Charge and Statistics of Dilute Laughlin Quasiparticles

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I. Introduction

- Laughlin Quasiparticle and Shot Noise

II. Transmission of Dilute Quasiparticles through a point contact

PRB 67, 45307 (03) w/ Matthew Fisher (ITP)

- Three Terminal Geometry for preparing "dilute quasiparticle beam"

  (Moty Heiblum et al. '02)
- Shot Noise Puzzle
- Luttinger Liquid Theory

III. Proposed measurement of fractional statistics: Telegraph Noise

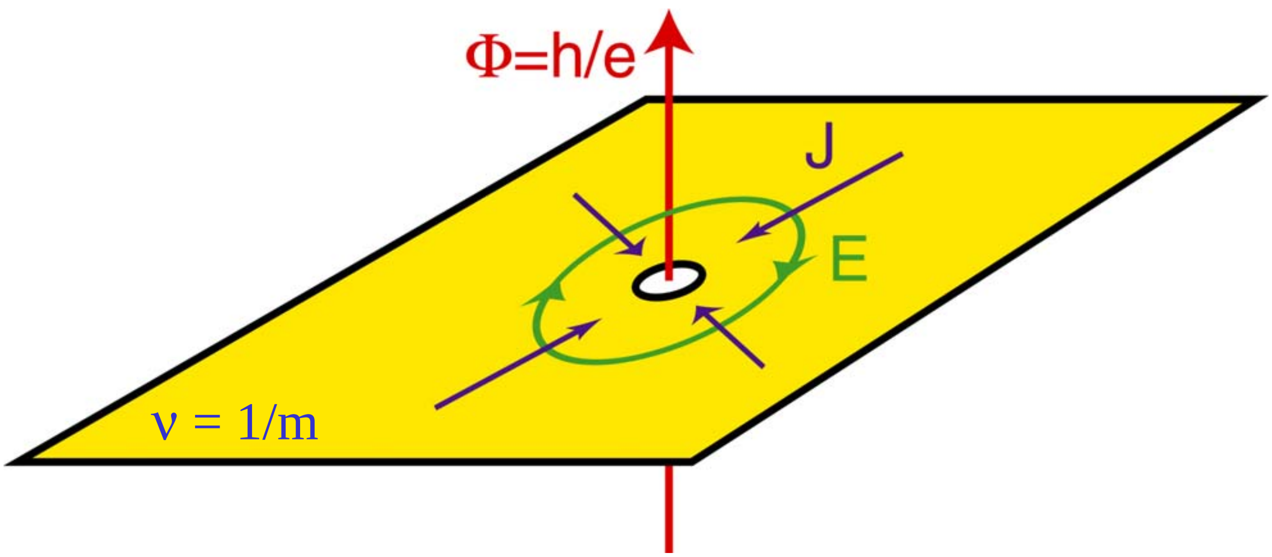
PRL 90, 226802 (03)

- Corbino Geometry (Moty Heiblum et al. '03)
- Telegraph Noise and fractional statistics
- Luttinger Liquid Theory

Laughlin Quasiparticle for ν = 1/m

- Energy Gap $\sigma_{xx} = 0$
- Quantized Hall Conductance $\sigma_{xy} = (1/m)e^2/h$

⟹ Fractionally Charged Excitations



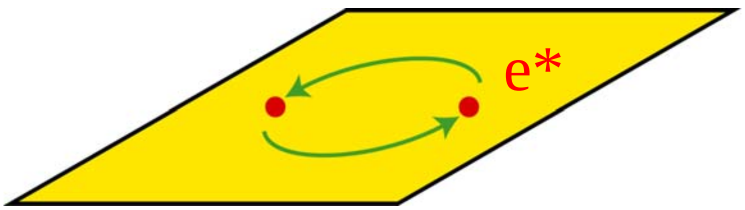
$$\frac{dQ}{dt} = \sigma_{xy} \oint E \cdot d\ell = \sigma_{xy} \frac{d\Phi}{dt}$$

$$Q = \sigma_{xy}(h/e) = e/m \equiv e^*$$

Fractional Statistics

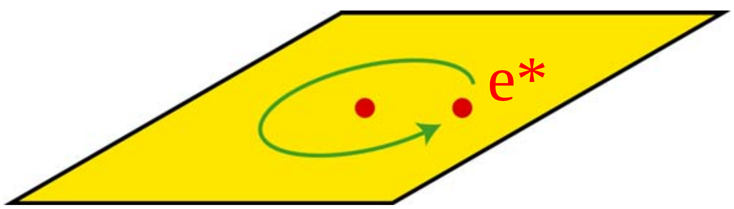
Halperin 1984;
Arovas, Schrieffer, Wilczek 1984

- Phase $\Theta = \pi/m$ under interchange:



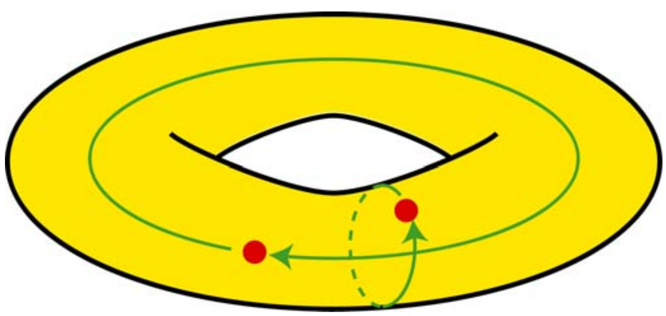
$$\Psi \to \Psi e^{i\Theta}$$

- Phase when one circles another:



