

Effects of Greenhouse Gases on Earth, Venus, and Mars: Beyond the One-Blanket Model

Philip C. Nelson

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia PA 19104 USA

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I present a discussion of the effect of increasing carbon dioxide on planetary climate, at a level suitable for insertion as a module into an upper-level Physics course. The treatment includes two key ingredients that are often missing from more elementary discussions, yet are amenable to analytic methods: First, that convection implies a dependence of surface temperature on the height of the outermost infrared-thick layer; and second, that increasing the level of CO₂ closes spectral windows of absorption. These themes are applicable not only to an industrializing Earth, but also to our neighboring planets.

I. INTRODUCTION

Many Physics students have been exposed to models of the warming influence of increasing CO₂, but often via a very simple “one-blanket” model. Although that is a good start, students may then be left wondering:

- “The most abundant greenhouse gas by far is H₂O, and we have no control over it, so how can reducing CO₂ emission be relevant?”
 - “In the CO₂ absorption bands the atmosphere is already completely opaque, and you can’t exceed 100% absorption, so how can adding more hurt?”
- (1)

Every citizen needs some crisp, qualitative answers to these questions. One sometimes gets the impression that to offer convincing replies requires considerable formal machinery, including solution of the Schwarzschild equation of radiative transport. Indeed, some textbook discussions jump directly from a highly simplified model to the conclusions of state-of-the-art general circulation models, which students must then take on faith.

This article will attempt to bridge the gap in a way that can be inserted as a short module into a standard existing course such as “Modern” Physics (or indeed nearly any upper-level course); or in specialized courses on the physics of energy or environment.¹ Even students who will later pursue detailed analysis can benefit from exploring models that are simple enough to expose *mechanisms*, and they may find the graphical reasoning here to be a useful guidepost (and useful when discussing with non-physicists). Students will also recognize that similar considerations also apply to exoplanets.²

The main points to be made here are available in more technical publications, for example, Ref. 3; the goal of the present article is to present them in a brief format. Features that are unusual for treatments at this level include discussion of the role of convection via graphical arguments (and dimensional analysis), as well as some observational data that are not usually shown. I also discuss how to avert potential confusion by spotlighting the simple mathematical fact that $\langle g \circ f \rangle \neq g(\langle f \rangle)$ if g is a nonlinear function (Sect. VB below, presented graphically in Fig. 7 and applied in Fig. 8). Finally, Sect. V addresses the motivating questions (1) above with a discussion of complex molecular spectra, leading up to a semiquantitative exercise for students. Many other important topics are omitted for brevity, including detailed radiative transport, full 3D modeling, the effects of oceans, cloud feedback, and so on. Interested readers will find some of these omissions rectified in intermediate-level books^{4–9} and articles^{10–15} and references therein.

Suggested answers to queries (1) above appear in the Conclusion.

II. BACKGROUND

The following sections will rely on some material from the Physics curriculum, summarized briefly here. Thermal radiation can be emitted even by a body that is not “black,” which can be characterized by a wavelength-dependent emissivity factor $\mathcal{E}(\lambda)$, equal to 1 for a perfectly absorbing body (Kirchhoff law). If the emissivity is roughly constant over the wavelength range where emission is nonnegligible, then the body’s total irradiance (energy emitted per area of surface) is given by the Stefan–Boltzmann law

$$\text{irradiance} = \mathcal{E}\sigma_{\text{SB}}T^4, \quad (2)$$

where $\sigma_{\text{SB}} = \pi^2k_{\text{B}}^4/(60\hbar^3c^2) \approx 5.7 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the Stefan–Boltzmann constant. That energy flux can be broken down as a density in wavelength λ (a modified Planck spectrum):

$$d(\text{irradiance}) = \mathcal{E}(\lambda)4\pi^2\hbar c^2\lambda^{-5}(e^x - 1)^{-1}d\lambda, \quad \text{where } x = 2\pi\hbar c/(\lambda k_{\text{B}}T). \quad (3)$$

A single molecule’s ability to absorb light can be described by the value of absorption cross-section, an area that we will call a_1 . The probability for a photon to be absorbed by a gas containing such molecules is then given by the Beer–Lambert–Bouguer rule:

$$\text{Prob}(\text{absorbed}) = 1 - e^{-a_1(\lambda)\rho b}, \quad (4)$$

where ρ is number density of absorbers and b is path length. (Sect. IV C will generalize to the case where ρ is not constant.)

The absorption cross-section spectrum for a molecular species in gas form generally consists of peaks corresponding to state transition energy differences. Those peaks broaden when gas pressure is increased, because molecular collisions start to become significant. Not every possible state transition will actually couple strongly to electromagnetic

radiation; those with a nonzero transition dipole moment have much higher mean rates than those without. Homonuclear, diatomic molecules such as O_2 or N_2 have no ground-state dipole moment, nor can they acquire one by bending or stretching, so their transitions in the infrared spectrum are decoupled from light. More complex molecules, such as CO_2 and H_2O , do have allowed transitions and are strongly IR-active. (For a lecture demonstration showing IR activity of CO_2 gas, see Refs. 16 and 17.)

III. NAKED PLANETS

We begin by thinking about the temperature of an object that is heated by a distant source of light, for example, a planet or moon in our solar system. Each of those objects has a light and a dark side at any moment, but each is rotating; we will make the rough approximation that each is subjected to a solar energy infall that is the average of day and night. Incoming light also lands at various angles depending on position on the surface: Generally the angle is more oblique at the poles than at the equator. We will make the even rougher approximation of neglecting all these effects, imagining that all energy intercepted from the Sun is spread uniformly over the planet’s surface.

Despite the huge energy infall, Earth’s average surface temperature is changing only slowly; *every watt of incoming solar energy must get sent back into space*. For a mechanical metaphor, imagine a system with a simple self-regulation mechanism. Water flows into a bucket at a fixed rate. A leak at the bottom of the bucket lets water out at a rate that depends on its pressure. The pressure increases with increasing depth of water in the bucket. Suppose that the bucket starts empty. Initially, inflow exceeds outflow, and the level rises. Eventually, however, the system arrives at a steady state, in which its water level sets an exit rate that matches the inflow. If we increase the inflow rate, the system will stabilize at a new, higher steady level. If instead we shrink the exit hole, we again find that the eventual steady level is higher than initially.

