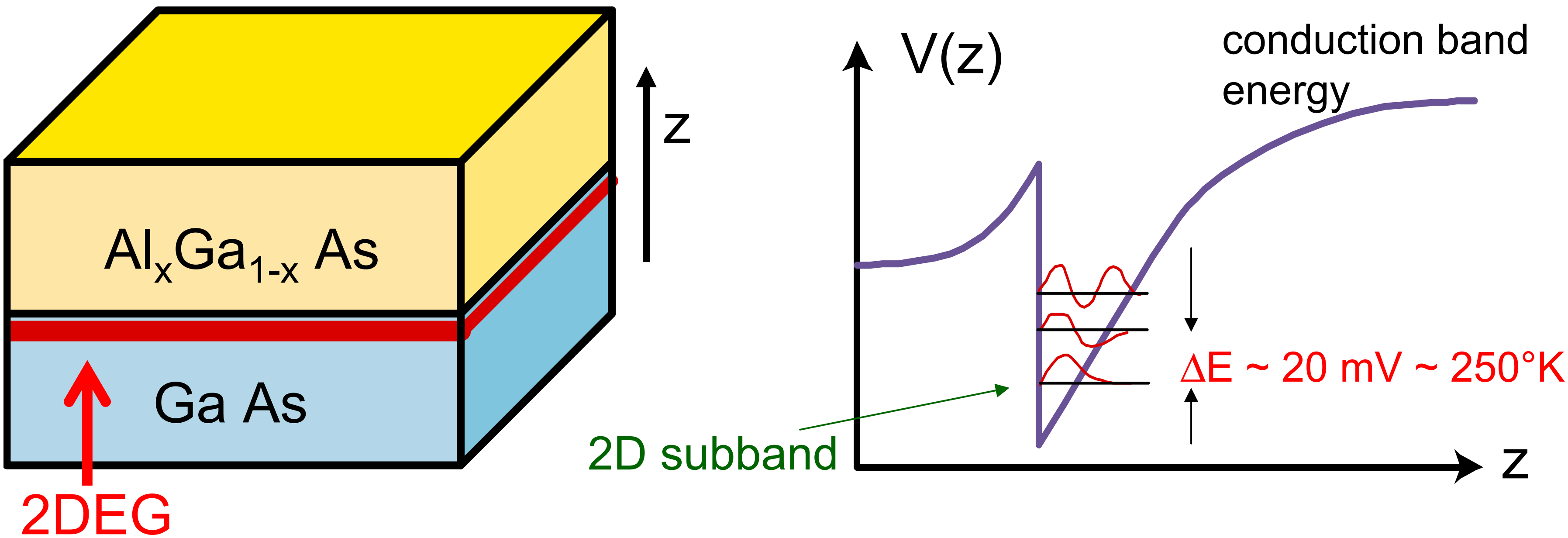
- Bohr radius :

$$a_0 = \frac{\hbar^2}{mke^2} = .5\text{Å}$$

Flatland ... (1980's)

Semiconductor Heterostructures : "Top down technology"

→ Two dimensional electron gas (2DEG)

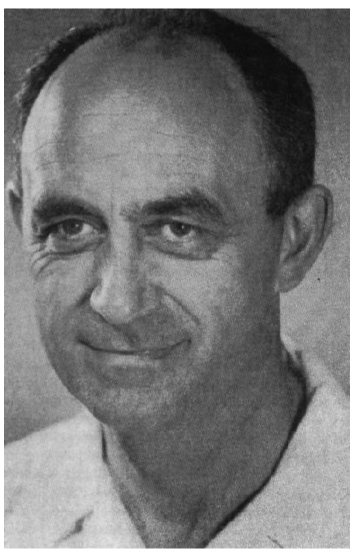


Fabricated with atomic precision using MBE.

1980's - 2000's : advances in ultra high mobility samples

Pauli Exclusion Principle

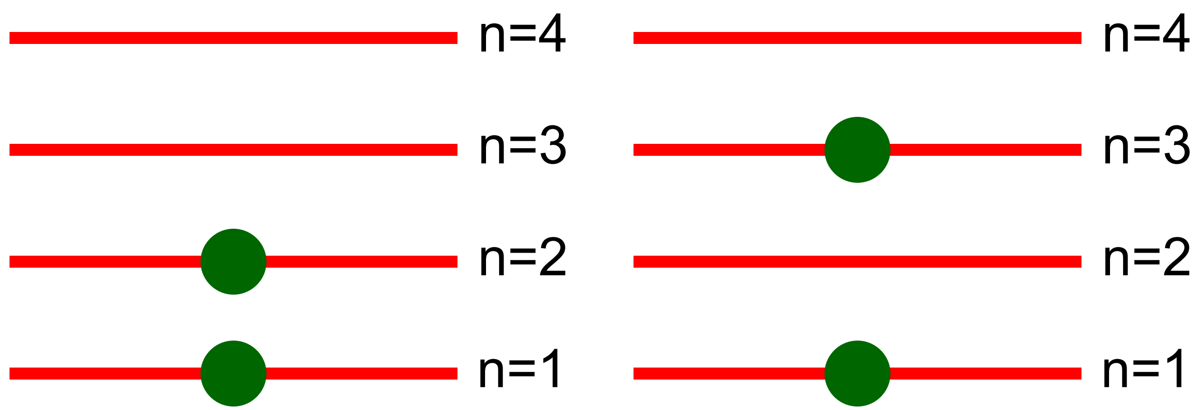
Wolfgang Pauli 1900-1958



Enrico Fermi 1901-1954

Electrons are “Fermions”:

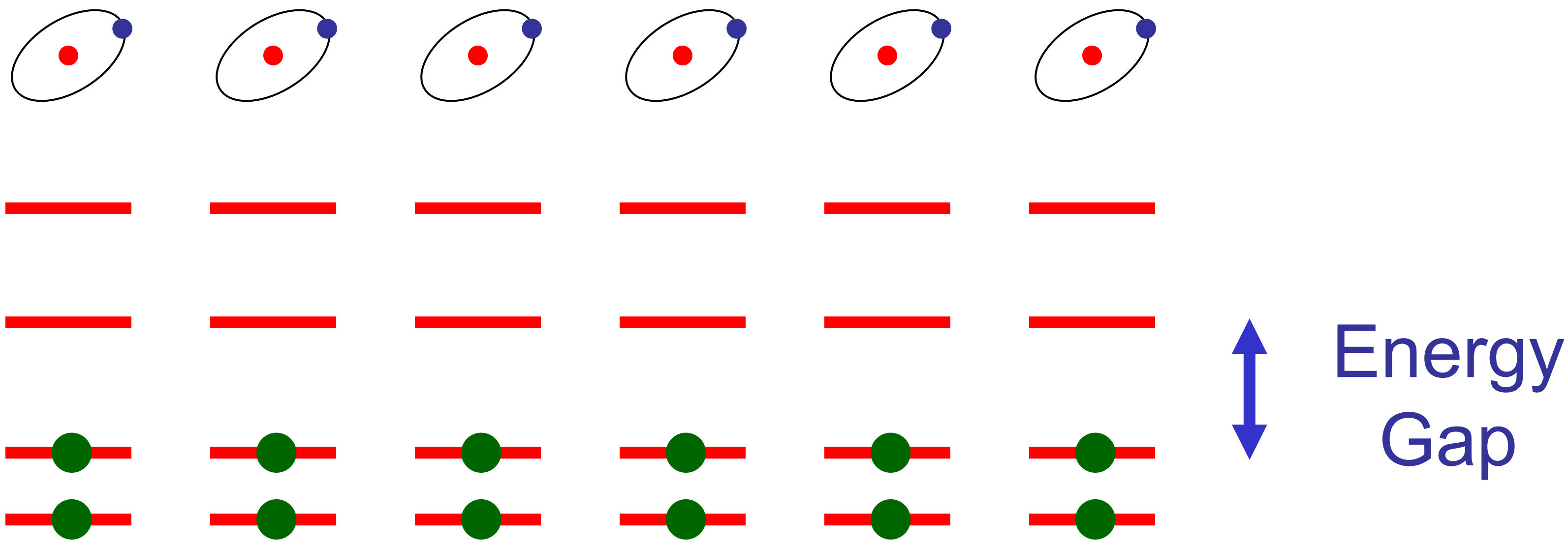
Each quantum state can accommodate at most one electron