$$\Psi \to \Psi e^{2i\Theta}$$

- Signature of topological order



Thouless 1989;
Wen, Niu 1990

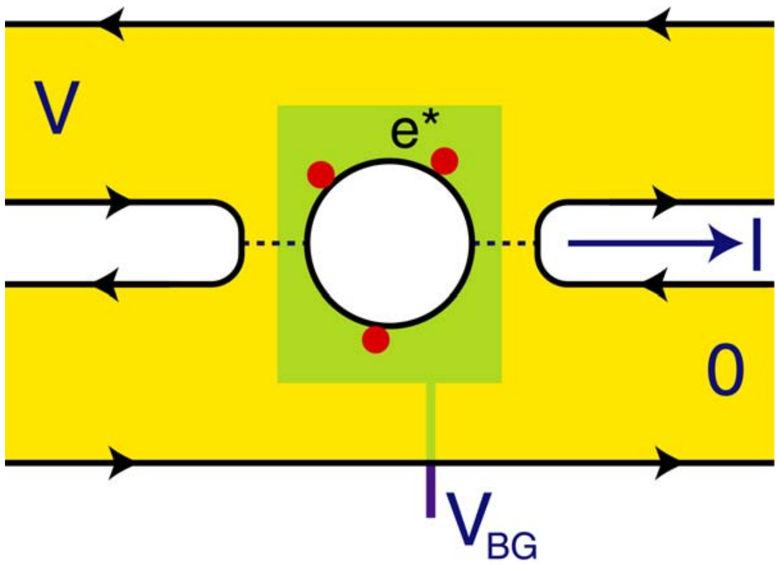
m-fold topological degeneracy on torus

Measurement of Fractional Charge

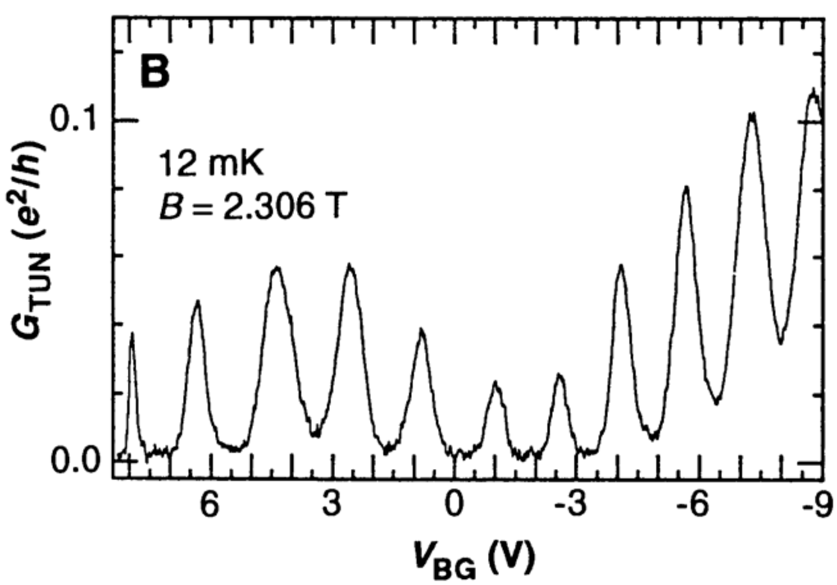
1. Capacitive Measurement

Resonant Tunneling Through Anti-dot

Goldman & Su (Science, 1995)



$$\Delta V_{BG} = e^*/C$$

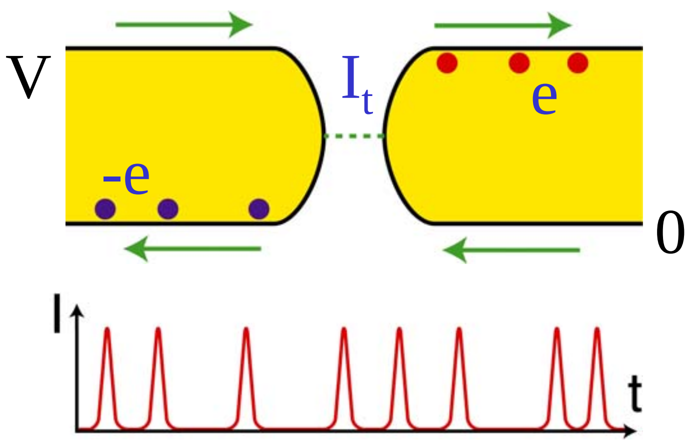


2. Shot Noise Measurement

Backscattering at Quantum Point Contact

Kane, Fisher (PRL 1994);
De Picciotto et al. (Nature 1997); Glattli et al. (PRL 1997)

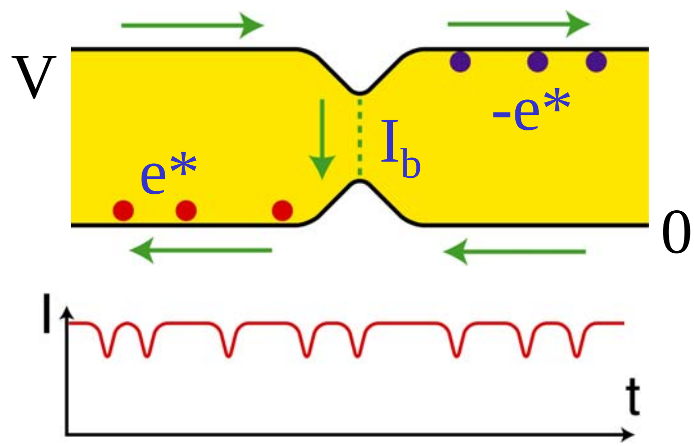
Strong Pinch Off



Electron Tunneling

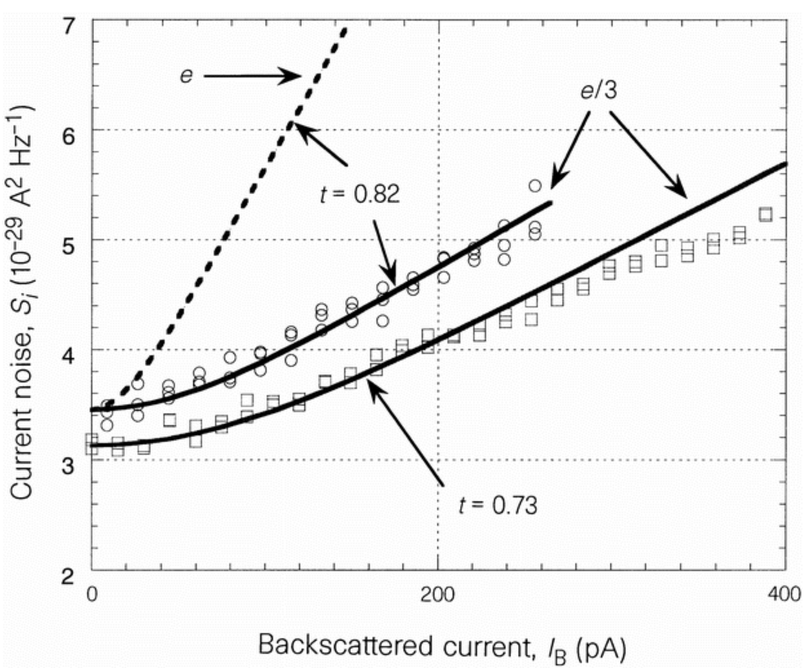
$$\langle I^2\rangle_{\omega\sim 0} = e\ I_t$$