A fixed radiant energy input to a planet’s surface also brings it to a steady state, in which its output of thermal radiation balances the input. Satellites measure the power radiated by the Sun as $\mathcal{P}_{\text{sun}} \approx 3.9 \cdot 10^{26}$ W. Most of this energy disappears into space, missing the planet. Only the fraction $(\pi R_p^2)/(4\pi D_p^2)$ intercepts it, where R_p is the planet’s radius and D_p its distance. Some of the incoming solar energy is reflected, for example, by clouds. We account for this by reducing the solar input by a factor $1 - \mathcal{R}$, where the reflectivity (also called “Bond albedo”) \mathcal{R} is a dimensionless number determined by observation.¹⁸ We will call this reduced total input the planet’s net insolation. Per planet surface area, it is given by¹⁹

$$\text{net insolation} = (1 - \mathcal{R}) \frac{\mathcal{P}_{\text{sun}}}{16\pi D_p^2} \approx 240 \text{ W/m}^2 \quad \text{for Earth.} \quad (5)$$

Table I gives some values for \mathcal{R} and D_p .²⁰

TABLE I.

planet	distance D_p [10^{11} m]	reflectivity \mathcal{R}	predicted naked-planet temperature [K]	observed mean surface temperature T_{surf} [K]
Earth	1.50	0.31	254	288
Venus	1.08	0.76	230	737
Mars	2.3	0.25	210	210

The planet’s temperature must rise until its thermal emission matches the net insolation, similarly to the stabilizing mechanism in the “bucket” metaphor earlier in this section. For a planet with little or no atmosphere (“naked planet”), the emission rate is $\mathcal{E}\sigma_{\text{SB}}T_{\text{surf}}^4$. Because the incoming energy flow is reduced by the factors in Eq. 5, the predicted surface temperature T_{surf} is far lower than T_{sun} . In fact, all three of the planets we are considering arrive at steady-state temperatures for which they emit most of their energy in the mid-infrared wavelength range ($\lambda = 10\text{--}20 \mu\text{m}$). In this range, their emissivities are $\approx 100\%$.²¹ The fourth column of the table shows the predictions from this model.

Our oversimplified approach has fortuitously done well at predicting the mean surface temperature of Mars. However, the other two planets are failures: For Earth, our estimate of mean surface temperature is below the freezing point of water.²² Venus, with its higher reflectivity, was predicted to be even colder, and yet its actual surface temperature is hot enough to melt lead! So we have missed something big; however, the following section will show that the overall strategy begun here can be retained once we acknowledge that missing piece.

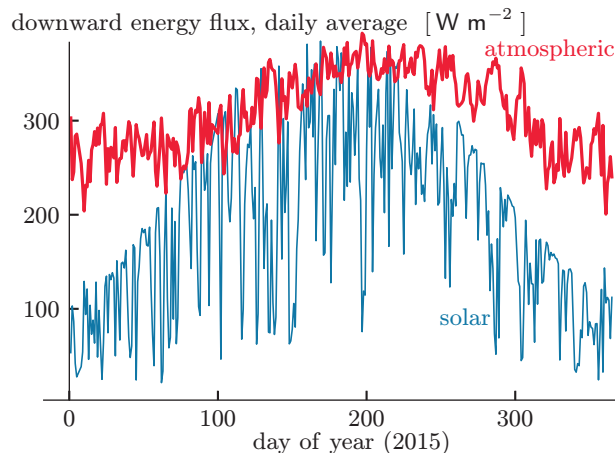


FIG. 1. [Observational data.] **Origins of infrared radiation arriving at Earth’s surface.** The thin lower curve (blue) shows the spectral irradiance integrated over all downward directions and over the band 300 to 2800 nm, which includes nearly all light coming directly from the Sun but excludes nearly all thermal radiation at Earth’s temperature. The day-to-day fluctuations arise from changes in cloud cover. The heavy upper curve (red) shows the downward irradiance integrated over the band 4500 to 42000 nm, which excludes nearly all light coming directly from the Sun but includes nearly all thermal radiation at Earth’s surface temperature. (Data courtesy Peter Pilewskie; see Ref. 24.)

IV. EARTH AND VENUS, DRESSED IN GRAY

We may guess that the predictions in Sect. III were unsuccessful in part because Earth and Venus are both “clothed” in mixtures of gases. But it’s puzzling, because:

- The dominant air molecules (including N_2 , O_2 , and CO_2) are all weakly absorbing throughout much of the frequency range of the incoming solar radiation (visible and near-IR). So incoming solar energy really does end up mainly warming the surface, as we assumed.
- Although water droplets absorb some infrared radiation present in the solar spectrum, gravity constrains them to be near the surface; we can lump this solar absorption with that of the surface. Water droplets also have a big effect on *visible* light, but mainly to reflect or otherwise scatter it; we already accounted for reflection.
- Although hard ultraviolet does get absorbed by oxygen, it is a minor fraction of the total incoming energy flux.

So it may not be obvious what is wrong with the calculations in Sect. III. In a nutshell, this section will argue that:²³

- On a clothed planet, the Stefan–Boltzmann law instead sets the temperature of a high atmospheric layer (the outermost IR-thick layer, Sect. IV A), *not* the planet’s surface.
- Temperature rises linearly as we descend below the outermost IR-thick layer, with a gradient set by the nature of convection (Sect. IV B).
- Thus, the surface temperature depends on the *height* of the outermost IR-thick layer, which in turn depends on atmospheric composition (Sect. IV D).

A. Infrared-active gases impede the loss of solar energy from a planet’s surface

Fig. 1 discloses a big omission in the discussion of Sect. III.²⁵ The total energy input to Earth’s surface is actually dominated by a new source that we have not yet considered: Our atmosphere itself emits infrared down toward the surface.

It may seem paradoxical that energy from our atmosphere, whose ultimate origin is the Sun, should be arriving at a higher rate than solar energy itself (Fig. 1)! But consider a vacuum flask designed to keep coffee hot. Suppose that instead of a drink, it contains an electric heating element that constantly delivers energy at some rate \mathcal{P} . If the reflection from the inner wall were perfect, the energy input would raise the interior temperature without limit. Even with realistic, imperfectly reflecting walls, a steady state can develop in which energy from the outward-directed thermal radiation is reflected back into the interior at a rate greater than \mathcal{P} . There is no contradiction with energy

