Sliding Luttinger Liquids

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I. Introduction

- The 1D Luttinger Liquid

II. The Sliding Phase

- A 2D Luttinger liquid

Mukhopadyay, Kane, Lubensky, PRB 2001;
Ohern, Lubensky, Toner PRL 1999;
Emery, Fradkin, Kivelson, Lubensky PRL 2001;
Vishwanath, Carpentier PRL 2001;
Sondhi, Yang PRB 2001.

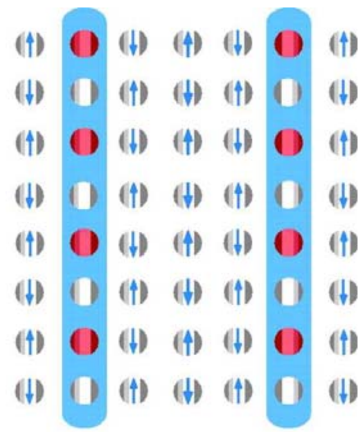
III. Instabilities of the Sliding Phase

- The Fractional Quantum Hall Effect from 1D Bosonization

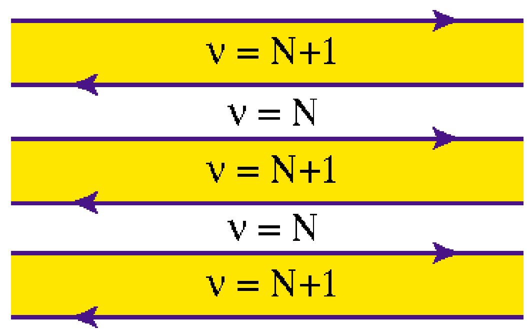
Kane, Mukhopadyay, Lubensky PRL 2002.

Weakly Coupled 1D Electron Systems

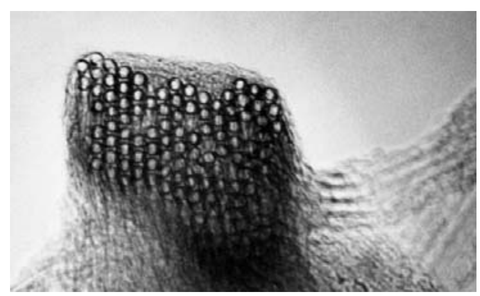
- Strip Phases of Cuprate Superconductors



- Quantum Hall Smectic Phases



- Weakly Coupled Wires
  e.g. Nanotube ropes



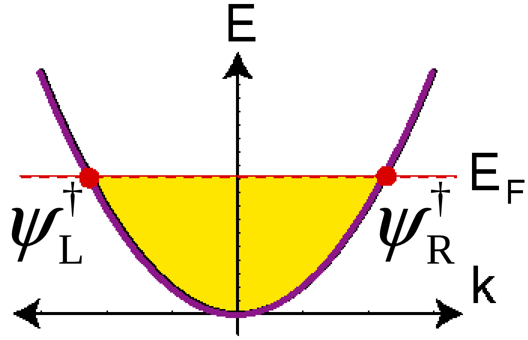
Theoretical Motivation:

Can the powerful techniques from 1D be used to understand strongly correlated states in higher dimensions?

Three Views of the 1D Electron Gas

1. Non Interacting

   Fermi Liquid



2. Repulsive Interactions

   “Almost” a crystal

3. Attractive Interactions

   “Almost” a superconductor $\quad \psi_L^\dagger \psi_R^\dagger \sim e^{i\varphi}$

Luttinger Liquid

- Power Law Correlations with exponent depending on interactions.
- Analogous to classical 2D XY model
- Bosonization:

$$L = \frac{1}{8\pi g}(\partial_\mu \theta)^2 = \frac{g}{8\pi}(\partial_\mu \varphi)^2$$

$$g = \begin{cases} 1 & \text{Non Interacting} \\ <1 & \text{Repulsive} \\ >1 & \text{Attractive} \end{cases}$$

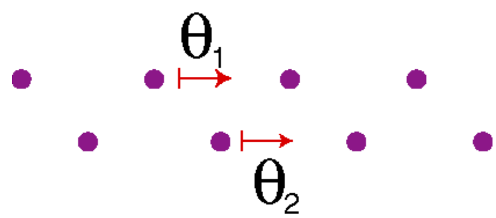
$$\psi_{L,R}^\dagger \sim e^{i(\varphi \pm \theta)/2}$$