Weak Pinch Off



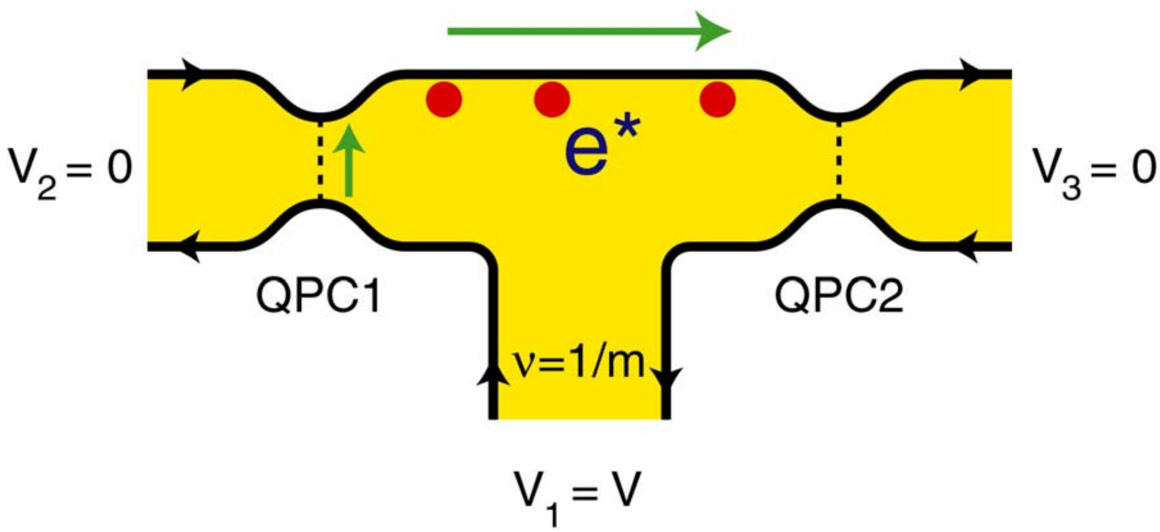
Quasiparticle Tunneling

$$\langle I^2\rangle_{\omega\sim 0} = e^*\ I_b$$

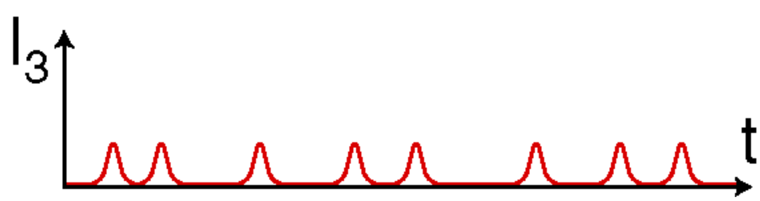


Three Terminal Configuration

Comforti et al. (Nature 2002)



- When QPC2 is open, a "Dilute Beam" of quasiparticles is isolated in lead 3.

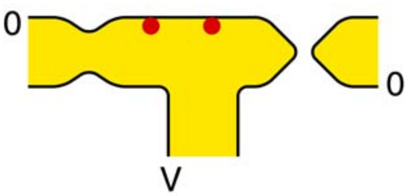


- Transport of Dilute beam through QPC2 probes transmission of individual quasiparticles.

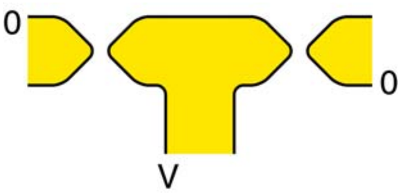
Transmission of Dilute Quasiparticles Through a Point Contact

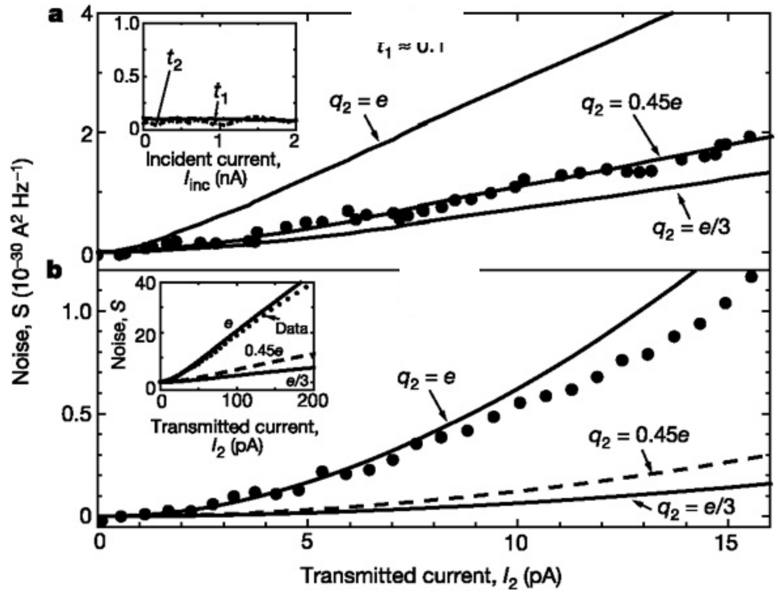
Comforti et al. (Nature 2002)