Ground state

Excited State

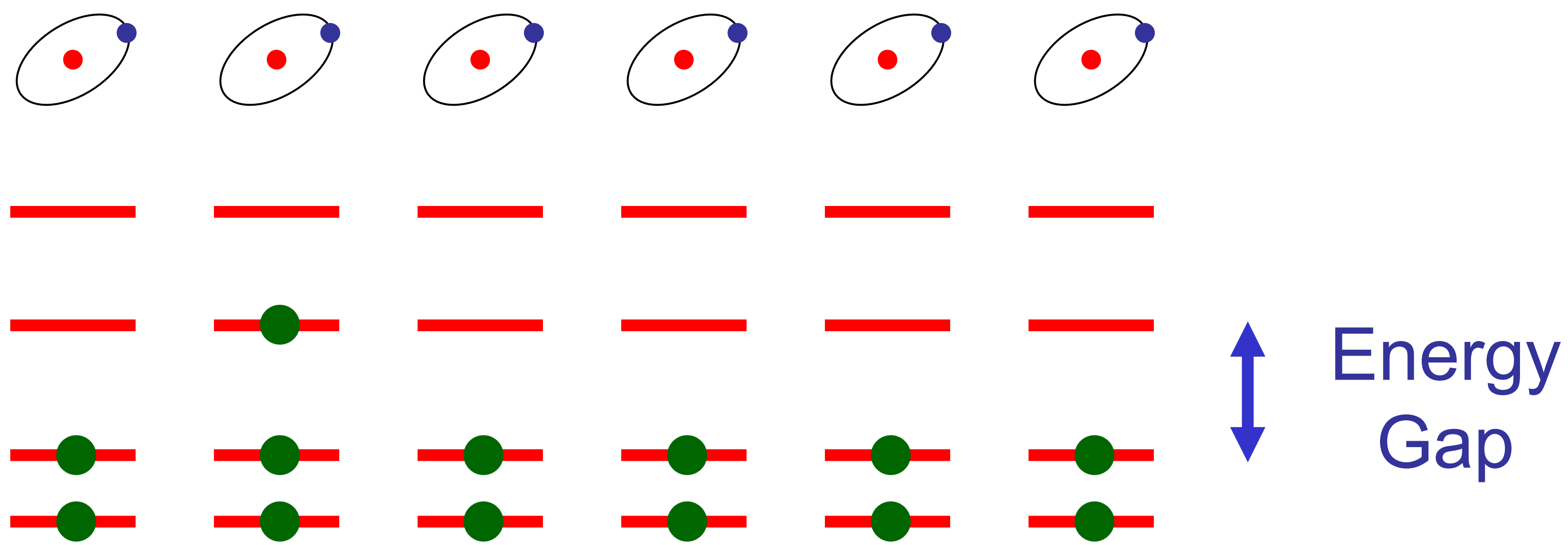
Insulator



An insulator is inert because a finite energy is required to unbind an electron.

A semiconductor is an insulator with a small energy gap.

Add electrons to a semiconductor

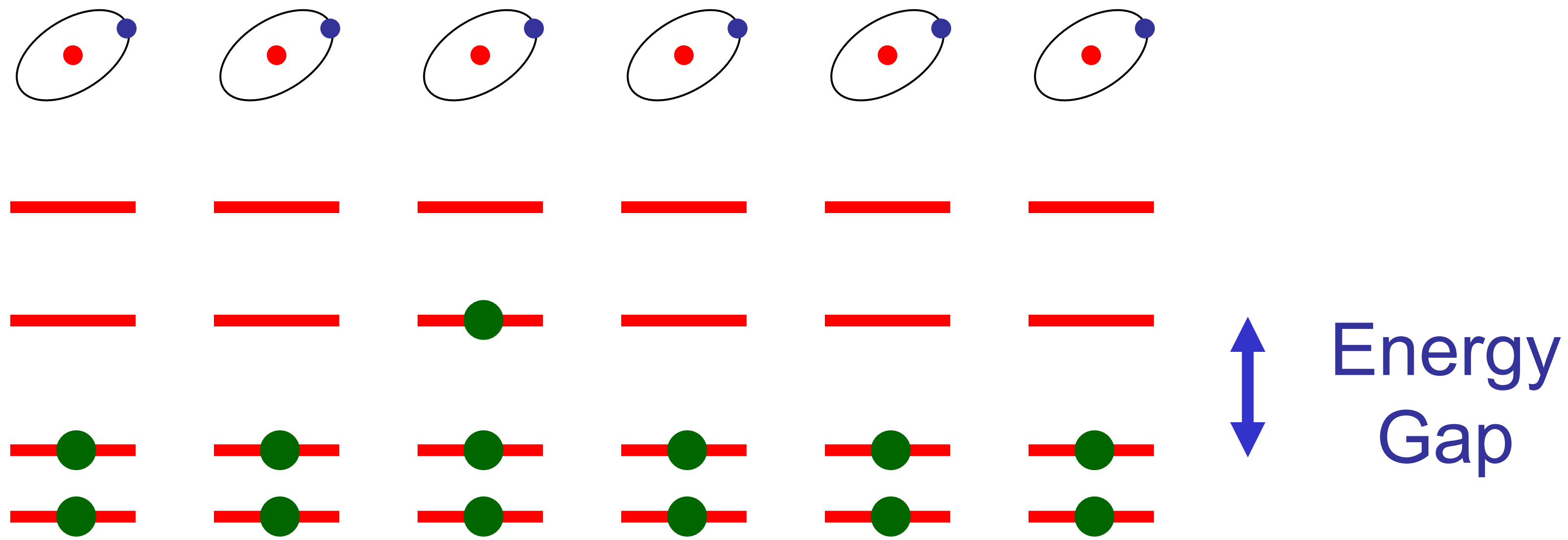


Accomplished by either

- Chemical doping
- Electrostatic doping (as in MOSFET)

Added electrons are mobile.