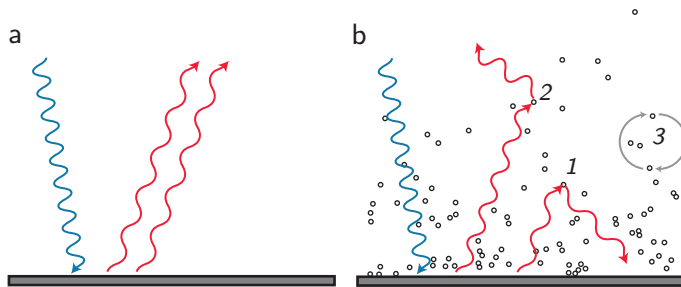


FIG. 2. **Energy transport modes.** (a) Solar radiation arrives at a naked planet (left) and warms the surface, which in turn emits a larger number of lower energy photons (right). (b) With an atmosphere, some of the outgoing IR radiation is absorbed by infrared-active gas molecules (circles). Those molecules in turn emit IR, partly back down toward the surface (1). The density of molecules decreases with height, so outgoing radiation can escape freely if it is emitted at high enough altitude (2), but the temperature at that altitude is lower than on the surface. Another energy-transport mechanism is available as well: the physical movement of molecules (convection), symbolized by the gray circulation at right (3).

conservation, because the hot interior also emits energy toward the walls at an equally large rate (plus \mathcal{P}). Similarly, the bottom of Earth’s atmosphere receives IR energy at a constant rate (originating when the short-wavelength incoming solar radiation heats the surface), but there is no reason why total upward and downward energy fluxes should not exceed that rate.

In short, a planetary atmosphere can obstruct the loss of energy by absorbing some of what is emitted from the surface and emitting new infrared radiation, some of which is directed back toward the planet (Fig. 2b). Some planets, including Earth and Venus, have atmospheres that are “optically thick” at mid-infrared wavelengths (that is, a photon emitted by the surface in this band is likely to be absorbed in the atmosphere before reaching space). In that situation, only the outermost IR-thick layer of the atmosphere will be able to emit freely to space. *That* high layer’s temperature may be approximately given by an estimate like the one made in Sect. III (see Eq. 7 below), but the planetary surface will in general be warmer.

B. The lapse rate characterizes energy transport in the troposphere

We would like to improve our planetary energy flow calculation by incorporating the effects of infrared-active gases, at least qualitatively. But immediately we face a difficulty: The transport of energy involves a mechanism not envisioned in the “blanket” model, namely, large-scale motion of air (3 in Fig. 2b).

Fortunately, Fig. 3 suggests a simple approach. Both on Earth and on Venus, the low, densest, part of the atmosphere shows a strikingly *linear* dependence of temperature on altitude, despite large changes in density and temperature. The temperature gradient is called the lapse rate,²⁶ and the zone over which it is roughly constant is called the troposphere.

If a planet’s atmosphere contains a significant amount of infrared-active molecules, then its lower layers will be opaque to infrared light, and photon emission will not be the most effective way for energy to flow away from the surface. Solids and liquids can transfer thermal energy by conduction, but this process is also not very effective in gases. Instead, warmer (lower) air will physically exchange places with cooler (higher) air, a process called convection, because gases become buoyant when heated.

Convection acts to reduce any temperature gradient, but only down to a certain point: Once the lapse rate decreases below a critical value, then an air layer will be stable against convection.²⁹ Thus, we expect a zone throughout which convection has reduced the temperature gradient to that critical value.

We can estimate the critical lapse rate via a dimensional argument.³⁰ We are seeking a quantity with units K/km. The combination of relevant constants g/c_p has the required units, where g is the acceleration of gravity at the surface and c_p is the specific heat at constant pressure, per mass. What makes this formula so useful is that it is nearly independent of gas temperature and density because c_p is constant for an ideal gas. For a gas such as dry air that roughly obeys the ideal gas law, without any component that can condense upon cooling,³¹ the formula predicts a lapse rate of about 10 K/km (see the Appendix).

A similar but more accurate formula can be written to account for an atmosphere with a condensing component, such as water vapor on Earth.³² The actual observed lapse rate in the troposphere is about 6.5 K/km (Fig. 3a). For Venus, the data again fall roughly on a straight line (Fig. 3b).

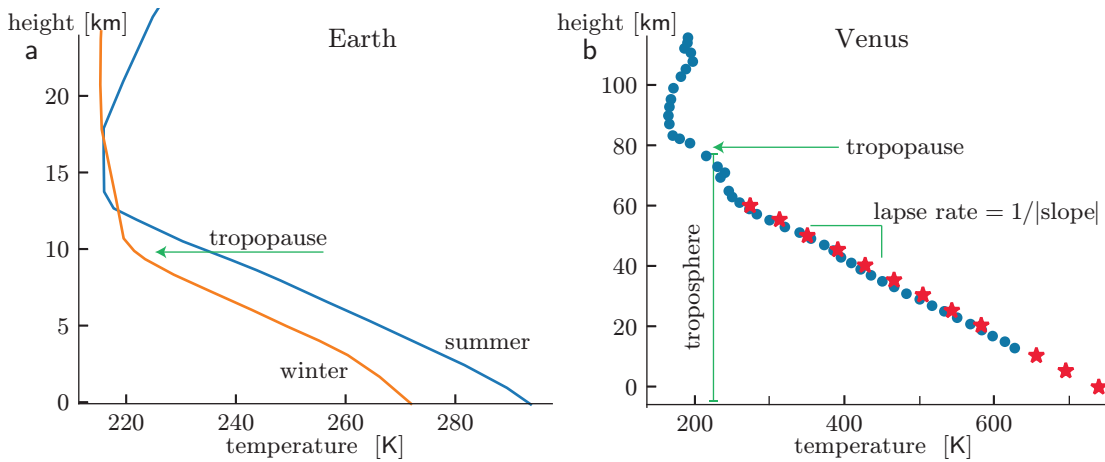


FIG. 3. [Observational data.] **Atmospheric temperature profiles.** (a) Temperature profiles for Earth measured at mid-latitudes. Summer is warmer than winter, but all seasons have a similar lapse rate. Above 30 km the temperature increases, but this has little relevance for climate, because most of the atmosphere lies in the troposphere. (b) Temperature profile for Venus. The two symbols show data from two different probes descending near the equator. The tropopause is at much higher altitude than on Earth. (Data from: (a) Ref. 27 and (b) Ref. 28.)