Dilute : q ~ .45 e

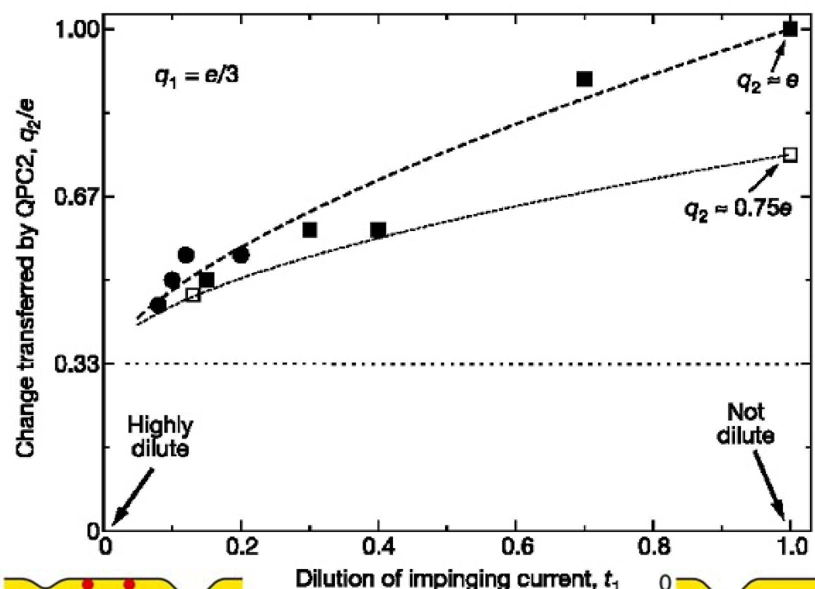


Not Dilute : q ~ e

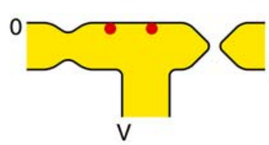


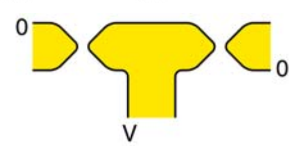


q



t_1





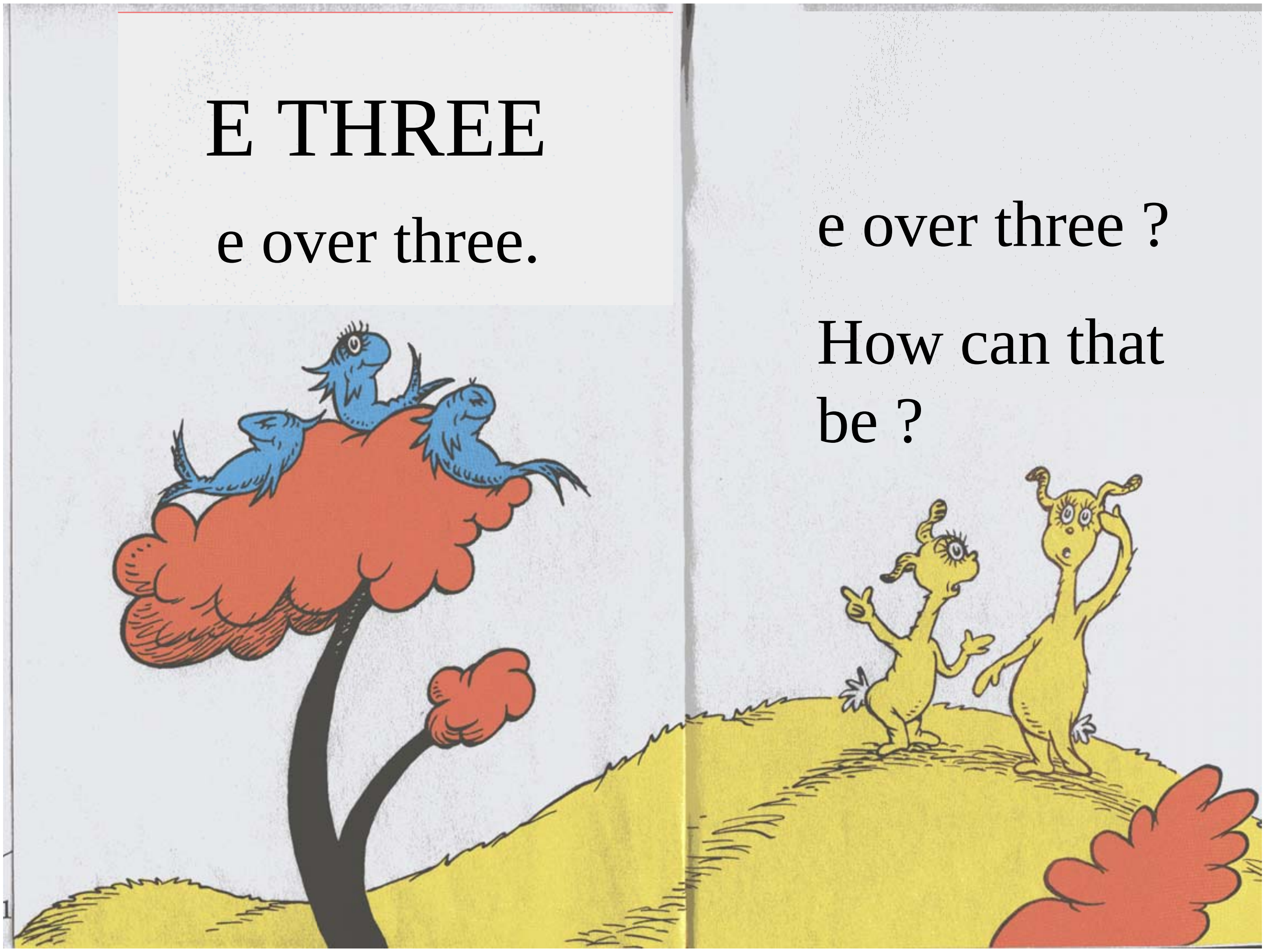
Can Dilute Charge e/3 Quasiparticles Traverse a Nearly Opaque Barrier ?

E THREE

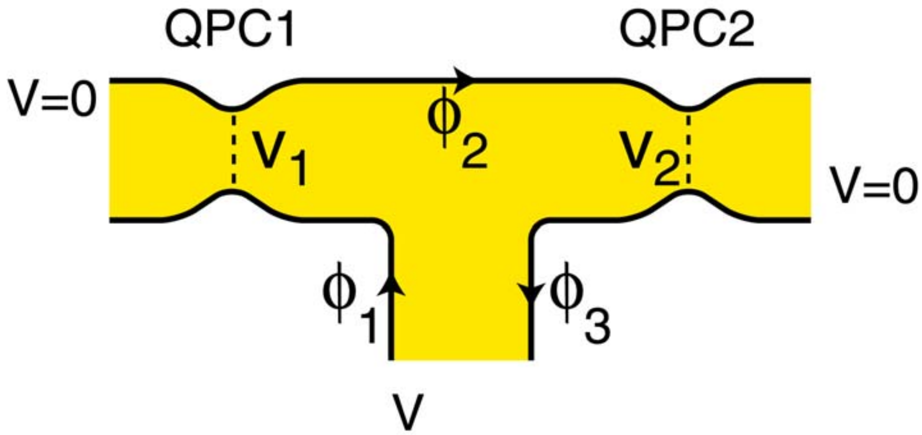
e over three.

e over three ?

How can that be ?