Add electrons to a semiconductor



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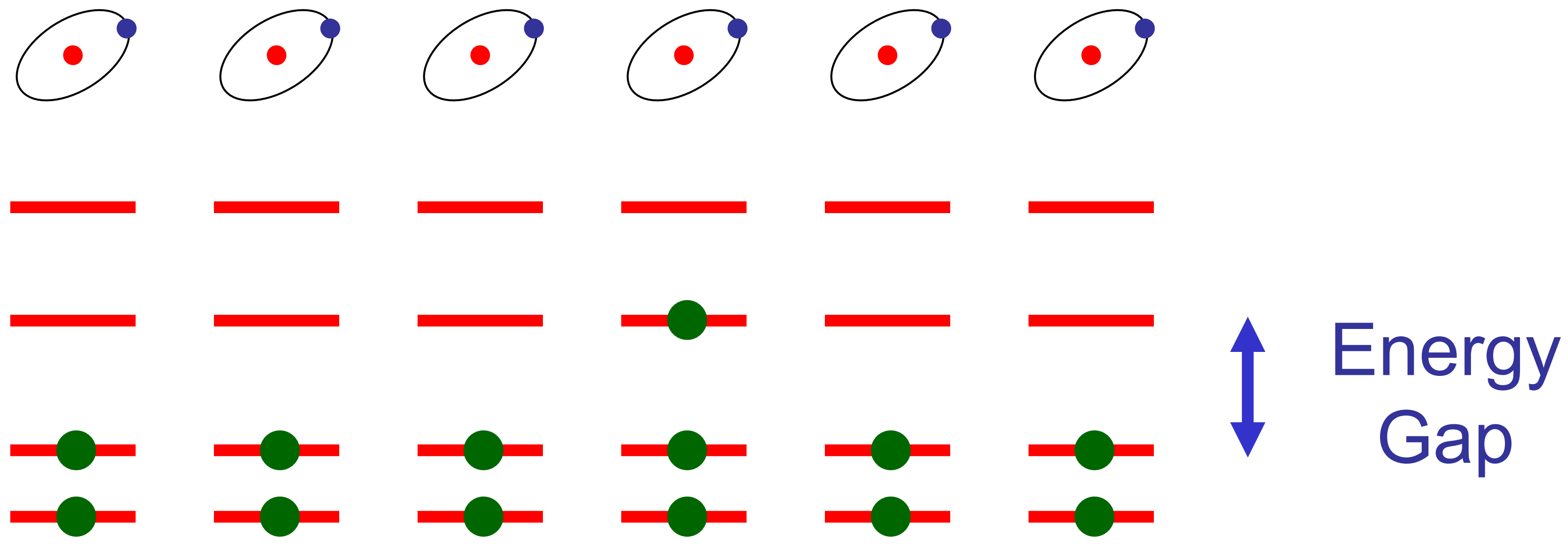

Accomplished by either

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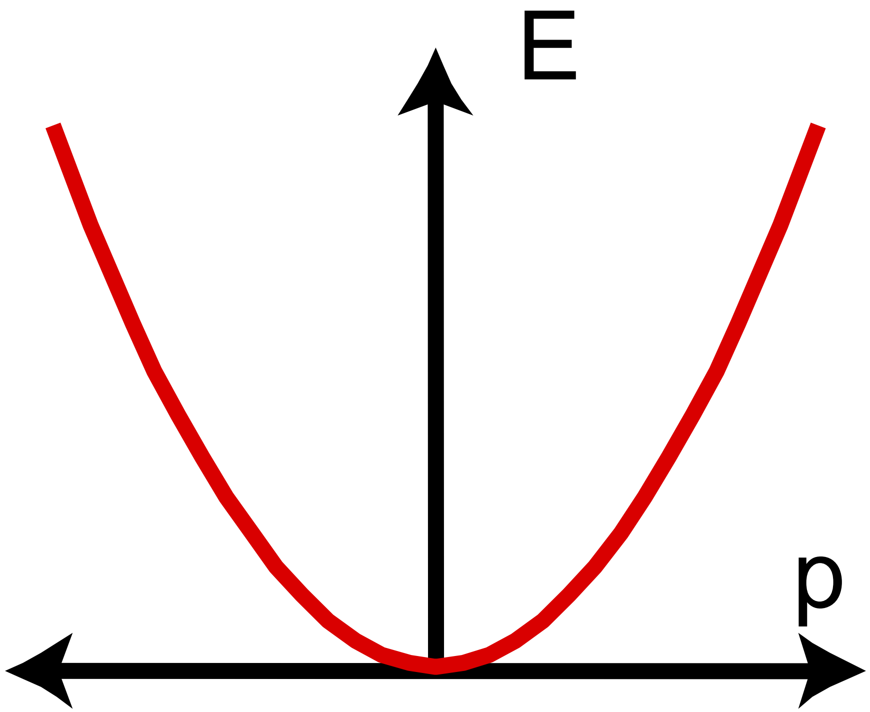
- Electrostatic doping (as in MOSFET)

Added electrons are mobile.