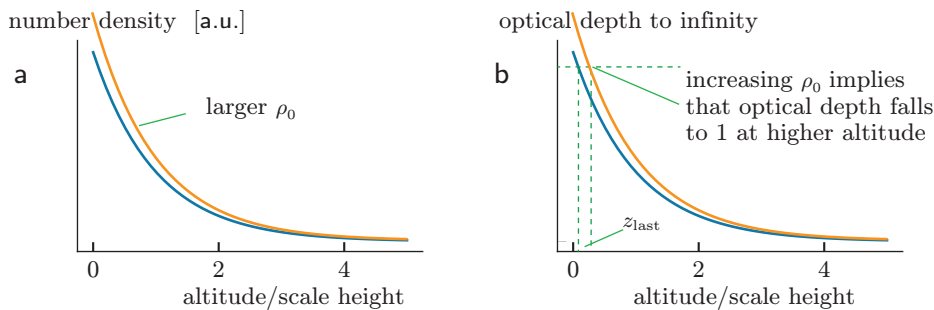


FIG. 4. **Effects of increasing density of a component gas.** (a) Exponentials representing number density $\rho(z)$ of a well-mixed molecular species (arbitrary units) as a function of altitude z . Two different values of ρ_0 , the density at the surface, are shown. (b) Corresponding optical depths from z to infinity (Eq. 6). A critical value, such as 1, is achieved at different altitudes z_{last} depending on ρ_0 . (For a condensing species such as water vapor, the density profile is different, but still there will be a z_{last} .)

C. The outermost IR-thick layer emits energy to space

Now that we have a simplified picture of convection, let's work upward from the planetary surface. Incoming radiation in the visible wavelength range arrives there, and the fraction that is not reflected is absorbed by oceans and/or land, then reemitted as IR. Sect. IV B said that in the troposphere, incoming energy is transported upward with fixed temperature gradient (lapse rate). At some altitude, however, the remaining atmosphere is so thin that radiation becomes the dominant energy transport mechanism. This crossover altitude controls the surface temperature, so its dependence on atmospheric composition is the next topic to consider.

In more detail, the absorption cross section is a measure of a molecule's ability to absorb light of a given frequency. In this section, we will temporarily suppose that the cross section is independent of wavelength throughout the infrared region, a simplification sometimes called the gray gas model.³³ A uniform sample of thickness b absorbs a beam of light by the exponential law (Eq. 4). More generally, if the density is nonuniform we replace $a_1 \rho b$ in Eq. 4 by the optical depth $\tau = a_1 \int dz \rho(z)$. Note that despite the word "depth," this quantity is dimensionless.

In a planetary atmosphere, the overall density of gas falls roughly exponentially with altitude.³⁴ The density of a well-mixed chemical species such as CO_2 then also falls exponentially: $\rho(z) = \rho_0 e^{-z/H}$, where ρ_0 is the density at the surface and H is a constant called the scale height (Fig. 4a). We are interested in the optical depth from a given z to infinity, because this quantity describes how much the atmosphere impedes the escape of photons:

$$\tau_\infty(z) = a_1 \int_z^\infty dz' \rho_0 e^{-z'/H} = a_1 \rho_0 H e^{-z/H}. \quad (6)$$

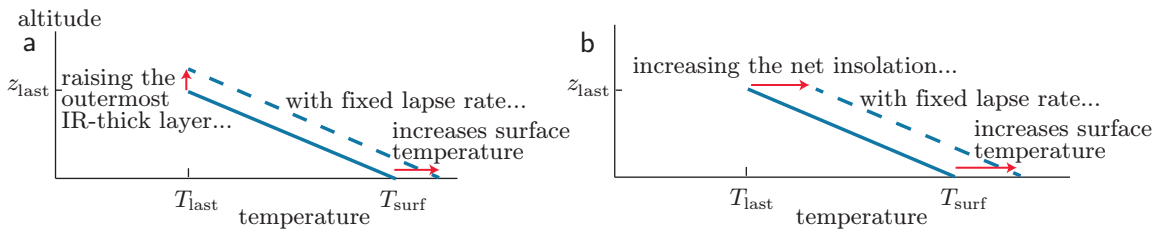


FIG. 5. **Effects of changes to atmospheric composition or surface reflectivity on surface temperature.** (a) A change in atmospheric composition, with no change in reflectivity, can raise surface temperature by moving the outermost IR-thick layer upward (Fig. 4b). (b) A decrease in reflectivity, with no change in composition, can raise surface temperature by increasing the net insolation and therefore T_{last} .

Thus, the optical depth to infinity is also exponential in form (Fig. 4b).

We loosely define the outermost IR-thick layer of the atmosphere by the altitude z_{last} at which IR optical depth to infinity falls to some critical value, for example, 1, corresponding to absorption of less than $1 - e^{-1} \approx 63\%$. We also simplify by assuming that³⁵

- Everything below z_{last} transports energy outward with fixed lapse rate (the tropopause is always at least as high as z_{last}); and
- Everything beyond z_{last} radiates energy to space, following Eq. 2.

Although the assumptions just made are crude, they do incorporate the key insight of the preceding discussion: *For a clothed planet, the estimate of surface temperature in Sect. III failed because it should have been applied to the outermost IR-thick layer, not to the surface.* Let T_{last} denote the temperature of the outermost IR-thick layer. Energy arrives at that layer from below at a total rate per area equal to the net insolation (absorbed solar input, Eq. 5). The layer emits infrared radiation outward from its top, with total rate per area given by the Stefan–Boltzmann law. Its emissivity is ≈ 1 because by assumption it is IR-opaque. Higher levels, however, were assumed to be weakly absorbing, and hence also weakly emitting by Kirchhoff’s law, so they have negligible effect on outgoing energy. Thus,³⁶

$$\text{net insolation} = (1 - \mathcal{R}) = \sigma_{\text{SB}} T_{\text{last}}^4, \quad (7)$$

which determines T_{last} .

D. A graphical argument shows the effects of changing atmospheric composition or reflectivity

Eq. 7 can now be combined with the lapse rate to predict qualitatively the temperature at the surface, once we know the altitude z_{last} of the outermost IR-thick layer. Increasing the amount of infrared-active gases (for example, in the aftermath of a massive volcano eruption or as a result of industrial activity) raises the outermost IR-thick layer (Fig. 4b) and so increases the surface temperature (Fig. 5a).³⁷ An increase of net insolation, for example, by a reduction of averaged reflectivity after the loss of polar ice, also warms the surface by a different route (Fig. 5b).

We now see the cause of the very high surface temperature on Venus: Its atmosphere is 96% carbon dioxide, a strongly infrared-active gas compared to Earth’s oxygen and nitrogen. Moreover, Venus has much *more* atmosphere than Earth: Its surface pressure is 92 times as great as ours. For both of these reasons, its atmosphere remains opaque to infrared photons even up to high altitudes, leading to a much higher outermost IR-thick layer than on Earth, and a correspondingly high surface temperature.

V. BEYOND THE GRAY GAS MODEL

A. Molecular IR absorption spectra are complex

Sect. IV C introduced the concept of the outermost IR-thick atmospheric layer by integrating the absorption cross section from altitude z out to infinity (Eq. 6) and defining z_{last} as the altitude below which a particular threshold value is exceeded. Our discussion assumed that the cross section was independent of wavelength (the “gray gas model”), but Fig. 6 shows this to be far from true. Nor is this a peculiarity of carbon dioxide: All IR-active gases have complex absorption spectra.