Coupled Luttinger Liquids

- Expect instabilities due to coupling between wires

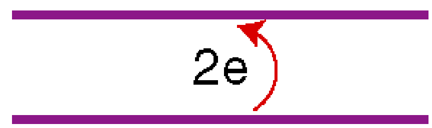
1. Charge Density Wave

$$O_{\mathrm{CDW}} = \sum_{<ij>} \cos(\theta_i - \theta_j)$$



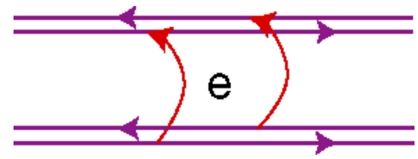
2. Superconductor

$$O_{\mathrm{SC}} = \sum_{<ij>} \cos 2(\varphi_i - \varphi_j)$$



3. 2D Fermi Liquid

$$O_{\mathrm{FL}} = \sum_{<ij>} \psi^\dagger_{Li}\psi_{Lj} + \psi^\dagger_{Ri}\psi_{Rj}$$



- Renormalization Group Analysis: $H_{\mathrm{int}} = \lambda_\alpha \hat{O}_\alpha$

$$\frac{d\lambda_\alpha}{d\ell} = (2 - \Delta_\alpha)\lambda_\alpha$$ Relevant if $\Delta_\alpha < 2$.

$$\Delta_{\mathrm{CDW}} = 2g$$
$$\Delta_{\mathrm{SC}} = 2/g$$
$$\Delta_{\mathrm{FL}} = (g + 1/g)/2$$

Is the Luttinger Liquid always unstable?

Two Kinds of Interactions:

1. Forward Scattering

$$H_{\mathrm{FS}} = \sum_{ij} V_{ij}^{\theta} \partial_x \theta_i \partial_x \theta_j + V_{ij}^{\varphi} \partial_x \varphi_i \partial_x \varphi_j$$

2. Interchannel Scattering

$$\hat{O}_{\mathrm{CDW}}, \hat{O}_{\mathrm{SC}}, \hat{O}_{\mathrm{FL}}, \ldots, \text{ many more}$$

- Responsible for Instabilities
- Dimensions depend on H_{FS}

Sliding Luttinger Liquid

- Choose H_{FS} to make "all" O_{α} irrelevant

"Smectic Metal"

- Anisotropic Electrical Conductivity
- Power Law correlations (like 1D L.L.)
- Collective Modes propagate in 2D

- An anisotropic 2D Luttinger Liquid
- Analogous to Sliding Phase of coupled classical 2D XY models

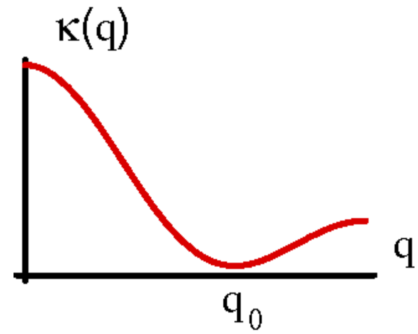
Ohern, Lubensky, Toner PRL 99

Model Interaction

Ohern, Lubensky, Toner PRL 99

$$\kappa(q_\perp) \equiv \sqrt{V^{\varphi}(q_\perp)/V^{\theta}(q_\perp)}$$

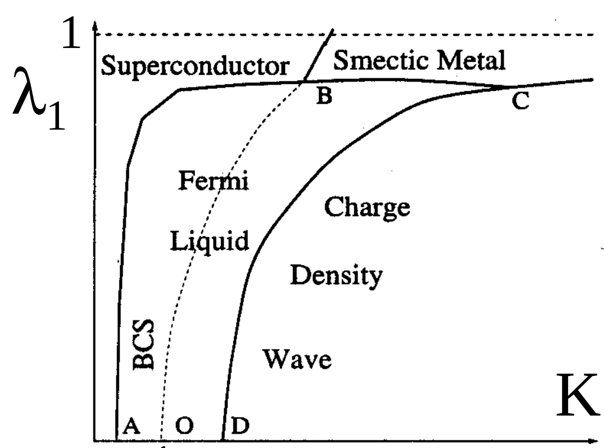
$$= K(1+\lambda_1 \cos q_\perp + \lambda_2 \cos 2q_\perp)$$



- Nearest neighbor model

Emery,Fradkin,Kivelson, Lubensky PRL 01

$\lambda_2 = 0$, nearest neighbor CDW, SC and FL terms only.