Added electrons form a band



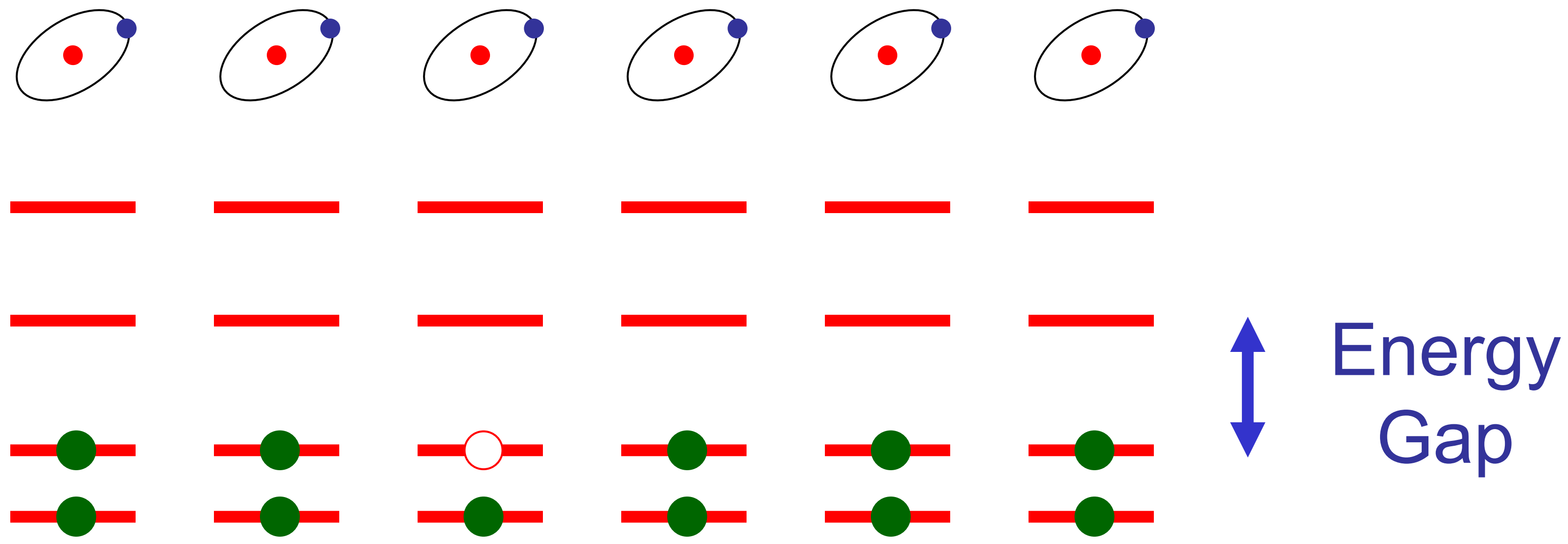
$$E = \frac{p^2}{2m^*}$$

m* = “Effective mass”

Electrons in the “conduction band” behave just like ordinary electrons with charge e, but with a renormalized mass.

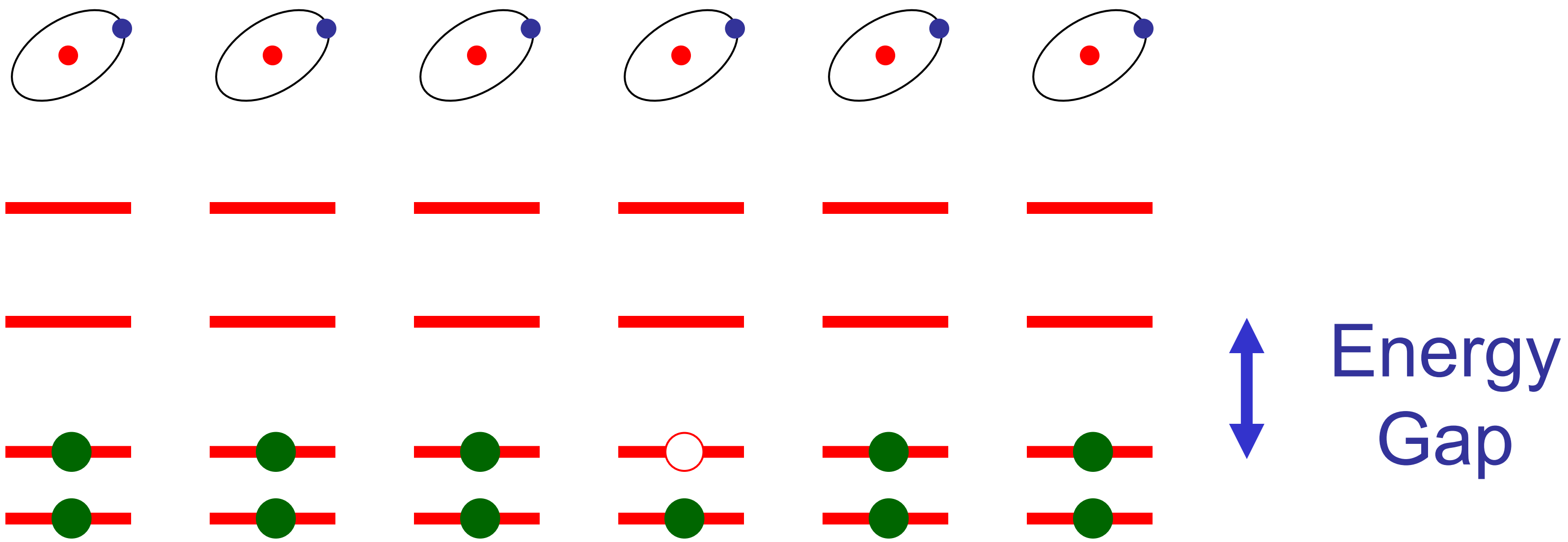
“Emergent quasiparticles at low energy”