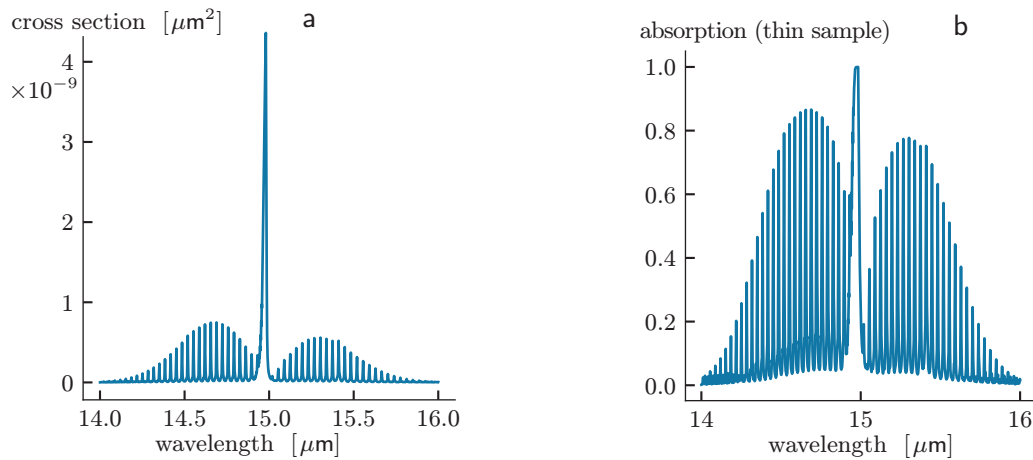


FIG. 6. **IR absorption spectrum of CO₂.** (a) [Experimental data.] Detail of the absorption cross section a_1 at sea-level pressure and density, as a function of wavelength, showing complex substructure. (b) [Calculation from data.] Absorption of a sample of pure CO₂ with depth 1 mm. The graph was obtained by evaluating Eq. 4 with the data in (a). (Data generated from the HITRAN2016 database by the Reference Forward Model, kindly supplied by David Romps; see Refs. 38 and 39.)

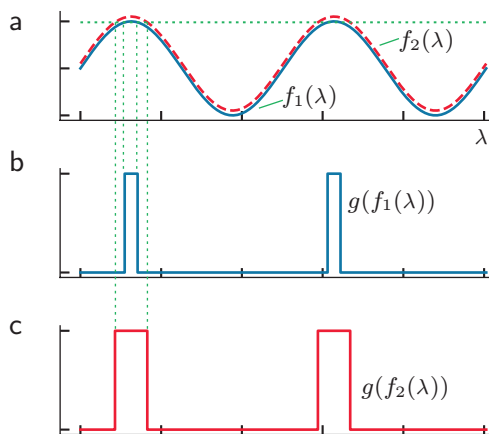


FIG. 7. **Effect of a nonlinear threshold function.** (a) Two functions f_1 and f_2 differ by a slight vertical shift. A threshold function (step function) g will be applied to each one at the horizontal dotted level. (b) The first function just barely peeks over the threshold. (c) The small shift greatly influences the thresholded result.

Optical depth depends on the absorption cross section, so it, too, will be different for different wavelengths. In particular, each wavelength has its own separate value for the altitude $z_{\text{last}}(\lambda)$ of its outermost IR-thick layer. The following subsections will outline how this fact affects planetary climate calculations.

B. Absorption windows depend on the density of IR-active gases

The complexity of absorption spectra may tempt us to simplify by replacing them with average values, and feeding the result into a gray-gas model. But we must do better than that. To see why, recall that if g is a nonlinear function, then the average of $g(f(\lambda))$ over a range of λ is different from g evaluated at the average of f .

Fig. 7 shows an extreme example, which nevertheless is relevant for our problem. Here we imagine a “clipping function” g that equals zero for values below a threshold and equals one for values above it. When two very similar-looking functions, with very similar average values, are fed into g , the results are quite different, and in particular they have quite different average values. Although our optical absorption function $1 - e^{-\tau}$ is not literally a clipping function, it does have a similar effect, transitioning from 0 to 1 over a limited range of τ values.

Fig. 8 shows the result of applying the absorption function to the carbon dioxide spectrum. Because optical depth depends on density as well as on cross-section, two different scenarios are shown: The first is a real historical moment, whereas the second is a possible future situation. Both scenarios agree that there is a band of total absorption, but

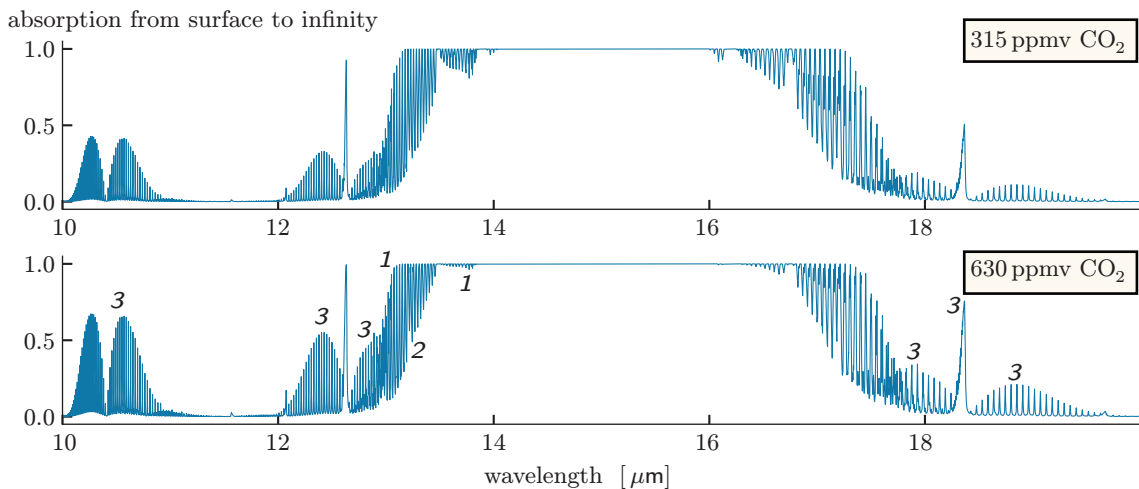


FIG. 8. **Absorption from surface to infinity.** Optical depth was computed by integrating CO₂ density from the planetary surface to infinity (Eq. 6) and multiplying by the cross-sections; then, total absorption was computed via Eq. 4. Because of the great depth of the atmosphere, features that were scarcely visible in Fig. 6b have large effects. Top: Absorption calculated with the measured atmospheric concentration of CO₂ in 1957. Bottom: Hypothetical concentration twice that in 1957. The significant differences are that: (1) The band of total absorption is wider; (2) absorption between peaks is stronger; and (3) bands of partial absorption have become more complete.