- More general model

Vishwanath, Carpentier PRL 01
Mukhopadyay, Kane, Lubensky PRB 01
Sondhi, Yang PRB 01

  - Higher order operators reduce region of stability

    - Further neighbor CDW, SC, FL
    - Correlated hopping, etc.

  - Sliding phase stable close to boundary of instability to transverse CDW with wavevector q_0

Nearest neighbor model

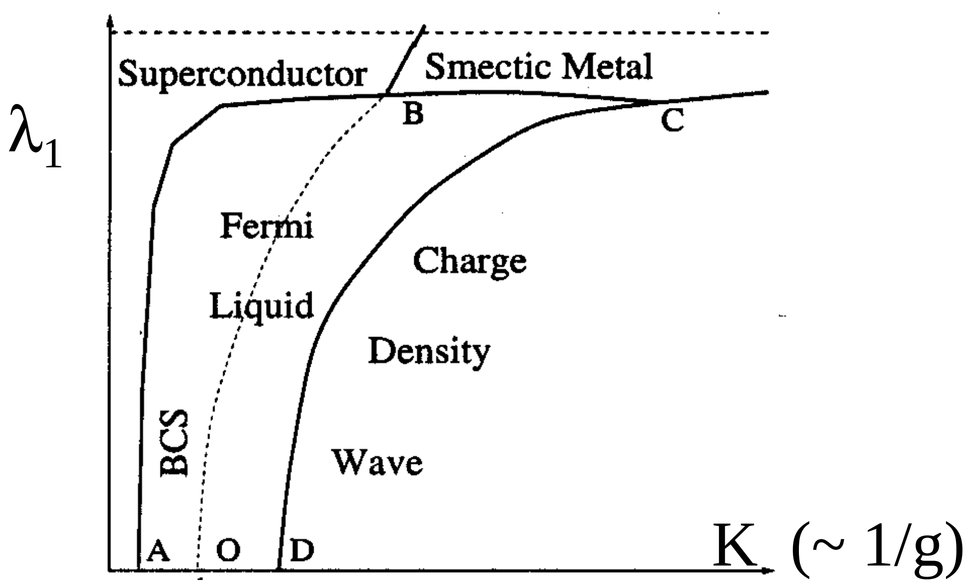
Emery,Fradkin,Kivelson, Lubensky PRL 01

- Nearest neighbor CDW, SC and FL terms only.

- Model Interaction:

$$\kappa(q_\perp) \equiv \sqrt{V^{\varphi}(q_\perp)/V^{\theta}(q_\perp)}$$

$$= K(1+\lambda_1 \cos q_\perp)$$



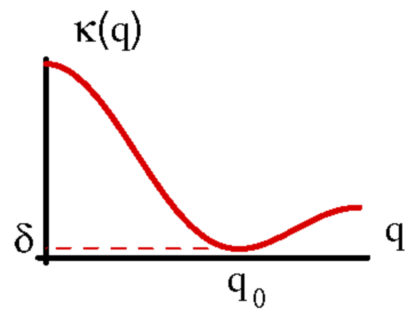
Higher order operators lead to instability

- Further neighbor CDW, SC, FL
- Correlated hopping, etc.

Stabilizing the Sliding phase

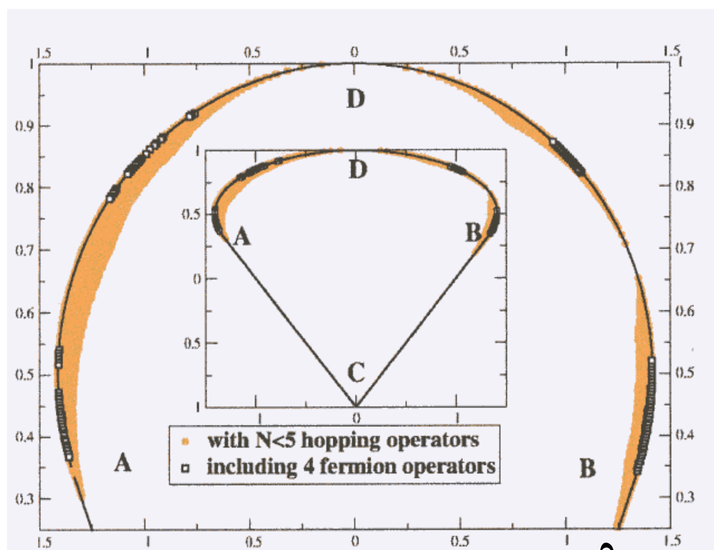
Model Interaction: $\kappa(q_\perp) = K(1+\lambda_1 \cos q_\perp + \lambda_2 \cos 2q_\perp)$

