Physics 516: Electromagnetic Phenomena

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27 January 1884. Thought about electromagnetic rays.
11 May. Hard at work on Maxwellian electromagnetics.
13 May. Nothing but electromagnetics.
8 July. Electromagnetics, still without success.
17 July. Depressed; could not get on with anything.
24 July. Did not feel like working.
7 August. Saw from Ries's book that most of what I have found so far is already known.
– From the Diary of Heinrich Hertz

Our goal in this course is to examine some electromagnetic phenomena relevant to the research being done in this department, then obtain them as mathematical consequences of classical electrodynamics. We want to look at the phenomena through the lens of mathematics, to see common themes. Finally, we want to consolidate many bits of information you've gathered in your previous courses into a more coherent whole.

Announcements: `http://courseweb.library.upenn.edu/`
Time: MWF 10:00–11:00 and Th 12:00–1:00 in DRL room A5. One session per week will be designated "recitation"; it will usually be a problem-solving session. Sometimes we will instead use this hour for makeup lectures or pre-exam review.
Additional office hours: We'll arrange these to suit the class.
Assignments: Biweekly problem sets, midterm, final.
General policies: see "Course Info" handout.

General prerequisites:
Familiarity with Maxwell equations in differential and integral form. Familiarity with electro- and magnetostatics. Familiarity with special relativity, some linear algebra, and some Lagrangian mechanics, at the levels of PHYS351 and 361–362 and MATH240. Facility with some computer-math package, such as *Matlab*.

Books (more are listed in the References):
Recommended:
Pollack and Stump, *Electromagnetism* (Addison Wesley, 2002). If you find any errors in the book, please bring them to my attention so I can distribute to the class.
Also useful (and available in the Math/Physics Reserve):
Landau and Lifshitz, *Classical theory of fields* and *Electrodynamics of continuous media*
Feynman lectures: The definitive and extended edition volumes 1–2
JD Jackson, *Classical electrodynamics*
E Purcell, *Electricity and magnetism*

Tentative Outline

We can't cover all of this, so let me know if there's a topic you really want to see. See the weekly Assignments for reading and homework assignments.

Prolog

Gladstone: "What is the practical worth of electricity?"
Faraday: "One day, Sir, you may tax it."

Demo: *Microwave polarizer paradox. Radio waves transport energy. Optical polarizers and birefringence. 3D movie glasses.*
Demo: *Stationary coil, moving magnet. Stationary magnet, moving coil.*

0. **Parable: The equations of Newtonian gravity**

Hello Maxwell

"Maxwell's theory is Maxwell's system of equations... One cannot escape the feeling that these equations have an existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers." – Heinrich Hertz

1. **What the equations say**
   Necessity of the field idea. Lorentz force law gives meaning to the quantities appearing in Maxwell. Charge density, current, and the continuity formula. Necessity of the "displacement current" term.
   Demo: *Lorentz force law in Crookes tube. Electrophoresis with a magnetic field, visualized using pepper in water. Button magnet motor.*
2. **Dimensions and units.**
   SI sí, CGS no.
3. **Static charges**
   The potential. Field energy; capacitors. Linear, polarizable media; dielectric constant. Electrostatic multipole expansion. Force and torque on a rigid multipole in an external field. Separation of variables in Laplace's equation: Fields outside a thin disk.
   Demo: *Fluid-filled capacitor. Corona discharge from handheld Tesla coil.*
   Demo: *Optical tweezers video.*
4. **Electrostatics in water**
   Relaxation time scale, quasi-static case. Electrocardiogram. Extracullular recording from individual neurons.
5. **Stationary but nonstatic sources**
   Vector potential. Current dipole field and magnetoencephalography. Magnetostatic multipole expansion. Force and torque on a dipole in external fields. Magnetic tweezers and magnetic cell sorting.
   Demo: *Levitating frog video.*

6. **Time dependence**
   Induced EMF means nonconservative electric fields.
   Demo: *Magnet falling through conducting tube*

