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# N

## Neural Spikes, Identification from a Multielectrode Array

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- 9 92C20 Neural biology, 92C05 Biophysics, 92C42  
 10 Systems biology, networks

### 11 Synonyms and Abbreviations

- 12 [Action potential \(Spike\)](#); [Multi-electrode array \(MEA\)](#);  
 13 [Ordering points to identify the clustering structure](#)  
 14 [\(OPTICS algorithm\)](#); [Retinal ganglion cell \(RGC\)](#);  
 15 [Spacetime pixel \(Stixel\)](#); [Spike identification \(Spike](#)  
 16 [sorting\)](#)

### 17 Definitions

- 18 The brain and other neural tissue contain many types of  
 19 cells, notably including *neurons*, cells that are special-  
 20 ized for information processing and communication.  
 21 The output of most neuron types consists of *spikes*,  
 22 that is, rapid changes in the electrical potential across  
 23 their outer membrane. Each spike creates a detectable

disturbance in electric potential in the medium sur- 24  
 rounding the neuron. *Extracellular recording* of spikes 25  
 attempts to detect and analyze those disturbances, a 26  
 task that is complicated by the fact that an extracellular 27  
 electrode typically picks up signals from many differ- 28  
 ent neurons. Such signals must therefore be decom- 29  
 posed into contributions from each of the underlying 30  
 neurons, a procedure called *spike sorting*. Unambigu- 31  
 ous spike sorting is made easier by the recent avail- 32  
 ability of large, high-density *multi-electrode arrays* 33  
 (MEAs) that simultaneously monitor dozens or even 34  
 thousands of electrodes. This entry describes a class 35  
 of methods for sorting MEA data based on Bayes’s 36  
 formula (“Bayesian” spike sorting methods). 37

### Overview

38

The vertebrate retina is a popular model system for 39  
 neuroscience, in part because it is so amenable to 40  
 detailed study. Similar recordings can now also be 41  
 made in other brain areas [2]. However, recordings 42  
 obtained in this way are useful only if every spike can 43  
 be correctly assigned to the neuron that generated it 44  
 (the “spike sorting problem”). Reviews of early work 45  
 on spike sorting can be found in Lewicki [5] and Qui- 46  
 an Quiroga [10]. 47

Spike sorting is possible in principle because each 48  
 neuron is located at a fixed position relative to each 49  
 electrode, generating a distinctive pattern of excitation 50  
 amplitudes on the array of electrodes; also, the ampli- 51  
 tude and time course of each neuron’s spikes are at 52  
 least partly similar to each other, and different from 53  
 those of neighboring neurons. Nevertheless, it is a non- 54  
 trivial task to determine each of the ideal waveforms 55

56 (the “templates”), separating them from each other and  
 57 from noise. Moreover, in practice there can be signif-  
 58 icant variation in the spike waveforms from a given  
 59 neuron (for instance in amplitude), complicating the  
 60 task of determining from data which templates are  
 61 present in a sample. That is, spike sorting is a problem  
 62 in probabilistic inference.

63 This entry outlines an algorithm that carries out  
 64 this program [9], combining elements from several key  
 65 articles (e.g., [3, 6–8, 11]). Other relevant approaches  
 66 include [4, 12, 13].

### 67 Typical Experiment and Data

68 All illustrative data were recorded at 10 kHz from  
 69 albino guinea pig retina, presented with a stan-  
 70 dard random visual stimulus. Data were taken with  
 71 a 30-electrode MEA from MultiChannel Systems  
 72 (MCS GmbH, Reutlingen, Germany), covering about  
 73 0.018 mm<sup>2</sup> of retina.

74 The black curves in Fig. Fig. 1a,b show some repre-  
 75 sentative data, as arrays of graphs each representing a  
 76 time series of recorded potentials on a particular elec-  
 77 trode (or “channel”). In addition to identifiable spikes,  
 78 each electrode has activity that we will collectively  
 79 refer to as “noise.”