Luttinger Liquid Model : ν = 1/m



$$H = \frac{mv_F}{4\pi} \sum_{i=1}^{3} \int dx_i \left[\partial_x \phi_i(x_i)\right]^2 +$$

$$v_1 \cos(\phi_1 - \phi_2 - Vt) + v_2 \cos(\phi_2 - \phi_3)$$

Analysis:

QPC1:

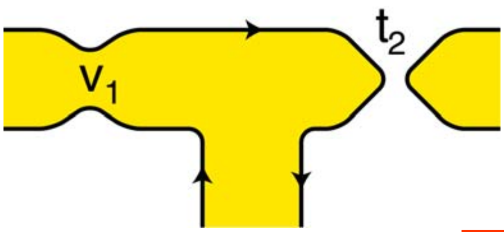
- Perturbative for small v_1 (Dilute Limit)

QPC2:

- Perturbative for small v_2
- Perturbative for large v_2 (small t_2)
- Exact Solution for m=2 via fermionization

Perturbative Analysis

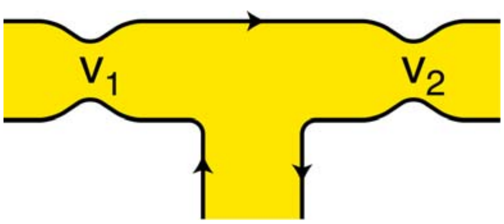
1. Small t_2 (T<<V)



$$I(V) = c v_1^2 t_2^2 V^{2m+2/m-3}$$
$$S(V) = e c v_1^2 t_2^2 V^{2m+2/m-3}$$

$$Q = e$$

2. Small v_2 (T<<V)



$$I(V,T) = c_1 \frac{v_1^2}{V^{1-2/m}} \left[1 - c_2 \frac{v_2^2}{T^{2-2/m}} \right]$$

$$S(V,T) = e^* c_1 \frac{v_1^2}{V^{1-2/m}} \left[1 - c_3 \frac{v_2^2}{T^{2-2/m}} \right]$$

$$Q = e^* + c \frac{v_2^2}{T^{2\text{-}2/m}}$$

Perturbation theory diverges for T=0 even for fixed finite V.

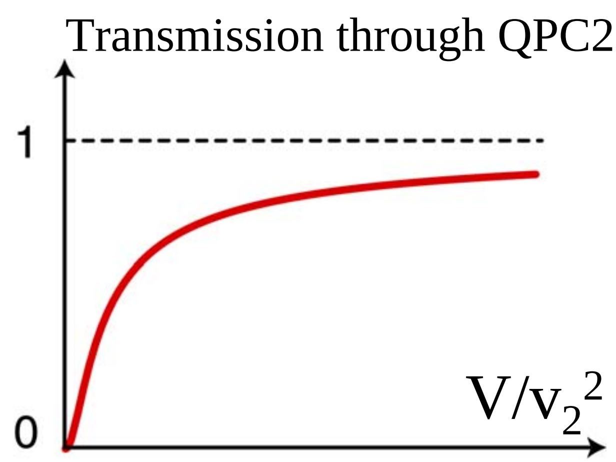
Exact Solution

For m=2 map problem to free fermions

1. Zero Temperature:

$$I(V) = I_{\text{in}}(V)\left(1 - \frac{2}{\pi}K\left(-\frac{V^2}{16\mathrm{v}_2^4}\right)\right)$$

$$S(V) = eI(V)$$



$Q = e$, independent of v_2, even when transmission through QPC2 is nearly perfect.

2. Finite Temperature:

Limits $T \rightarrow 0$ and $v_2 \rightarrow 0$ do not commute:

$Q(T \rightarrow 0, v_2 = 0) = e/m$

$Q(T = 0, v_2 \rightarrow 0) = e$

A nasty integral

$$\frac{8}{\pi^3}\int_R d^4y\,du\,\Theta(\{y_k\},u)\sin\left(\frac{u}{2X^2}\right)\frac{e^{-2(y_{12}+y_{34})}}{u y_{13} y_{24}}\frac{(y_1(y_3-u)+y_3(y_1-u))(y_2(y_4-u)+y_4(y_2-u))}{|y_1(y_1-u)y_2(y_2-u)y_3(y_3-u)y_4(y_4-u)|^{1/2}}$$

with $$\Theta(\{y_k\},u)=\begin{cases} 1 & \text{for} \quad y_1>u>y_2>y_3>0>y_4 \\ -1 & \text{for} \quad y_1>y_2>y_3>u>0>y_4 \\ -1 & \text{for} \quad y_1>u>0>y_2>y_3>y_4 \\ 0 & \text{otherwise.} \end{cases}$$

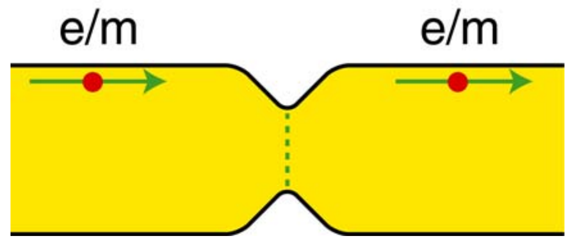
= 1 (independent of X) !