7. **Simplest wave solutions**
   Plane waves in vacuum. Linear and circular polarization. Reflection interference contrast microscopy. Waves transport momentum and energy.
   Demo: *Newton Rings; RICM video.*

8. **The ray-optics limit**
   Plane waves hitting a plane interface between two homogeneous media; Snell's rule. Total internal reflection: fiber-optics, endoscopy, TIRF microscopy. Generalization to arbitrary nonuniform medium; Eikonal approximation. Applications: mirages; reflection from the ionosphere.
   Demo: *Light pipes. TIR angle.*

Relativistic classical fields

"Oh, that Einstein, always cutting lectures—I really would not have believed him capable of it." – Einstein's former teacher Minkowski, upon reading Einstein's relativity paper.

9. **Lorentz invariance**
   Rotational invariance and tensors in 3D. Relativistic invariance and tensors in 4D. Doppler shift, aberration of starlight, Fizeau/Zeeman experiment. Dipole anisotropy of the cosmic microwave background radiation. An interlude on other tensor analyses: flavor, weak SU(2), and grand unified multiplets.
   Demo: *Light travels in vacuum but not sound. Doppler shift for sound.*

10. **Why Maxwell's equations look like that**
    Invariant form of Maxwell equations; need for the displacement term, and its sign. The essential role of charge conservation. 4-vector potential.

11. **Variational formulation**
    Lagrangian for EM fields, and for particles+fields. Noether theorem.

12. **Energy and momentum of fields**
    Energy–momentum tensor. Faraday's imagery and its mathematical implementation.
    Demo: *The jumping ring. The cryogenic jumping ring. Levitation of a magnet by a superconducting plate.*

13. **Wave polarization, full and partial**
    Spin of the photon. Linear and circular polarization. Stokes parameters.
    Lorentz-invariant meaning of circular polarizations.
    Demo: *Insert quarter-wave plate into Michelson interferometer and rotate it to destroy, then restore, interference fringes.*

14. **Waveguides and transmission lines**
    A ladder of lumped elements. Dispersion and cutoff. Boundary conditions at a perfect conductor. Modes in a rectangular or circular guide. Zero-mode guides and single-molecule biophysics.
    Demo: *Marshmallows in microwave.*

Fiat Lux

"It is difficult to deal with an author whose mind is filled with a medium of so fickle and vibratory a nature. . . ; We now dismiss. . . the feeble lucubrations of this author, in which we have searched without success for some traces of learning, acuteness, and ingenuity, that might compensate his evident deficiency in the powers of solid thinking. . . " – Henry Brougham, 1803 [Criticizing Thomas Young's wave theory of light]

15. **Radiation**
An accelerated charge develops kinked field lines. Retarded potentials. Jefimenko formulas. Application to terahertz generators.
Demo: *Ogle the web-based field line generator:*
`http://www.cco.caltech.edu/~phys1/java/phys1/MovingCharge/MovingCharge.html` *and*
`http://webphysics.davidson.edu/applets/Retard/Retard.html`

16. **Slowly-moving sources**
Far fields of an oscillating electric dipole; magnetic dipole; higher multipoles. Angular dependence of Förster resonance transfer (FRET). Defocused orientation and position imaging (DOPI). Antennas. How pulsars radiate. Near fields: near-field scanning optical microscopy.

17. **Rapidly-moving sources**
Relativistic beaming. Liénard–Weichert potentials and fields. Bremsstrahlung and synchrotron radiation. Wigglers and undulators.

18. **Free electrons and waves**
Single electrons: Thomson scattering. Its polarization. Mean free path of radiation in the Sun. Polarization of CMBR reveals quadrupole moment of the cosmic inhomogeneity.