Ohern, Lubensky, Toner PRL 99
Vishwanath, Carpentier PRL 01
Mukhopadyay, Kane, Lubensky PRB 01



- Large density fluctuations at wavevector q_0 for small δ frustrate CDW formation.

Values of λ_1, λ_2 for which SLL phase is stable to a large class of operators for some K

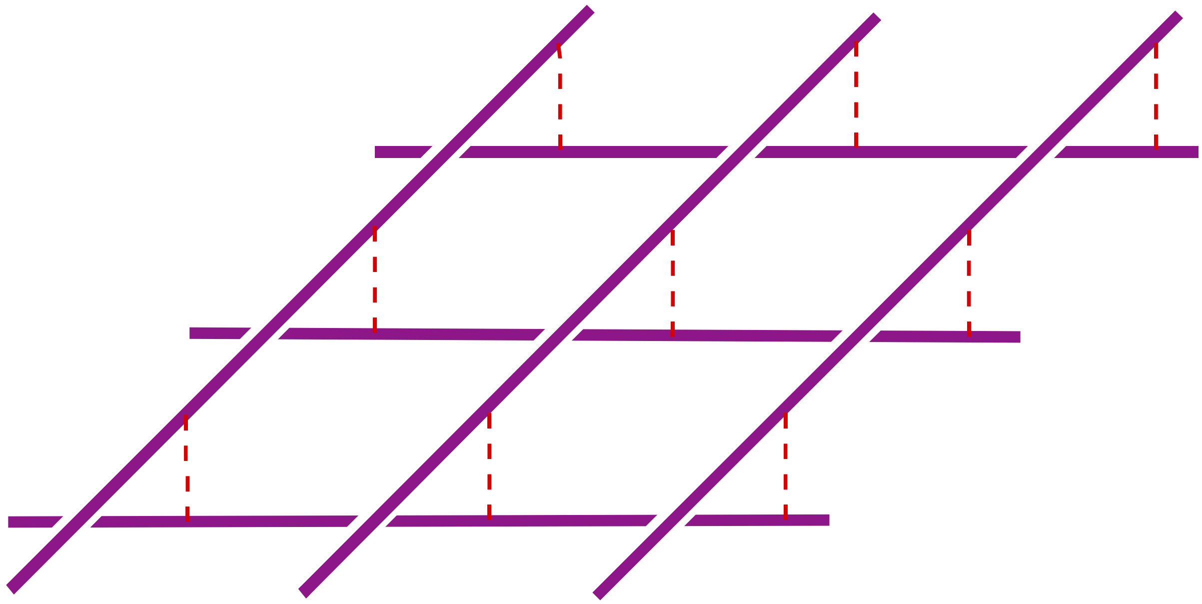


Vishwanath, Carpentier PRL 01

- Perpendicular Magnetic field increases region of stability by eliminating superconducting instability.

Sondhi, Yang, PRB 01

Crossed Sliding Luttinger Liquid



- Interactions between perpendicular wires are marginal but do not affect dimensions of operators.

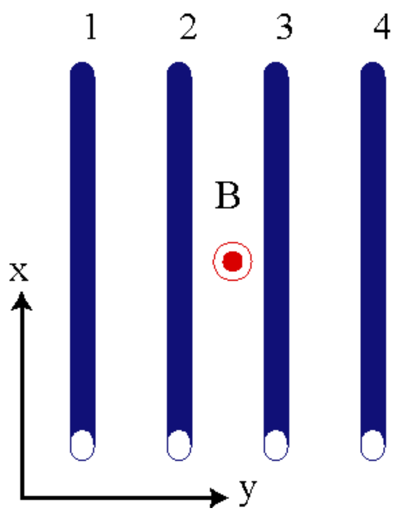
- Tunneling between perpendicular wires is irrelevant in sliding phase.