### 80 Spike Identification Method

81 Figure 2 summarizes the steps described below. After  
 82 data acquisition and high-pass filtering, the data are  
 83 packaged into two types of 3.2 ms clips: (a) “noise  
 84 clips,” in which the potential never crosses a threshold,  
 85 and (b) “spike events,” each surrounding a moment at  
 86 which the potential crosses (falls below)  $-4$  times the  
 87 standard deviation of the potential in the noise clips  
 88 [11]. A small subset of the spike events was extracted  
 89 to speed up the analysis steps shown in dashed lines in  
 90 Fig. Fig. 2.

### 91 Clustering and Template Building

92 Each spike event consists of  $N = 3.2 \text{ ms} \times 10 \text{ kHz} \times$   
 93  $5 \times 6 = 960$  numbers, the potentials on a  $32 \times 5 \times 6$  grid  
 94 of spacetime pixels (“stixels”). Each event involves  
 95 a superposition of spikes drawn from an unknown  
 96 number of classes corresponding to distinct neurons.

97 The first step is to find those classes, including charac-  
 98 terizing each class’s mean waveform and its variability.  
 99 That is, we must *cluster* the spike events.

100 A powerful algorithm well suited to this task is  
 101 OPTICS [1]. Strictly speaking, OPTICS does not clus-  
 102 ter data; instead, it reorders a given set of points into  
 103 a single linear sequence in which similar elements are  
 104 placed close to each other. If a feature such as overall  
 105 amplitude varies continuously among exemplars, they  
 106 are grouped together; if that feature is bunched into  
 107 two or more clusters, they will be visibly separated in  
 108 the sequence. A human operator can then rapidly scan  
 109 the ordered list of exemplars and cut it into batches  
 110 corresponding to distinct clusters [9].

111 The steps described above produce clusters, that is,  
 112 collections of similar events (“exemplars” of the clus-  
 113 ter). The next step is to create a consensus waveform  
 114 (“template”) summarizing each cluster, and character-  
 115 ize the deviations from that consensus. Figure Fig. 3  
 116 shows the result of taking the template to be the point-  
 117 wise median of the aligned exemplars in a cluster. A  
 118 particular exemplar may contain other activity besides  
 119 the spike of interest. Choosing the median prevents  
 120 such chance collisions from influencing the template,  
 121 because at any particular stixel most exemplars do *not*  
 122 display any additional spike.

123 Individual instances of a particular spike type will  
 124 deviate from the template. However, at least in guinea  
 125 pig retina, the most significant sources of variation are  
 126 (a) additive noise and (b) overall multiplicative rescal-  
 127 ing of the spike’s amplitude. To quantify (b), for each  
 128 exemplar the method finds the overall rescaling factor  
 129  $A$  that optimizes the overlap of that exemplar and the  
 130 template, then stores the mean and variance of those  
 131 factors in a lookup table for later use as a prior prob-  
 132 ability (2). Finally, it logs the number of exemplars in  
 133 each template, converts to an approximate firing rate,  
 134 and saves those rates, again for later use as a prior.

135 In the discussion below, the index  $\mu$  represents tem-  
 136 plate type; the symbol  $F_{\mu;x,y,t}$  refers to the potential  
 137 of template  $\mu$ , on the electrode with address  $x, y$ , at  
 138 time  $t$ .

### Spike Fitting

139 The preceding steps yield templates of various discrete  
 140 types, indexed by  $\mu$ . Within each type, there are also  
 141 continuous variations in amplitude, which we express  
 142 as an overall multiplicative factor  $A$  relative to the  
 143

144 template; there is also a choice of firing time  $t_1$ . A  
 145 “spike descriptor” is a specification of all these vari-  
 146 ables. The next stage of spike sorting is to identify what  
 147 spike(s) are present in each event of the full dataset  
 148 (“spike fitting”). The strategy is to evaluate the pos-  
 149 terior probability of each spike descriptor given that  
 150 event, marginalize over uninteresting variations within  
 151 that type (the value of  $A$ ), then maximize over the  
 152 remaining variables ( $\mu$  and  $t_1$ ).

### 153 Generative Model

154 To obtain the posterior probability, one must find for-  
 155 mulas for the prior probability of a spike descriptor,  
 156 and for the likelihood (probability that a particular  
 157 waveform would occur if that spike were present). That  
 158 is, we must specify an explicit generative model of the  
 159 data [8].