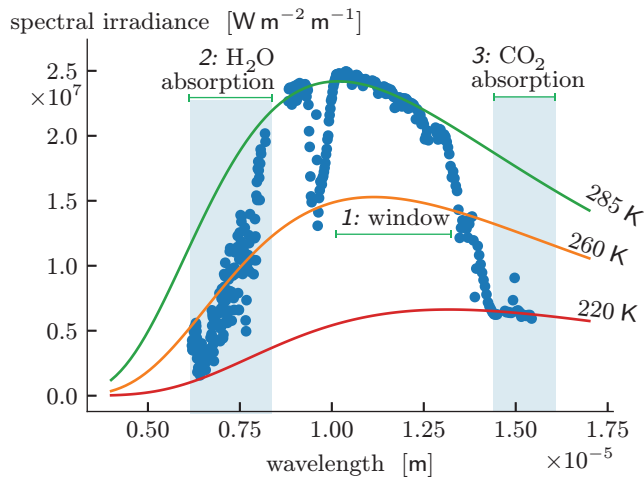


FIG. 9. [Observational data; mathematical functions.] **Earth’s emission spectrum viewed from space.** The dots are data from the Atmospheric Infrared Sounder probe; for comparison, the curves show thermal radiation spectra at various temperatures (Eq. 3). 1: In an IR-transparent window, $z_{\text{last}} = 0$ and energy streams out freely at the surface temperature. 2: In a wavelength band where water vapor absorbs strongly but CO₂ does not, Earth emits at an effective temperature corresponding to the altitude at which water vapor condenses out; higher than this, there is very little H₂O. 3: In a wavelength band where CO₂ absorbs strongly but water vapor does not, Earth emits at a still lower temperature corresponding to z_{last} in the upper troposphere. (Data courtesy Yi Huang; see also Ref. 40.)

they differ in its effective width. The following subsection explores the implications of this observation for climate.

C. Narrowing an absorption window also increases surface temperature

We can summarize the message of Fig. 8 by saying that the contribution of carbon dioxide gas to atmospheric IR absorption consists of absorption bands separated by “windows.” Even when we add in the other relevant IR-active atmospheric constituents (mainly water vapor), we still find windows in which the entire atmosphere is nearly transparent, for example, the range 11–12 μm. In this range, there is *no* value $z_{\text{last}}(\lambda)$ satisfying $\tau_{\infty}(z_{\text{last}}, \lambda) > 1$; that is, the altitude of the “last” IR-thick layer is zero.

In such windows, thermal radiation at Earth’s surface temperature streams freely away to space, unlike the scenario in Sect. IV C.⁴¹ Moreover, because emissivity is equal to absorptivity (Kirchhoff’s law, Sect. II), in those windows no

additional IR radiation from the atmosphere joins the outgoing surface photons. In these bands, an extraterrestrial observer thus sees thermal radiation at the surface temperature. Indeed, region 1 in Fig. 9 shows such behavior.

Our problem may seem far more complicated than in the gray gas model, but recall that at any altitude in the troposphere, all wavelength bands *share a common temperature* set by the lapse rate and a single number, the surface temperature. Imagine subdividing the spectrum into wavelength slices and rank-ordering them in increasing order of z_{last} :

- For the absorption windows (bands with $z_{\text{last}}(\lambda) = 0$), the discussion in Sect. III is valid. In these windows, the flux of energy is of Planck form, at the surface temperature T_{surf} (region 1 in Fig. 9).
- The bands with the next lowest value of $z_{\text{last}}(\lambda)$ also emit photons with the Planck spectrum, but with a temperature $T(z_{\text{last}}(\lambda))$ that is lower than T_{surf} by $z_{\text{last}}(\lambda)$ times the lapse rate (region 2 in Fig. 9).
- The bands with the next higher value of $z_{\text{last}}(\lambda)$ are emitted from still cooler layers (region 3 in Fig. 9), and so on.

The value of T_{surf} is now determined by the requirement that the sum of all the outward energy flows must balance the net insolation, just as with a naked planet (Sect. III). As in the simpler gray gas model (Sect. IV), we again see that the effect of the atmosphere is to set a higher surface temperature than that of the corresponding naked planet.

We can now return to this article’s opening queries (Sect. I and Conclusion): Let us revisit the question of what happens when the density of an IR-active gas such as CO_2 is raised. For wavelengths deep within a strong absorption band, the story is similar to what was said in Sect. IV D: $z_{\text{last}}(\lambda)$ increases, so the surface temperature must go up in order to emit the required amount of energy.

However, there is now an additional effect not present earlier. An absorption band will get *wider* as we increase the density of an IR-active component, and the spaces between its individual peaks will also narrow (Fig. 8). Thus, at the edges of a band (the “wing” regions⁴²) z_{last} will increase from zero to some nonzero altitude if the density rises high enough to create an IR-thick layer. The wavelength zones that are able to impede outgoing IR radiation thus enlarge (windows shrink), and so the quasisteady surface temperature rises faster than would have been predicted by the mechanism in the preceding paragraph acting alone.⁴³

Let’s make a quantitative estimate based on the considerations in the preceding paragraph. The observational data in Fig. 9, and other sources, suggest a rough division of the IR spectrum into several bands, each radiating to space at a distinct temperature (Table II). The total energy output can be obtained by integrating Eq. 3 over each band,

TABLE II.

wavelength band [μm]	T_{band} [K]
6–8.1	250
8.1–9.4	285
9.4–10	260
10–13	285
13–17	220
17–29	260
29–45	230

generalizing Eq. 2:

$$\text{band contribution} = \frac{15}{\pi^4} \sigma_{\text{SB}} T_{\text{band}}^4 \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{x^3 dx}{e^x - 1} \quad \text{where} \quad x = \frac{2\pi \hbar c}{\lambda k_{\text{B}} T_{\text{band}}}.$$

Substituting the values in the table and carrying out the integrals numerically yields that the total irradiance of all bands roughly equals the net insolation given in Eq. 5.