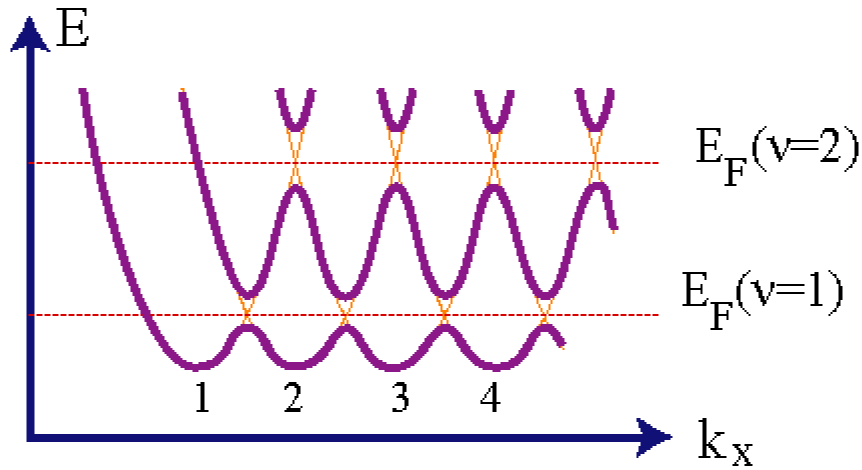
- Electrical conductivity isotropic at low (but finite) temperature.

An "isotropic" 2D Luttinger Liquid

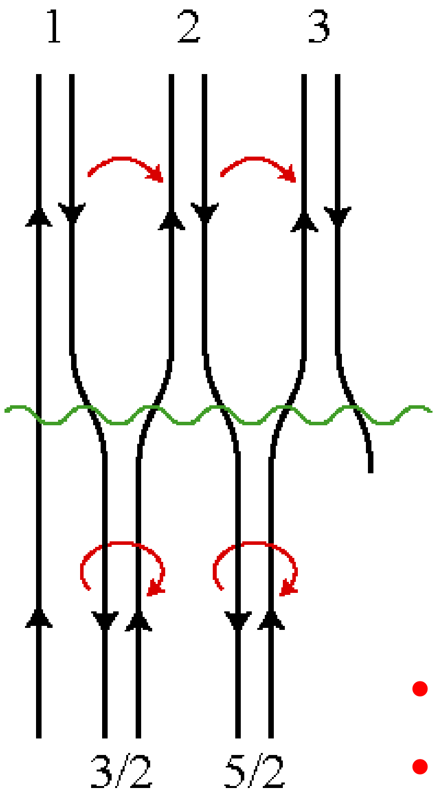
Instabilities of the Sliding Phase

- Integer Quantum Hall Effect





- From Bosonization:



$$H_{\text{int}} = \lambda \sum_{i=1} \cos[\varphi_i - \varphi_{i+1} + \theta_i + \theta_{i+1}]/2$$

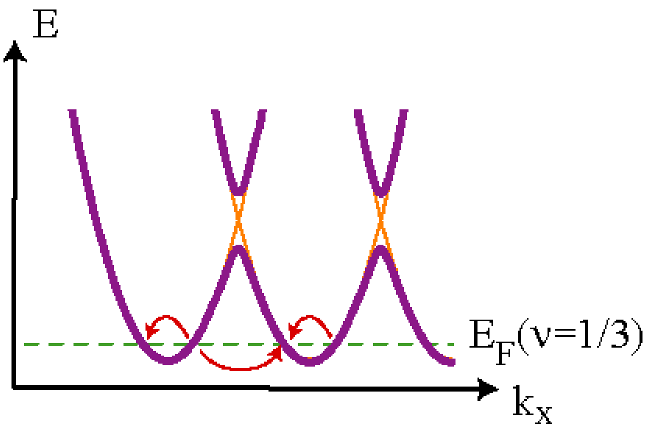
"Switch Partners"