"Holes" in the valence band



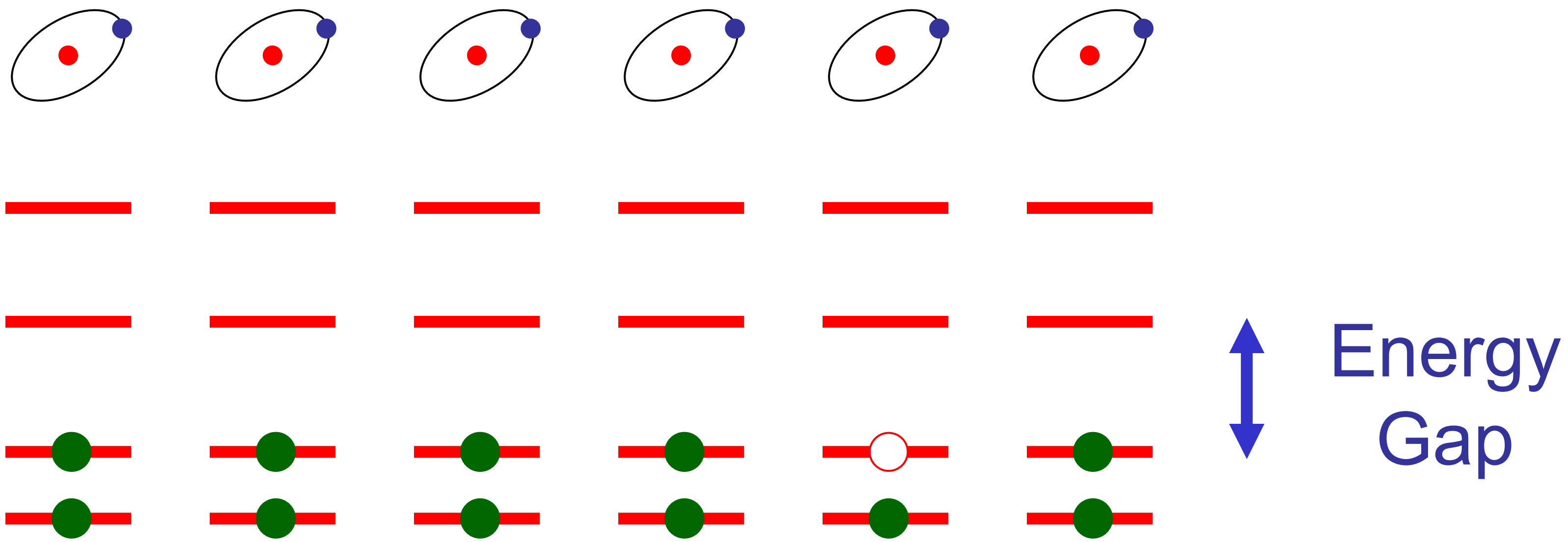
Added holes are mobile

“Holes” in the valence band



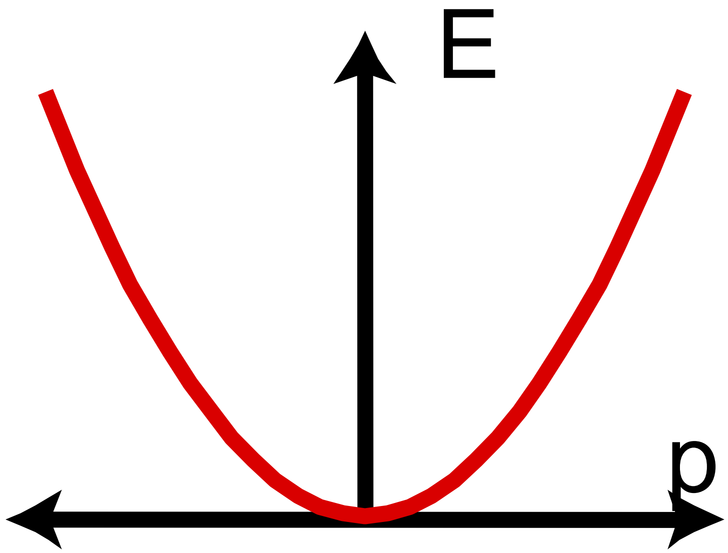
Added holes are mobile

“Holes” in the valence band



Added holes are mobile

Holes are particles too



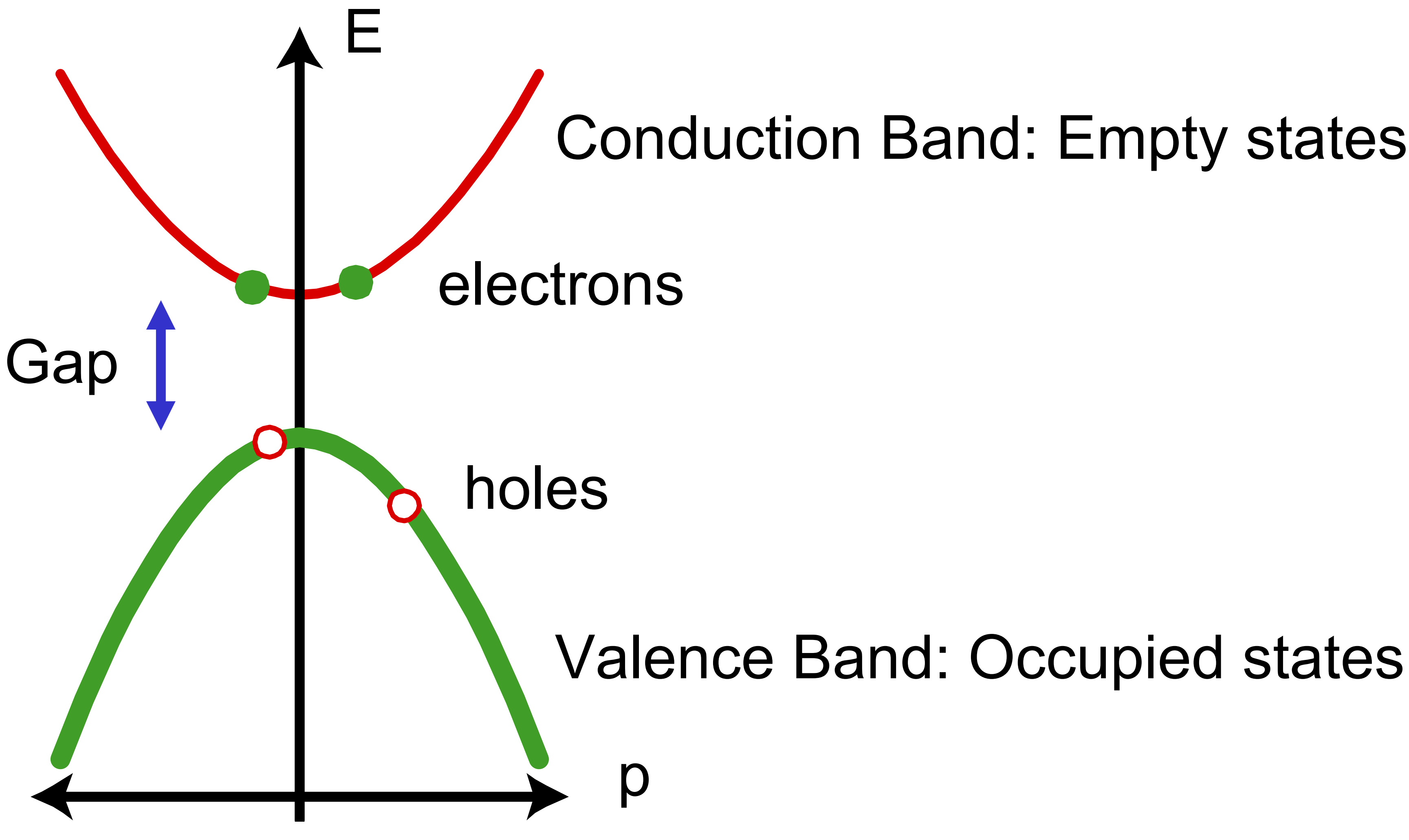