Interpretation: Andreev Reflection

Three possible scattering products
for an incident quasiparticle

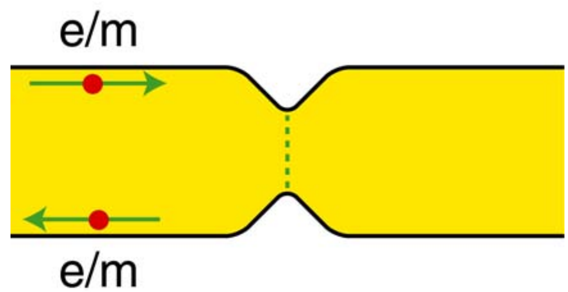
1. Transmission

Probability T



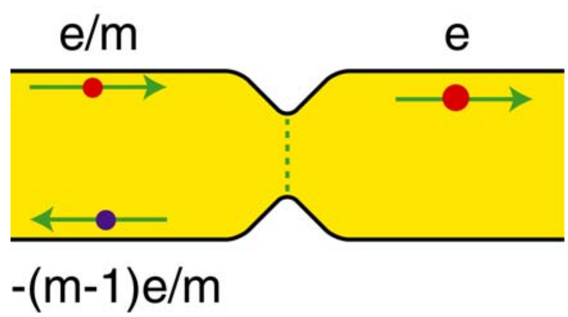
2. Reflection

Probability R



3. Andreev Reflection

Probability A

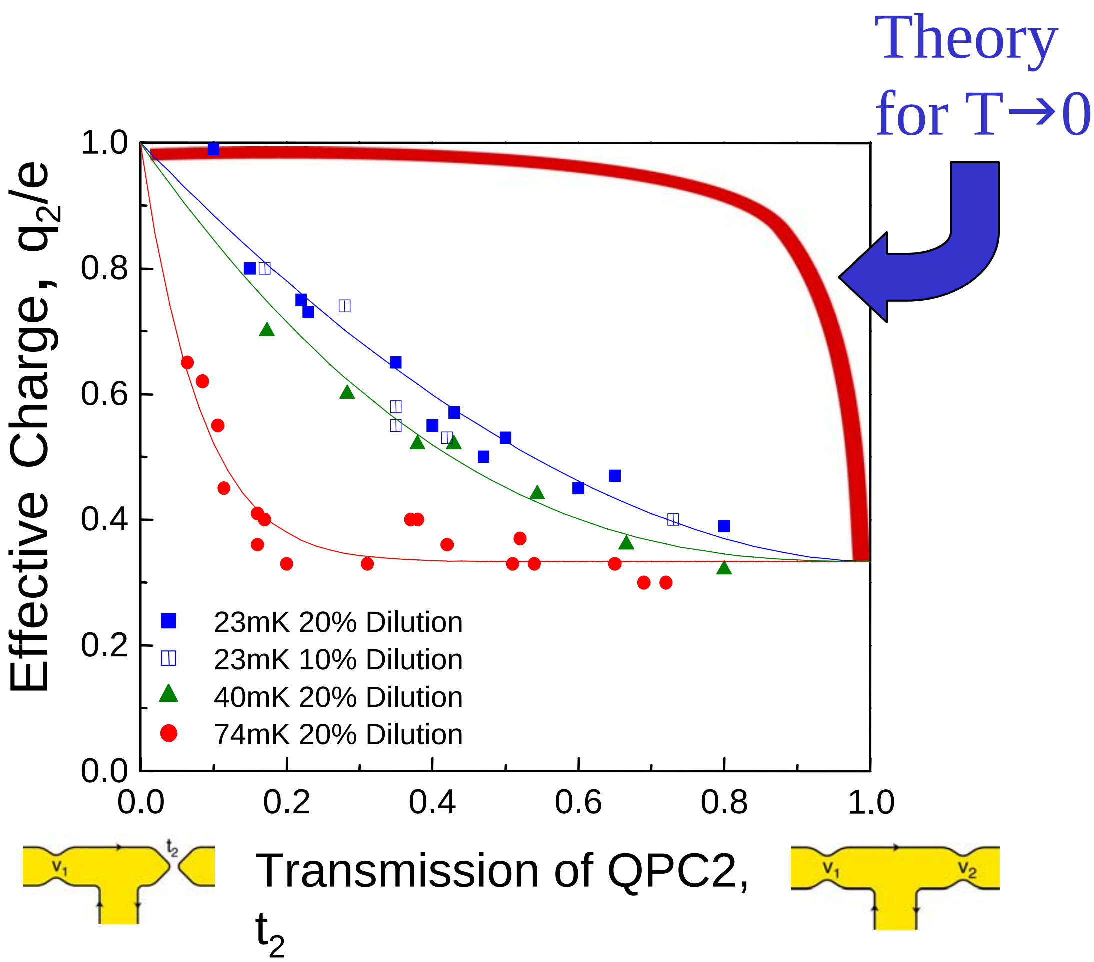


Theory predicts T=0 at zero temperature

- Strong Pinch-off: R~1, 0<A<<1
- Weak Pinch-off: R=1-1/m, A=1/m

Observe Andreev processes by measuring correlations between transmitted and reflected currents.

What about the experiments?



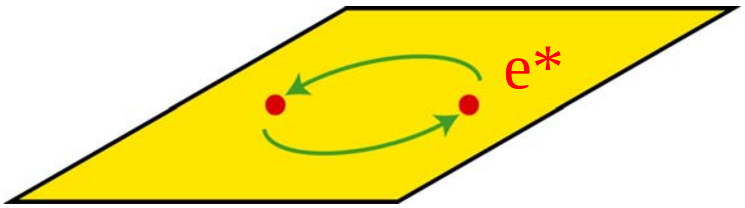
The measured charge does approach e for small t_2 and low temperature, but “high” temperature data remains unexplained.

- Role of irrelevant operators, eg. $(d\phi/dx)^3$
- Role of smooth edges near point contact.

Fractional Statistics

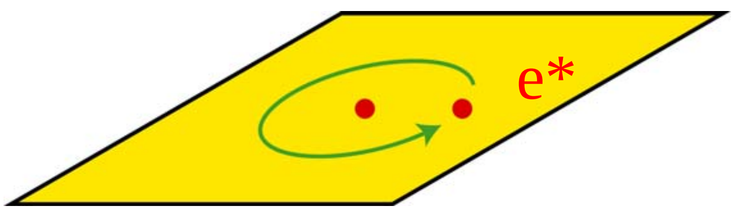
Halperin 1984;
Arovas, Schrieffer, Wilczek 1984

- Phase Θ = π/m under interchange:



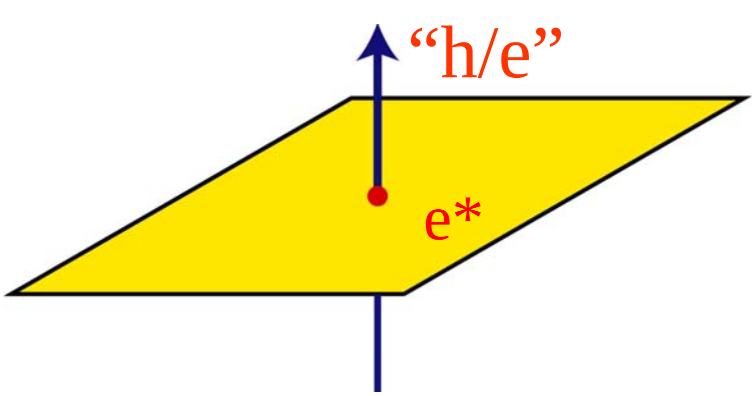
$$\Psi \rightarrow \Psi e^{i\Theta}$$

- Phase when one circles another:



$$\Psi \rightarrow \Psi e^{2i\Theta}$$

- Model: Bosons + Statistical Flux



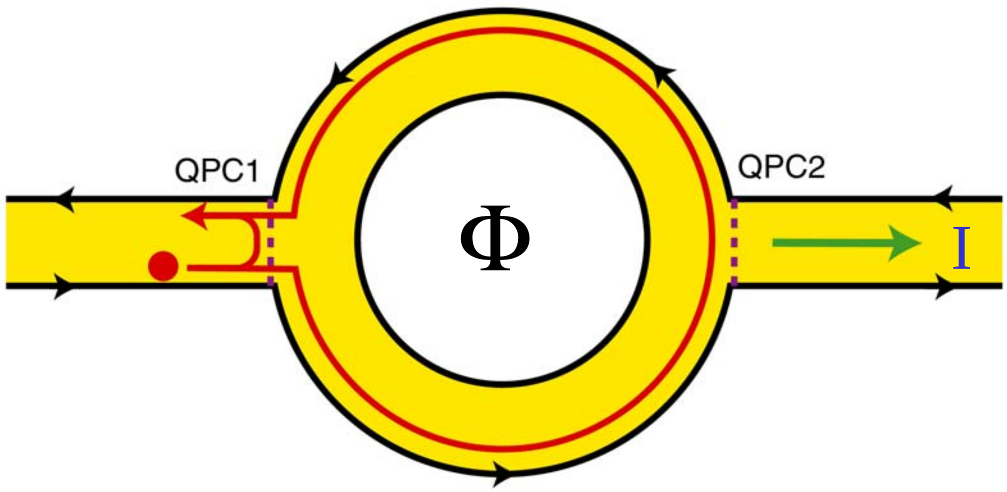
$$2\Theta = \frac{e^*}{\hbar}(h/e)$$
$$= 2\pi/m$$

Measurement of Fractional Statistics

Quantum Interference Necessary To
Measure Statistical Phase

Two Point Contact Interferometer

Chamon et al. (1997)



$$I = I_0 + \Delta I \cos[e^* \Phi / h + 2\Theta N_{qp}^{enc}]$$

Aharonov Bohm Phase — Statistical Phase

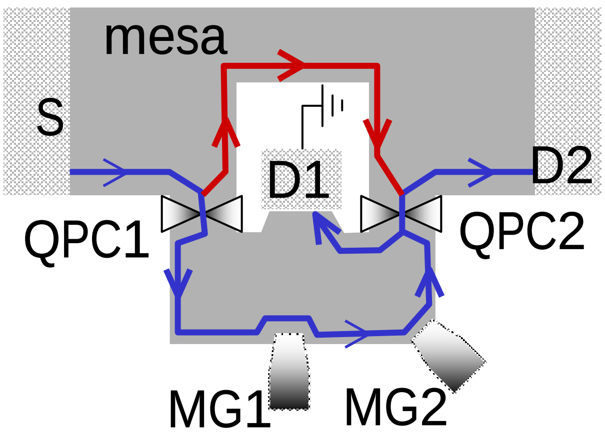
- Net phase changes by $2\Theta = 2\pi/m$ when qp tunnels from outside ring to inside of ring.

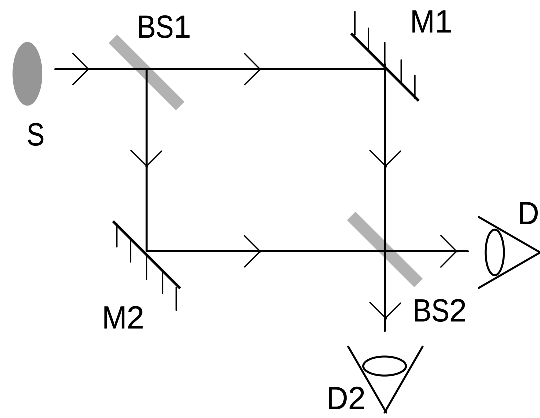
- Equilibrium h/e^* oscillations eliminated by qp tunneling.

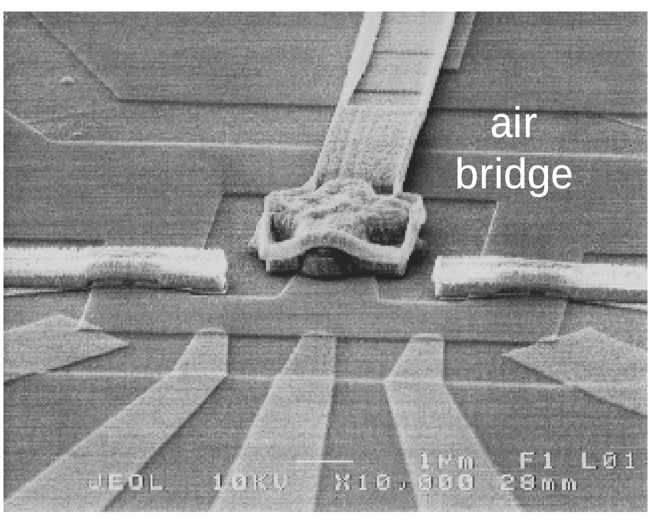
An Ohmic contact in the middle of a ring

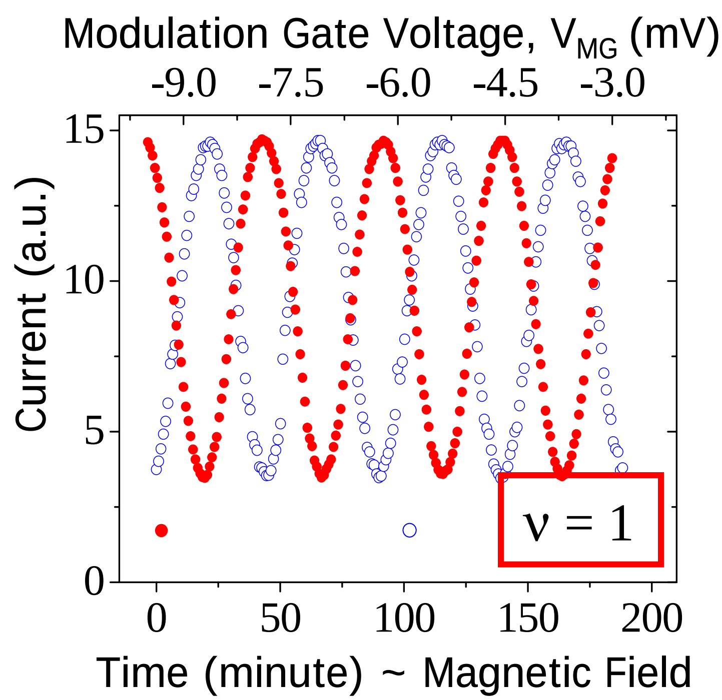
Y. Ji, Y. Chung, D. Sprinzak, M. Heiblum,
D. Mahalu and H. Shtrikman, Nature (03)