19. **Waves in plasmas**
Mean field approximation. Dispersion relation. The ionosphere: Why you can hear Europe. EM waves in metals; skin depth. Why pulsars seem to chirp. Faraday rotation in magnetized plasma; measurement of cosmic magnetic fields.
Demo: *Distortion of flame by an electrostatic field.*

Light in matter

May: "Have the crystals faults like us?" Lily: "Certainly, May. Their best virtues are shown in fighting their faults. And some have a great many; and some are very naughty crystals indeed." – John Ruskin, 1866

20. **Bound electrons**
Atomic polarizability and its classical representation. Permanent and induced moments. Anisotropy: the symmetric polarizability tensor. Single-dipole scattering, its frequency dependence, and its polarization. Return to the microwave polarizer and polaroid sheets. Blue, polarized sky versus white, unpolarized clouds. Mie scattering. Vibrating scatterer: Classical Raman effect.
Demo: *Scatter white light from milk suspension.*

21. **Transmission by dilute matter**
Plane wave incident on a polarizable sheet. Origin of refraction. How can matter slow down light, when matter is mostly empty space? Absorption.

22. **Magnetic polarizability**
The full polarizability tensor. Isotropic, but parity-noninvariant contributions; scattering from a helical wire. Optical activity.
Demo: *Optical activity: syrup of various depths; sucrose vs fructose; limonene vs its enantiomer. Rotation angle increases with thickness.*

23. **Applications to biophysical chemistry**
Circular dichroism and optical rotatory dispersion.

24. **Scattering by dilute matter**
Many isotropic dipoles: Density fluctuations. Masses and sizes of particles in solution can be measured using Rayleigh scattering. Diffusion constant can be measured by dynamic light scattering.

25. **Effective form of Maxwell equations in dense matter**
Polarization density. Bound charge and current densities. Spatial, not time averaging. Clausius–Mossotti relation. Energy of fields+media. Energy–momentum flux tensor.
Demo: *Diamagnetic levitation of graphite. Parallel-plate capacitor with fluid dielectric; fluid creeps up despite gravity.*

26. **Waves in linear dielectric material**
Dichroism and birefringence. The half-wave plate. Phase contrast microscopy.
Demo: *Dichroism: polaroid sheets. Birefrigence: Calcite. Polystyrene. Home-grown crystals. Quartz. Polyethylene baggie or food wrap: stretch it.*

27. **Discontinuities in homogeneous, linear media**
Reflection and refraction at interfaces. Optical tweezers. Optical torque wrench. Total internal reflection. Evanescent wave. Total internal reflection fluorescence microscopy. Fresnel equations; Brewster angle.
Demo: *Optical tweezers:*
`http://www.rowland.harvard.edu/labs/bacteria/showmovie.php?mov=laser_trap`.
Brewster angle; evanescent wave probed in microwave using paraffin prism.

28. **Transport along optical fibers.**
Example: the TE mode.

29. **Optical band gap materials**
The infrared laser scalpel. Other dielectric mirrors: Cat's eyes.

30. **Waves in metamaterials**
"Negative" materials. Superlenses. Mathematical equivalence between exotic inhomogeneous material and curved space. Cloaking.

Optional 1: Quantum electrodynamics

"You boil it in sawdust, you salt it with glue, you condense it with locusts and tape,
Still keeping the principal object in view: To preserve its symmetrical shape." — Lewis Carroll

31. **Quantum fields**
Scalar fields become spinless particles. Nonlinear terms in field equations become interactions among particles. Noether charges become conserved quantities carried by particles.

32. **Charged scalar field**
Global versus gauge invariances. The covariant derivative. Gauge invariance gives massless photons with no self-interaction.

33. **Scattering of light by light**

Optional 2: And now for something completely nonlinear

"The objective of physics is to establish new relationships between seemingly unrelated, remote phenomena." — L. D. Landau

34. **Gravitation**
Is the metric on spacetime really given *a priori*? Mach's paradox. If the metric is not fixed, can we find any reasonable equation of motion for it? What sort of properties must such an equation have? Gauge-type invariance of the metric. There's a unique relativistic, gauge-invariant, linearized gravitation theory. Vistas: Beyond linearized approximation.

35. **Nonabelian gauge invariance**
How and why to generalize gauge invariance. The Yang–Mills lagrangian and field equations.

36. **The Standard Model and beyond**
Spontaneous symmetry breakdown; Higgs phenomenon. Electroweak and grand unification. An exact classical solution: The 't Hooft–Polyakov magnetic monopole.