160 Before writing formulas, we first summarize in  
 161 words the general assumptions of the generative  
 162 model. The assumptions are: (1) Each neuron gener-  
 163 ates spike waveforms that are all identical, apart from  
 164 overall amplitude scale and additive noise; (2) The sig-  
 165 nal (spikes) and the noise are statistically independent  
 166 of each other; (3) The signal and noise sum linearly;  
 167 (4a) The noise, and (4b) the variability of spike ampli-  
 168 tudes, are well described by Gaussian distributions;  
 169 and (5) The prior probability that each neuron will fire  
 170 is independent of its, and the others’, histories, and of  
 171 the stimulus.

172 Assumption (4a) implies that the noise is char-  
 173 acterized by a covariance matrix,  $C$ . Evaluating  $C$   
 174 empirically on noise clips shows that: It is approx-  
 175 imately diagonal, and translation-invariant, in space;  
 176 and it is approximately stationary, that is, invari-  
 177 ant under time shifts. Moreover, its dependence on  
 178 time is roughly exponential:  $C(x, y, t; x', y', t') =$   
 179  $\eta \delta_{x,x'} \delta_{y,y'} e^{-|t-t'|/\tau}$ . That is,  $C$  is determined by just  
 180 two empirical quantities, the strength  $\eta$  and correla-  
 181 tion time  $\tau$  of the noise. In this formula,  $\delta_{x,x'}$  is the  
 182 Kronecker symbol.

183 We can now express the content of assumption  
 184 (4a). We regard  $x, y, t$  as a single  $N$ -valued index  
 185 and describe a noise clip by an  $N$ -component vec-  
 186 tor  $\mathbf{V}$  of potentials. Then the noise model states that  
 187 the probability density function for noise samples is  
 188  $P_{\text{noise}}(\mathbf{V}) d^N \mathbf{V} = (2\pi)^{-N/2} (\det C)^{-1/2} e^{-\mathbf{V}^T C^{-1} \mathbf{V} / 2} d^N \mathbf{V}$ .

### Fitting a Single Spike

189 Given an event, we wish to know if it contains any  
 190 spikes, and if so to identify them. First, temporarily  
 191 suppose that we know that the event contains exactly  
 192 one spike. We wish to know the spike’s type  $\mu$  and time  
 193 of occurrence  $t_1$ . Our best estimate of these quantities  
 194 comes from maximizing the posterior probability den-  
 195 sity  $\mathcal{P}(\mu, t_1 | \text{event})$ , where “event” is the recorded time  
 196 series of potentials on each electrode. This density is in  
 197 turn obtained by marginalizing  $\mathcal{P}(\mu, t_1, A | \text{event})$  over  
 198  $A$ , the amplitude scale factor of the spike relative to the  
 199 template.  
 200

Using Bayes’s formula, we can obtain  $\mathcal{P}$  as a  
 201 constant times a likelihood times a prior, or  
 202

$$\begin{aligned} & \mathcal{P}(\mu, t_1, A | \text{event}) dt_1 dA & 203 \\ & = K P(\text{event} | \mu, t_1, A) P(\mu, t_1, A) dt_1 dA, \quad (1) & 204 \end{aligned}$$

where  $K$  is independent of  $\mu, A, t_1$ . The differen-  
 205 tial  $dA dt_1$  reminds us that  $\mathcal{P}$  is a probability density  
 206 function, with units  $s^{-1}$ .  
 207

The likelihood function describes the distribution of  
 208 actual observations given the ideal spike. The assump-  
 209 tions outlined earlier amount to supposing that the  
 210 observed signal will differ from the rescaled ideal by  
 211 additive noise, so we simply write the likelihood as  
 212  $P(\text{event} | \mu, t_1, A) = P_{\text{noise}}(\delta \mathbf{V})$ , where  $\delta \mathbf{V} = \mathbf{V} -$   
 213  $A \mathbf{F}_{\mu, t_1}$ . In this formula, the shifted template vector  
 214  $\mathbf{F}_{\mu, t_1}$  has  $x, y, t$  component equal to  $F_{\mu; x, y, t(t-t_1)}$ .  
 215