Now suppose that an increase in CO_2 widens the fifth absorption band in the table at the expense of its neighbors by just $0.25 \mu\text{m}$ on each side. Then the energy output goes down, and hence out of balance with the input. The system must respond by raising its surface temperature by an unknown ΔT_{surf} . Due to the fixed lapse rate in the troposphere, the temperature at every altitude also increases by the same ΔT_{surf} (Fig. 5). A good homework problem at this point is to have students redo the calculation with various new temperature values and show that $\Delta T_{\text{surf}} \approx 1.3 \text{ K}$ is required to restore energy balance. This contribution to temperature change is in addition to the one from raising the altitudes of existing outermost IR-thick layers.

This section has amended the gray-gas model while leaving its lessons mainly intact. For many technical details, see Ref. 10.

D. Pressure broadening also affects the absorption windows

Sect. III obtained a reasonable answer by treating Mars as “naked,” but actually its atmospheric CO₂ density is much larger than Earth’s! To resolve this apparent contradiction, we need to appreciate another physically interesting aspect of molecular absorption, that is, pressure broadening (Sect. II). The pressure relevant to this phenomenon is the total gas pressure,⁴⁴ not just the part attributable to the IR-active gas, so it can be significant even though the density of CO₂ in Earth’s current atmosphere is very small.

Sect. VB argued that the “wings” of a molecule’s absorption spectrum are critical for determining its surface-warming effect. The preceding paragraph argued that those wings are strongly influenced by ambient pressure. This observation helps explain why lots of CO₂ on Mars does not create a large warming effect: Oxygen and nitrogen, the main constituents of Earth’s atmosphere, are largely absent on Mars because its lower surface gravity cannot hold onto them as well as Earth’s does. Because pressure broadening from those gases is absent, the CO₂ on Mars is less effective at blocking IR radiation than that on Earth.¹⁴ Less warming from CO₂ (as well as greater distance to the Sun) means a cooler planet. Moreover, another greenhouse gas, H₂O, is even lighter than N₂ or O₂, so little of it remains in the Martian atmosphere. All of these reasons combine to render Mars nearly as cold as if it had no atmosphere at all (Sect. III).

E. Positive feedbacks

The effects outlined in the preceding sections contribute to the climate sensitivity (change of temperature in response to an increase of CO₂). Climate sensitivity is enhanced by *feedback*:

- On Earth, warming caused by the two mechanisms just described drives more water vapor into the atmosphere from the oceans. Because water vapor is itself IR-active, its increase leads to still more warming (Fig. 5a).
- An increase in mean surface temperature also shrinks the polar icecaps, in turn reducing average reflectivity, increasing net insolation, and leading to further temperature increase (Fig. 5b).

Positive feedback can lead to bistability, so it is critically important to know how close the Earth system is to a bifurcation into a hot state. Conversely, in past epochs Earth has gotten stuck in a “snowball” state that was too cold to permit any appreciable water vapor content.

F. A road not (yet) taken on Earth

In contrast to Mars, Venus’s very high atmospheric pressure makes its CO₂ *more* effective than on Earth.⁴⁵ But *why* is Venus’s atmosphere so different from ours? Earth and Venus have similar size and composition and move in similar orbits, so it’s reasonable to suppose that they were similar in other respects when they first formed. For example, the *total* fraction of carbon (atmospheric plus that bound in rocks) is probably similar for these two planets (Ref. 46, Table 1). On Earth, carbon continually cycles between the atmosphere and the planet’s interior, providing a stabilizing feedback, but part of this loop seems to have broken on Venus, leaving much of the carbon in the atmosphere.

One clue is that *Venus no longer has any oceans*. In light of that fact, a possible scenario runs as follows. At some point, the temperature on Venus rose higher than Earth’s. That rise evaporated more water from the existing oceans, leading to the first positive feedback outlined in Sect. VE—in this case, a “runaway” scenario.⁷ Eventually *all* surface water was in vapor form. Liquid water is needed for the physical process of weathering rocks, which on Earth slowly removes atmospheric CO₂ and ultimately returns it to the planet’s interior, so losing liquid water broke that “deep carbon cycle.”⁴⁷ Meanwhile, volcanic activity continued, adding CO₂ from the interior to the atmosphere.

Gradually, solar UV photons broke up atmospheric water vapor and the light hydrogen escaped to space. The rising carbon dioxide level then took over the role of impeding infrared energy loss from the lost water vapor. Evidence for this scenario comes from the ratio of deuterium (D) to ordinary hydrogen (H), which is 150 times larger on Venus than on Earth or other solar-system objects: DH molecules, which are heavier than ordinary H₂, got left behind (fractionated) when the hydrogen escaped.⁴⁸

The foregoing scenario is based on known processes, but it is still speculative. What is not in dispute is that however Venus arrived at its current atmosphere, it is now stuck in a permanently hot state.

VI. CONCLUSION

Planetary climates are determined in part by a chain of energy transformations: Incoming light, primarily in the visible range, converts its energy to thermal motion of Earth's surface materials. That energy exits back into space by the net effect of convection and atmospheric absorption and emission, ultimately as radiation in a wavelength band different from the one that dominated the incoming light. Details of the matter/light interaction, which is inherently quantum mechanical in character, determine the different effects of various gases, with huge implications for life.

To summarize the limited discussion of this article, here are replies to the queries in the Introduction (see also Sects. V C and V E):

- CO₂ absorbs in a wavelength band where H₂O does not. Moreover, warming from CO₂ increase does indirectly affect H₂O vapor.
- The wings of each individual peak become important as CO₂ concentration increases. The lower peaks themselves also begin to matter. Moreover, even in the gray gas model, and even at wavelengths where the atmosphere is opaque, what matters is *how high up* it remains opaque, which does change with additional CO₂.

The discussion in this article was not detailed enough to make quantitative predictions of climate sensitivity. We have neglected lateral air flows and the effects of continents and oceans;^{49–53} we used a shortcut instead of solving the radiative-convective equations; we neglected the reduction of pressure broadening in the upper atmosphere, cloud feedback, plant transpiration; and so on. Nevertheless, it should now be understandable why more complete calculations predict that doubling CO₂ relative to preindustrial times will increase Earth's temperature significantly.

Appendix

Here are details about the ideal-gas lapse rate. To get degrees Kelvin in the numerator, we need the factor (specific heat)⁻¹. To eliminate seconds, we then need the acceleration of gravity. Then we already have the desired units; there is no room for any additional factors such as mass density. Each parcel of air is free to expand at the ambient pressure, so the relevant specific heat is at constant pressure. These considerations lead to the formula g/c_p proposed in Sect. IV B.