$$E = \frac{p^2}{2m_h*}$$

m_h* = Hole Effective mass

Holes in the “valence band” behave just like ordinary particles with charge + e, with a mass m_h* .

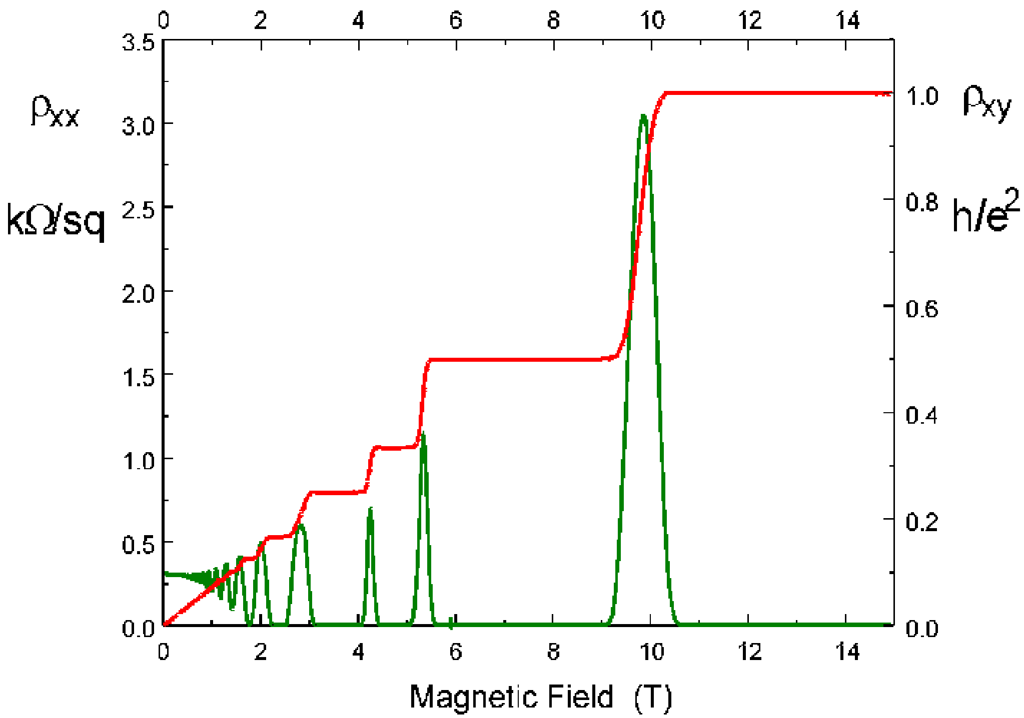
The sign of the charge of the carriers can be measured with the Hall effect.