Turning to the prior, assumptions (4b) and (5) give  
 216 it as  
 217

$$\begin{aligned} & P(\mu, t_1, A) dt_1 dA & 218 \\ & = (r_\mu dt_1) ((2\pi \sigma_\mu^2)^{-1/2} e^{-(A-\gamma_\mu)^2 / 2\sigma_\mu^2} dA), \quad (2) & 219 \end{aligned}$$

where  $\gamma_\mu$  is the mean and  $\sigma_\mu^2$  the variance of the scale  
 220 factor for cluster  $\mu$ ;  $r_\mu$  is the estimated overall rate of  
 221 firing for this cluster. Combining with the likelihood  
 222 function gives the posterior probability density, which  
 223 can readily be marginalized (integrated) over all values  
 224 of the amplitude scale factor  $A$ , because it is a Gaus-  
 225 sian function of  $A$  [9]. Maximizing over  $\mu$  and  $t_1$  then  
 226 identifies the most probable spike and its firing time.  
 227

### Multiple Spikes

228 In principle, one could extend the method of  
 229 the preceding subsection to compare the probabil-  
 230 ities of all possible combinations of two or more  
 231

232 spikes. Such an exhaustive approach, however, quickly  
 233 becomes impractical. Instead, note that even if an  
 234 event contains multiple spikes, the steps in outlined  
 235 above still identify that template whose subtraction  
 236 would lead to the largest increase in the probability  
 237 that the remaining waveform is noise. Thus, instead  
 238 of the exhaustive approach, one can use an iterative  
 239 (matching-pursuit or “greedy”) approach [9, 11]: Start-  
 240 ing with a spike event, find the absolute peak, fit it,  
 241 subtract the fit, and then repeat the process.

242 Any such iterative process must determine when to  
 243 stop fitting spikes. After marginalizing the expression  
 244 for the posterior probability over  $A$  and  $t_1$ , one can  
 245 simply divide by a similar expression for the prob-  
 246 ability that *no* additional spike was present (namely  
 247  $K P_{\text{noise}}(\mathbf{V}) P(\text{no spike})$ ). The unknown constant  $K$   
 248 cancels in this probability ratio, as do the rate factors  $r_\mu$   
 249 for all spikes found up to this point. We can then say  
 250 that fitting an additional spike is justified if the ratio  
 251 exceeds unity for some  $\mu_*$  and terminate the fitting  
 252 loop when that significance test fails.

### 253 Cluster Reliability

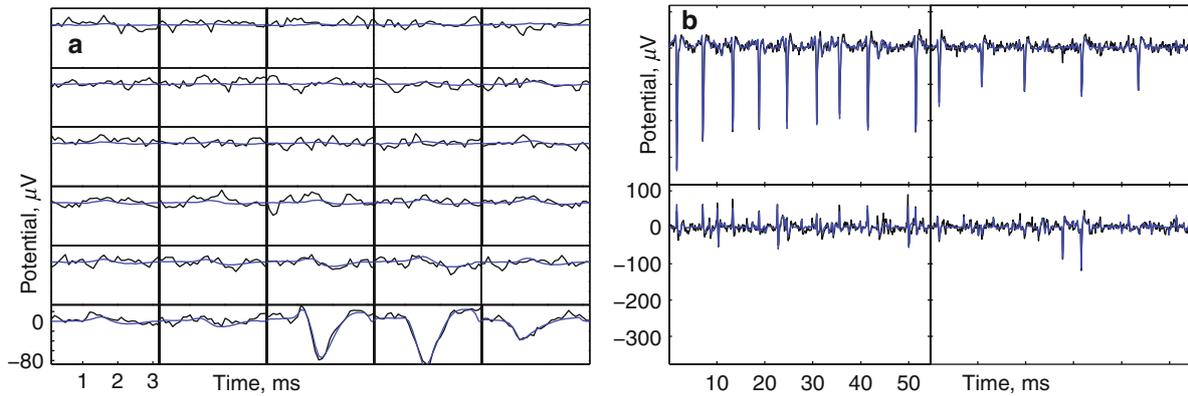
254 The last step in Fig. Fig. 2 is to determine which neu-  
 255 rons’ activities have been reliably captured. No method  
 256 will succeed in identifying spikes from every neu-  
 257 ron; for example, some will generate spikes whose  
 258 amplitude is too low relative to the noise. Also, some  
 259 neurons are gradually dying, or otherwise changing  
 260 character, during an experiment. Various criteria can  
 261 be imposed at this point to determine which of the  
 262 templates’ inferred spike trains should be trusted and  
 263 retained for later analysis [9].