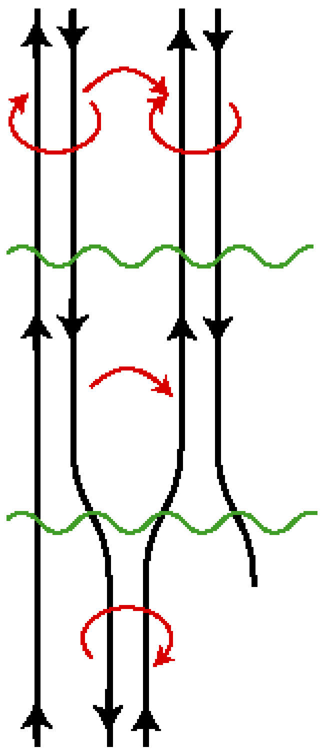
$$H_{\text{int}} = \lambda \sum_{i=1} \cos \tilde{\theta}_{i+1/2} \quad \sim \quad \lambda \sum_{i=1} (\tilde{\theta}_{i+1/2})^2$$

- Gap in bulk
- Edge Mode $\phi = \varphi_1 - \theta_1$ remains gapless

Laughlin State: ν = 1/3

3 particle correlated tunneling process





$$H_{\text{int}} = \lambda \sum_{i=1} \cos[\varphi_i - \varphi_{i+1} + 3(\theta_i + \theta_{i+1})]/2$$

Rescale: θ′ = 3 θ

$$H_{\text{int}} = \lambda \sum_{i=1} \cos[\varphi_i - \varphi_{i+1} + \theta'_i + \theta'_{i+1}]/2$$

"Switch Partners"

$$H_{\text{int}} = \lambda \sum_{i=1} \cos \tilde{\theta}'_{i+1/2} \sim \lambda \sum_{i=1} (\tilde{\theta}'_{i+1/2})^2$$

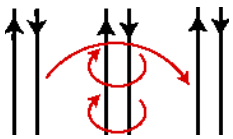
- Edge Mode: g=1/3 Chiral Luttinger Liquid
- 2π soliton in $\tilde{\theta}'$: Charge e/3 quasiparticle

Three Categories

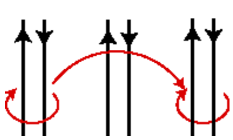
$$H_{\rm int} = \lambda \sum_{i=1} \cos[\sum_p m_p \theta_{i+p} + n_p \varphi_{i+p}]$$

I. Quantum Hall States

- Allowed at special magnetic fields $\nu = 2\frac{\sum_p p n_p}{\sum_p m_p}$
- Generalized Hierarchy
- $\cos\Theta$ ✂ Θ^2 : Bulk Gap + Edge states



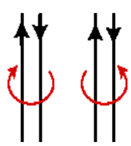
$\nu = 2/3$ $K = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$ (Haldane-Halperin Hierarchy)



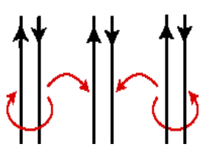
$\nu = 2/3$ $K = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ ("Bilayer state")

II. Crystals

- Allowed at any magnetic field: $\sum_p m_p = \sum_p p n_p = 0$
- $\cos\Theta$ ✂ Θ^2 : 2D Phonon mode



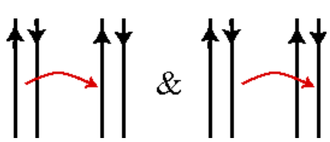
Crystal of Electrons



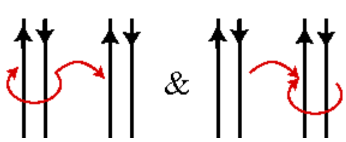
Crystal of Laughlin Quasiparticles

III. Degenerate Operators

- Difficult to analyze : ~~$\cos\Theta$ ✂ Θ^2~~



Fermi Liquid (B=0)



$\nu = 1/2$: Composite Fermi Liquid?

Conclusion

I. Sliding Luttinger Liquid

- Anisotropic 2D phase with power law correlations characteristic of 1D Luttinger Liquid.
- Residual Couplings "irrelevant"

II. Instabilities of Sliding Phase

- 1D bosonization offers a new, concrete framework for describing the fractional quantum Hall effect.

III. Can this be used to describe other strongly correlated states?

- Non Abelian Quantum Hall States
- Spin Liquid states, spin/charge separation
- ….