A “Mach-Zehnder Interferometer”



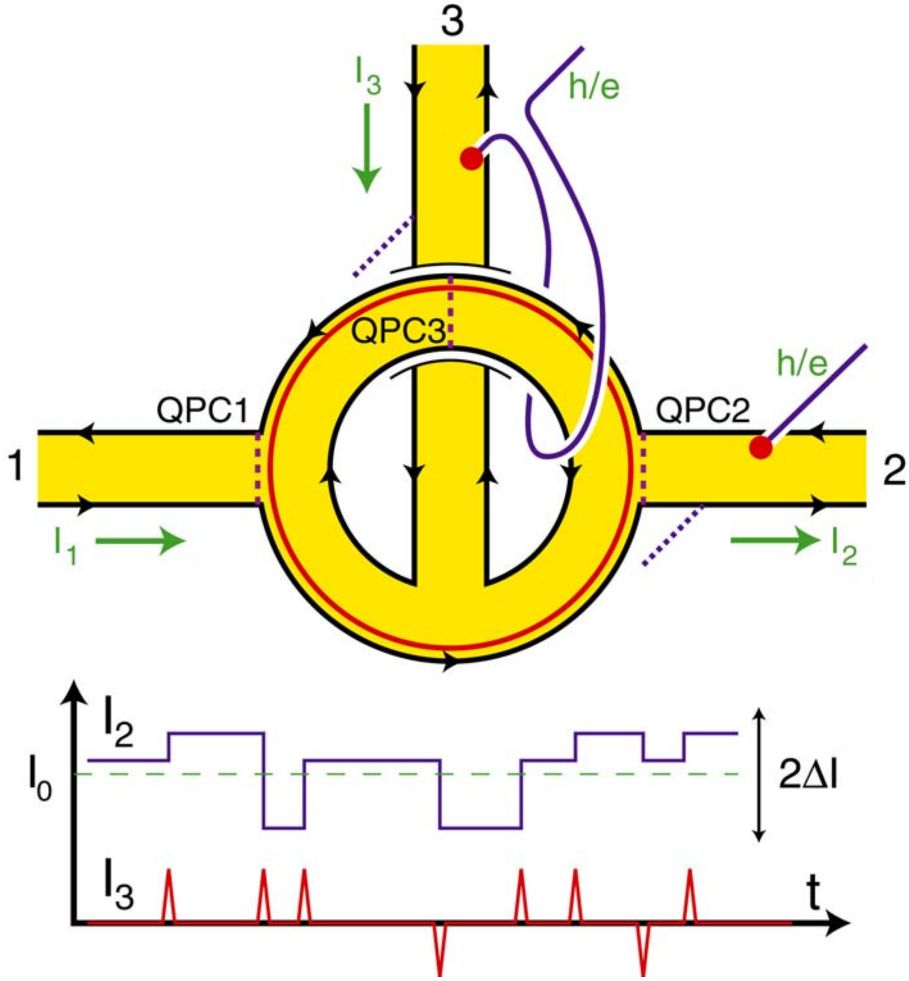






Telegraph Noise in ring with inner contact:

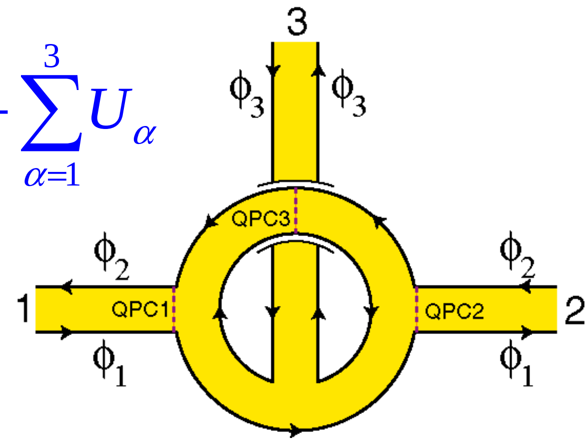
A Direct Signature of Fractional Statistics



- Net phase changes by $2\Theta = 2\pi/m$ when qp passes from lead 1 or 2 to lead 3.
- m-state telegraph noise for n = 1/m
- Related to m-fold "topological degeneracy"
- $\langle I^2 \rangle_{\omega\sim 0} = e^* \, \Delta I^2 / I_3$

Chiral Luttinger Liquid Model

$$H = \frac{mv_F}{4\pi}\sum_{i=1}^{3}\int dx_i\left[\partial_x\phi_i(x_i)\right]^2 + \sum_{\alpha=1}^{3} U_\alpha$$



Tunneling at QPCα :

$$U_\alpha = \mathrm{v}_\alpha \kappa_\alpha^{+} e^{i(\Delta\phi_\alpha - e^* V_\alpha t)} + \mathrm{v}_\alpha^{*} \kappa_\alpha^{-} e^{-i(\Delta\phi_\alpha - e^* V_\alpha t)}$$

Klein Factors

Low Frequency Noise :

$$\left\langle I^2 \right\rangle_{\omega\sim 0} = \frac{e^*\Delta I^2}{2I_3}\frac{\coth\left(e^* V_3/2T\right)}{1+\sin^2\Theta/\sinh^2\left(e^* V_3/2T\right)}$$

$$\sim \frac{e^*\Delta I^2}{2I_3} \qquad e^*V_3 >> kT$$

$$\sim \frac{e^{*2}\Delta I^2}{4G_3 T\sin^2\Theta} \qquad e^*V_3 << kT$$

Conclusion

- Fractionally charged quasiparticles can not traverse a nearly opaque barrier. At T=0 they can't even get past a nearly perfectly transmitting barrier!

- Telegraph Noise is predicted for a ring with an inner contact. It is a direct consequence of a fractional statistical phase, and has a unique signature in the low frequency noise.

Questions

- Origin of observed suppression of transmitted charge for moderately weak tunneling and moderately low temperature?
  Smooth edges?
  Irrelevant operators?
- Observation of Andreev processes?

- Exact Solution for n=1/3?

- Telegraph Noise for Hierarchical Quantum Hall States?

- Effect of dephasing on edge state transport?