### 264 Value of Bayesian Approach

265 The preceding discussion may have given the impres-  
 266 sion that the key elements in spike sorting are mathe-  
 267 matical. On the contrary, it is the resolving power of  
 268 the MEA approach itself, combined with the planar  
 269 geometry of the retina, that permit such thorough spike  
 270 identification. The Bayesian method described here  
 271 merely helps to use this resolving power to greatest  
 272 advantage.

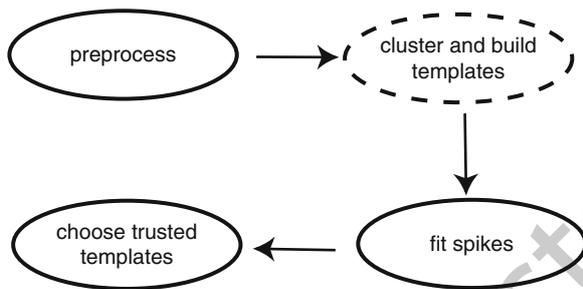
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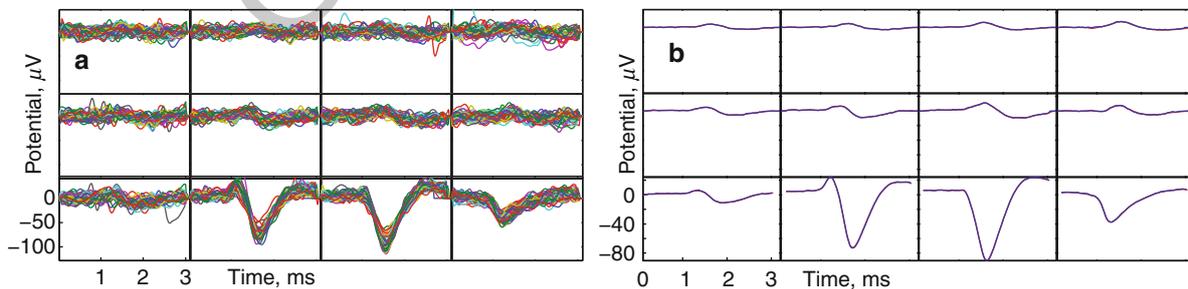


**Neural Spikes, Identification from a Multielectrode Array, Fig. 1** (a) Example of a single-spike event. Each sub-panel shows the time course of electrical potential (in  $\mu\text{V}$ , *black curves*), on a particular electrode in the  $5 \times 6$  array. The electrodes are separated by  $30 \mu\text{m}$  (similar to RGC spacing). *Blue curves* show the result of spike sorting, in this case a single

template waveform representing an individual neuron. (b) Detail of a more complex event and its fit, in which a single neuron fires a burst of nine spikes of varying amplitudes (*upper left channel*), while a different neuron fires five other spikes (*upper right channel*)



**Neural Spikes, Identification from a Multielectrode Array, Fig. 2** Schematic of spike sorting method



**Neural Spikes, Identification from a Multielectrode Array, Fig. 3** (a) Detail of 40 of the aligned exemplars used to compute a template, showing the potential on 12 neighboring electrodes. Some outlier traces reflect events in which this neuron fired together with some other neuron; the unwanted peaks

occur at random times relative to the one of interest, and thus do not affect the template. (b) *Blue*, detail of template waveform generated from (a). *Red*, for comparison, the pointwise mean of the 430 waveforms used to find this template. (The red and blue traces are too close to discriminate visually)