Earth's atmosphere is mostly nitrogen. Nitrogen is a diatomic molecule with molar mass 28 kg/mole, so in the ideal gas limit the specific heat per mass is

$$c_p = \frac{7}{2} \left(\frac{1}{0.028 \text{ kg/mole}} \right) \left(\frac{8.3 \text{ J}}{\text{mole K}} \right) \approx 1000 \text{ J/(kg K)}.$$

Then $g/c_p = 9.8 \text{ m s}^{-2}/(1000 \text{ J/(kg K)}) \approx 10 \text{ km}^{-1} \text{ K}$ as stated in the main text.

The atmosphere of Venus is mostly CO₂, another linear molecule but with molar mass 44 kg/mole. So we have

$$c_p = \frac{7}{2} \left(\frac{1}{0.044 \text{ kg/mole}} \right) \left(\frac{8.3 \text{ J}}{\text{mole K}} \right) \approx 660 \text{ J/(kg K)}.$$

The surface gravity of Venus is about 9 m s^{-2} , so $g/c_p = 9 \text{ m s}^{-2}/(660 \text{ J/(kg K)}) \approx 14 \text{ km}^{-1} \text{ K}$. Although the atmosphere is not a very ideal gas at the surface pressure of Venus, nevertheless this figure is not too far from the inverse slope $\approx 8 \text{ km}^{-1} \text{ K}$ that we read off from Fig. 3.

More importantly than the numerical values, the discussion in the main text rested on the empirical observation that the lapse rates on both Earth and Venus are roughly constant in the troposphere.

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- ¹⁷ <https://www.youtube.com/watch?v=0eI9zxZoipA>.
- ¹⁸ In the case of Earth, there are also reflective polar icecaps, aerosols, and so on. Although we are not yet including the effects of atmosphere, we have incorporated such reflection in the data of the table via the \mathcal{R} values. Specifically, \mathcal{R} is the reflectivity averaged over the frequency range typical of the solar spectrum.
- ¹⁹ Each planet (and even some moons) also has internal energy production (geothermal energy), but this source is negligibly small for the three we have chosen to study (Ref. 4, §2.5).
- ²⁰ The table lists distance values computed simply as the average of the closest and farthest points on each planet's orbit.
- ²¹ Perhaps surprisingly, even snow and ice have high IR absorptivity, and hence also high emissivity.
- ²² Note that if Earth were literally stripped of atmosphere, then its reflectivity, currently dominated by clouds, would change, affecting the prediction. The point of the estimate made here is simply the inadequacy of neglecting absorption in the atmosphere.
- ²³ The outlines of this story were already clear in the 19th century, shortly after W. Herschel's discovery of IR radiation. J. Fourier understood the role of energy balance between incoming short-wavelength and outgoing IR radiation in 1827, as well as the potential role of the atmosphere. J. Tyndall then measured the absorption spectra of many gases and correctly identified carbon dioxide and water vapor as critical for climate in 1863. At the very end of the century, S. Arrhenius constructed a simplified, but self-consistent model based in part on Tyndall's data. Modern discussions can be found in Refs. 3, 13, and 10.
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- ²⁷ K Stamnes, G E Thomas, and J J Stamnes. *Radiative transfer in the atmosphere and ocean*. Cambridge Univ. Press, Cambridge UK, 2nd edition, 2017.
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- ²⁹ Rising warm air expands and hence cools, losing its buoyancy, so there is a minimal value for the gradient before convection turns on.
- ³⁰ For a more detailed derivation, see Ref. 6, §34.2.2. Ref. 3, p.150 gives another detailed derivation, concluding $T_{\text{surf}}/T_{\text{last}} = (p_{\text{surf}}/p_{\text{last}})^{R/c_p}$, where R is the gas constant and $c_{p,\text{mol}}$ is the specific heat at constant pressure, per mole. Substituting the exponential dependence of pressure with altitude, Taylor expanding the exponential, and keeping up to linear order recovers the linear relation in the main text.
- ³¹ A convecting region of atmosphere with the critical lapse rate is also said to follow the “dry adiabat”.
- ³² For details, see Ref. 3. S. Manabe shared a Nobel Prize in 2021 in part for early work on radiative transport, on the role of

convection, and on incorporating the effects of water condensation into a climate model.

- ³³ Sect. V will improve upon this simplification. We continue to assume that absorption cross section is negligibly small in the *visible* range.
- ³⁴ Ref. 15 gives an improved formula, which however is numerically close to the exponential for Earth’s troposphere.
- ³⁵ We are avoiding a full radiative transport treatment; for that, see Ref. 27.
- ³⁶ See Refs. 3 and 9. T_{last} is sometimes called the “emitting” temperature, in contrast with a lower “skin” temperature.
- ³⁷ Changing the amount of a noncondensing trace molecule, such as carbon dioxide in Earth’s atmosphere, makes a negligible change in the specific heat, and hence in the critical lapse rate.
- ³⁸ D M Romps, J T Seeley, and J P Edman. Why the forcing from carbon dioxide scales as the logarithm of its concentration. *J. Climate*, 35(13):4027–4047, 2022.
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- ⁴¹ Some students will be interested to hear that this “decoupling” phenomenon is analogous to the epoch of recombination in the early Universe, after which thermal photons go out of equilibrium with matter.
- ⁴² M G Mlynczak, T S Daniels, D P Kratz, D R Feldman, W D Collins, E J Mlawer, M J Alvarado, J E Lawler, L W Anderson, D W Fahey, L A Hunt, and J C Mast. The spectroscopic foundation of radiative forcing of climate by carbon dioxide. *Geophys. Res. Lett.*, 43(10):5318–5325, 2016.
- ⁴³ The addition of gases not naturally present in the atmosphere, such as chlorofluorocarbons, can also create entirely new absorption bands in what was previously a window region.
- ⁴⁴ Thus, the spectral properties of greenhouse gases change with altitude, another complication that must be addressed in a full treatment. Indeed, on other planets with extremely high pressures, even O_2 and N_2 become IR-active.
- ⁴⁵ Although carbon dioxide has a window of IR transparency, droplets of sulfuric acid forming clouds over Venus trap emission even in that spectral region.
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- ⁴⁷ Confusingly, there are at least two different processes referred to as “weathering.” Here we mean the conversion of silicate rocks via the Urey reaction.² On Earth, photosynthesis by marine organisms also removes CO_2 from air; they eventually fall to the bottom of the ocean, carrying their fixed carbon into sediments that ultimately reenter Earth’s crust, another part of the carbon cycle that relies on liquid water (and mild temperatures). (Terrestrial plants are much less effective; much of their carbon is released back to the atmosphere after they die and decompose.).
- ⁴⁸ The remaining free oxygen may in turn have been removed from the atmosphere, for example, by oxidizing crustal elements such as iron. Earth, too, had very little free oxygen prior to the rise of photosynthetic organisms.
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