Band Structure of a Semiconductor



The Quantized Hall Effect

Hall effect in 2DEG MOSFET at large magnetic field

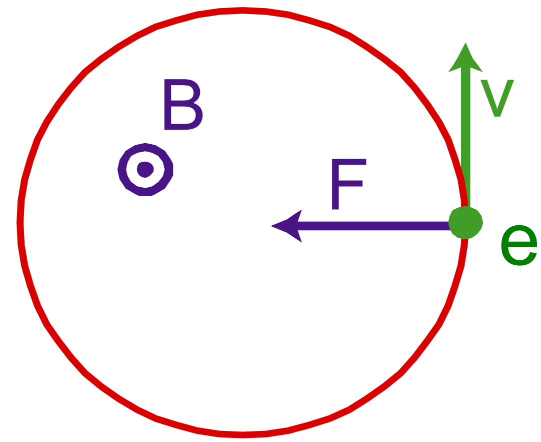


Von Klitzing,1981 (Nobel 1985)

- Quantization: $\rho_{xy} = R_Q / N$ N = integer accurate to 10^{-9} !
- Quantum Resistance: $R_Q = h / e^2 = 25.812\ 807\ \ \mathrm{k}\Omega$
- Explained by quantum mechanics of electrons in a magnetic field

Quantization in a magnetic field

Cyclotron Motion: $\vec{F} = -e\vec{v} \times \vec{B}$



1D Quantization argument:

$$F = evB = \frac{mv^2}{r} \qquad L = mvr = n\hbar$$

Solve for r_n and $E_n(r_n, v_n)$

$$E_n = \frac{eB}{2m} n\hbar = \frac{n}{2}\hbar\omega_c \qquad r_n = \sqrt{\frac{n\hbar}{eB}}$$

Cyclotron frequency $\omega_c = \frac{eB}{m}$

Magnetic flux enclosed by orbit $\pi r_n^2 B = \frac{n\pi\hbar}{e} = \frac{n}{2}\phi_0$

Magnetic Flux quantum $\phi_0 = \frac{h}{e}$

$$= 4.1 \times 10^{-15}\, Tm^2$$

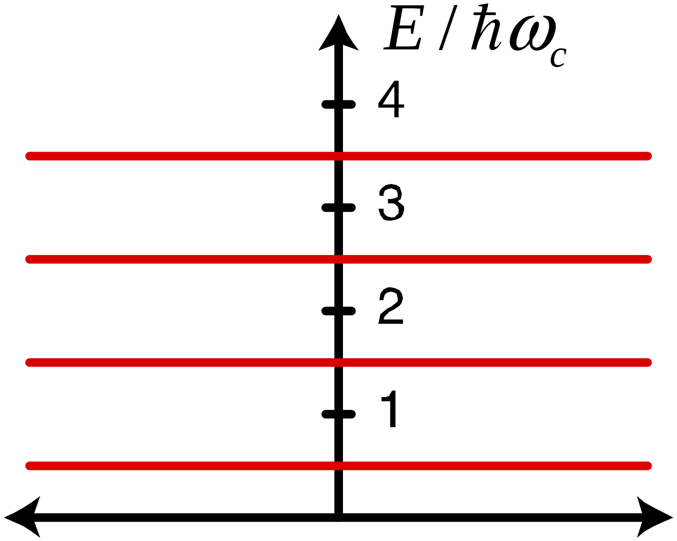
Landau Levels

Lev Landau 1908-1968

Landau solved the 2D Schrodinger Equation for free particles in a magnetic field

Closely related to the harmonic oscillator

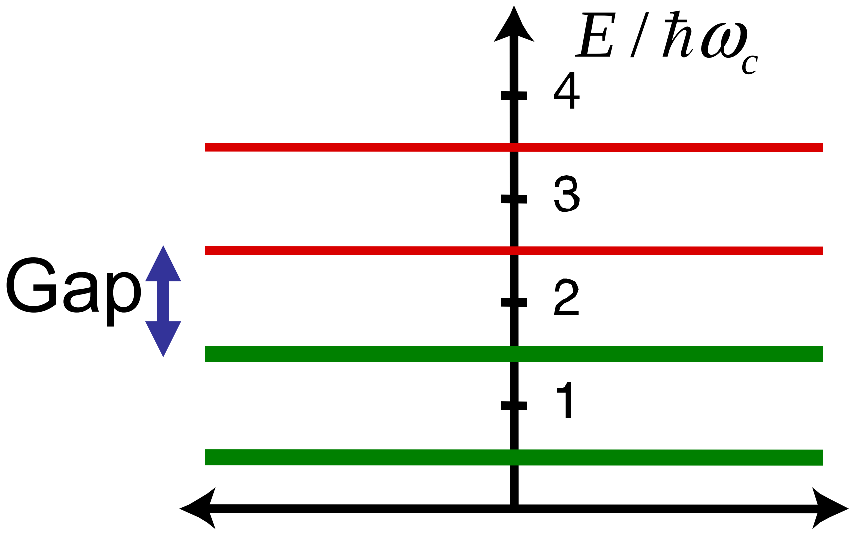
$$E_n = \hbar\omega_c (n + \frac{1}{2})$$



Each Landau level has one state per flux quantum

$$\#\ \text{states} = \frac{\text{total flux}}{\text{flux quantum}} = \frac{B \times \text{Area}}{(h/e)}$$

Quantized Hall Effect



N filled Landau levels:

particles/area:

$$n = N \times \frac{\text{\# flux quanta}}{\text{area}}$$

$$= N\frac{B}{\phi_0} = N\frac{eB}{h}$$

From last time: $$\rho_{xy} = \frac{E_y}{J_x} = \frac{B}{ne}$$

$$\rho_{xy} = \frac{1}{N}\frac{h}{e^2}$$