Kane Replies: Goldhaber *et al.* [1] argue that the proposal to probe fractional statistics in an edge state experiment [2] is flawed because fractional statistics are not meaningful for gapless edge states. To support this claim they present a numerical Berry's phase calculation for a quasihole in a closed disk. We begin by addressing the applicability of fractional statistics to edge states and then assess the relevance of the numerical calculation to this issue.

We agree that the calculation of an adiabatic Berry's phase is problematic in the presence of gapless excitations. However, this does not rule out the possibility of interference due to a statistical phase in a low energy transport experiment. The simplest implementation of this is the two point contact interferometer (2PCI) proposed by Chamon *et al.* [3]. Consider a Hall bar with two weakly backscattering point contacts (analogous to QPC1,2 in Fig. 1 of the Letter [2] without the inside edge). If the backscattering of quasiholes at the two point contacts is coherent the two processes interfere. The phase of interference determines the net backscattering amplitude which can be probed by measuring the conductance of the 2PCI. We thus take the change in the interference phase when a single quasihole is added between weakly backscattering point contacts to be an operational definition of the statistical phase.

Does the statistical phase defined in this way coincide with the adiabatic Berry's phase of bulk quasiholes? The answer requires a description of the low energy states responsible for transport. In the years since Laughlin proposed his celebrated wave function, a complementary low energy description of the Laughlin state has emerged in the form of a Chern Simons field theory [4]. The topological order of the Laughlin state is characterized by the coefficient (or more generally the "K matrix" of coefficients) of the Chern Simons term. The K matrix describes the statistical interactions between quasiholes and directly determines their adiabatic Berry's phase. Wen extended this to include gapless edge states and showed that their dynamics is governed by the K matrix [4]. In this Chiral Luttinger liquid (CLL) theory, the K matrix determines the interference phase in the 2PCI, and that phase is identical to the adiabatic phase for bulk quasiholes [2,3].

This correspondence is not a coincidence. The key assumption of the edge state theory is that the backscattered current is carried by individual quasiholes which tunnel between the edges through the quantum Hall fluid. The two tunneling trajectories along with propagation along the edge make a closed loop that is sensitive to the statistics of enclosed quasiholes. Gapless charge fluctuations on the edges are built into the CLL description, but they do not destroy interference at low energy. The CLL theory predicts that in many ways the edge behaves like a degenerate gas of quasiholes.

Goldhaber *et al.* calculate the Berry's phase for adiabatic transport of a quasihole around a circular loop in a closed disk at $\nu = 1/3$. Comparing the phase with and without an additional quasihole at the center they find that the phase difference varies smoothly between $2\pi/3$ and 0 when the loop crosses the edge of the disk. As they explain, the reason is simply that the added quasihole induces a quasi electron at the edge of the closed disk. When the loop encloses both the quasielectron and the quasihole, the statistical phases of the two cancel each other. The observed crossover does not imply that the statistics of the quasiparticles have somehow frayed. Rather it shows that the change in the number of quasiparticles enclosed by the chosen loop is not equal to one.

The authors are incorrect when they assert that the open geometry of the 2PCI will also suffer from this problem. The key point is that the loop relevant to the interference crosses through the quantum Hall fluid at the two point contacts. Since the inside and the outside of the loop are connected by quantum Hall fluid, the charge inside need not be a multiple of e. When a quasihole is added, the induced quasi electron will not be localized inside the loop. The change in phase acquired by a quasihole going around the loop will not include the contribution from the quasielectron. This could be demonstrated in a concrete calculation similar to [1]. Consider a strip bounded by $-L < x < L$ and $-a < y < a$, with $L \gg a$. Compute the Berry's phase for transporting a quasihole around a rectangle with sides $-b < x < b$ and $-c < y < c$ in the presence of and in the absence of a quasihole at $x = y = 0$. For $L \gg a, b$, the induced quasi electron will live predominantly outside the rectangle, $|x| > b$. We thus predict the phase difference will be $2\pi/3 + \mathcal{O}(b/L)$ even when c approaches the strip width a.

The weak backscattering point contact is an essential ingredient for proposals to probe the statistical phase via interference. The weak backscattering limit is quite subtle, and it would be interesting if it could be described using a more microscopic approach to assess the validity of the quasihole tunneling Hamiltonian of the CLL. The calculation on a closed geometry, however, sheds no light on this issue.

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