

Introduction to the Standard Model



- Origins of the Electroweak Theory
 - Gauge Theories
 - The Standard Model Lagrangian
 - Spontaneous Symmetry Breaking
 - The Gauge Interactions
 - Problems With the Standard Model
- (“Structure Of The Standard Model,” hep-ph/0304186)

Spontaneous Symmetry Breaking

Gauge invariance implies massless gauge bosons and fermions

Weak interactions short ranged \Rightarrow spontaneous symmetry breaking for mass; also for fermions

Color confinement for QCD \Rightarrow gluons remain massless

Allow classical (ground state) expectation value for Higgs field

$$v = \langle 0 | \varphi | 0 \rangle = \text{constant}$$

$\partial_\mu v \neq 0$ increases energy, but important for monopoles, strings, domain walls, phase transitions (e.g., EWPT, baryogenesis)

Minimize $V(v)$ to find v and quantize $\varphi' = \varphi - v$

$SU(2) \times U(1)$: introduce Hermitian basis

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2) \\ \frac{1}{\sqrt{2}}(\varphi_3 - i\varphi_4) \end{pmatrix},$$

where $\varphi_i = \varphi_i^\dagger$.

$$V(\varphi) = \frac{1}{2}\mu^2 \left(\sum_{i=1}^4 \varphi_i^2 \right) + \frac{1}{4}\lambda \left(\sum_{i=1}^4 \varphi_i^2 \right)^2$$

is $O(4)$ invariant.

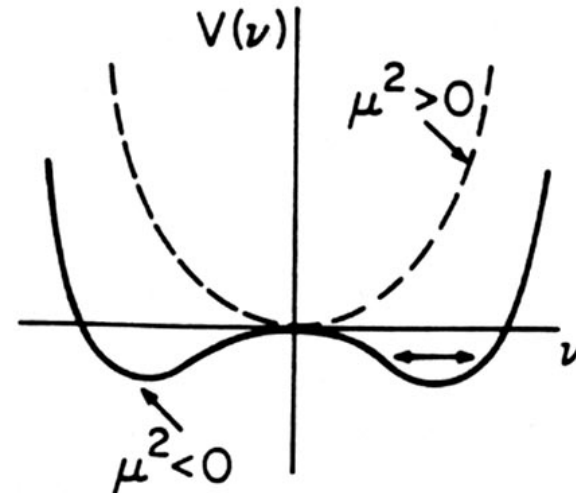
w.l.o.g. choose $\langle 0|\varphi_i|0\rangle = 0$, $i = 1, 2, 4$ and $\langle 0|\varphi_3|0\rangle = \nu$

$$V(\varphi) \rightarrow V(v) = \frac{1}{2}\mu^2\nu^2 + \frac{1}{4}\lambda\nu^4$$

For $\mu^2 < 0$, minimum at

$$V'(\nu) = \nu(\mu^2 + \lambda\nu^2) = 0$$

$$\Rightarrow \nu = (-\mu^2/\lambda)^{1/2}$$



SSB for $\mu^2 = 0$ also; must consider loop corrections

$$\varphi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \equiv v \Rightarrow \text{the generators } L^1, L^2, \text{ and } L^3 - Y$$

spontaneously broken, $L^1 v \neq 0$, etc ($L^i = \frac{\tau^i}{2}$, $Y = \frac{1}{2}I$)

$$Qv = (L^3 + Y)v = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} v = 0 \Rightarrow U(1)_Q \text{ unbroken} \Rightarrow$$

$$SU(2) \times U(1)_Y \rightarrow U(1)_Q$$

Quantize around classical vacuum

- Kibble transformation: introduce new variables ξ^i for rolling modes

$$\varphi = \frac{1}{\sqrt{2}} e^{i \sum \xi^i L^i} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

- $H = H^\dagger$ is the Higgs scalar
- No potential for $\xi^i \Rightarrow$ massless Goldstone bosons for global symmetry
- Disappear from spectrum for gauge theory (“eaten”)
- Display particle content in unitary gauge

$$\varphi \rightarrow \varphi' = e^{-i \sum \xi^i L^i} \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

+ corresponding transformation on gauge fields

Rewrite Lagrangian in New Vacuum

Higgs covariant kinetic energy terms

$$\begin{aligned}(D_\mu\varphi)^\dagger D^\mu\varphi &= \frac{1}{2}(0 \ \nu) \left[\frac{g}{2}\tau^i W_\mu^i + \frac{g'}{2}B_\mu \right]^2 \begin{pmatrix} 0 \\ \nu \end{pmatrix} + H \text{ terms} \\ &\rightarrow M_W^2 W^{+\mu} W_\mu^- + \frac{M_Z^2}{2} Z^\mu Z_\mu \\ &+ H \text{ kinetic energy and gauge interaction terms}\end{aligned}$$

Mass eigenstate bosons: W , Z , and A (photon)

$$\begin{aligned}W^\pm &= \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \\ Z &= -\sin\theta_W B + \cos\theta_W W^3 \\ A &= \cos\theta_W B + \sin\theta_W W^3\end{aligned}$$

Masses

$$M_W = \frac{g\nu}{2}, \quad M_Z = \sqrt{g^2 + g'^2} \frac{\nu}{2} = \frac{M_W}{\cos \theta_W}, \quad M_A = 0$$

(Goldstone scalar transformed into longitudinal components of W^\pm, Z)

Weak angle: $\tan \theta_W \equiv g'/g$

Will show: Fermi constant $G_F/\sqrt{2} \sim g^2/8M_W^2$, where $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ from muon lifetime

Electroweak scale

$$\nu = 2M_W/g \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV}$$

Will show: $g = e / \sin \theta_W$, where $\alpha = e^2 / 4\pi \sim 1 / 137.036 \Rightarrow$

$$M_W = M_Z \cos \theta_W \sim \frac{(\pi\alpha / \sqrt{2}G_F)^{1/2}}{\sin \theta_W}$$

Weak neutral current: $\sin^2 \theta_W \sim 0.23 \Rightarrow M_W \sim 78 \text{ GeV}$, and
 $M_Z \sim 89 \text{ GeV}$ (increased by $\sim 2 \text{ GeV}$ by loop corrections)

Discovered at CERN: UA1 and UA2, 1983

Current:

$$M_Z = 91.1876 \pm 0.0021$$

$$M_W = 80.403 \pm 0.029$$

The Higgs Scalar H

Gauge interactions: $ZZH, ZZH^2, W^+W^-H, W^+W^-H^2$

$$\varphi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_\varphi &= (D^\mu \varphi)^\dagger D_\mu \varphi - V(\varphi) \\ &= \frac{1}{2} (\partial_\mu H)^2 + M_W^2 W^{\mu+} W_\mu^- \left(1 + \frac{H}{\nu}\right)^2 \\ &\quad + \frac{1}{2} M_Z^2 Z^\mu Z_\mu \left(1 + \frac{H}{\nu}\right)^2 - V(\varphi) \end{aligned}$$

(quartic and induced cubic interactions)

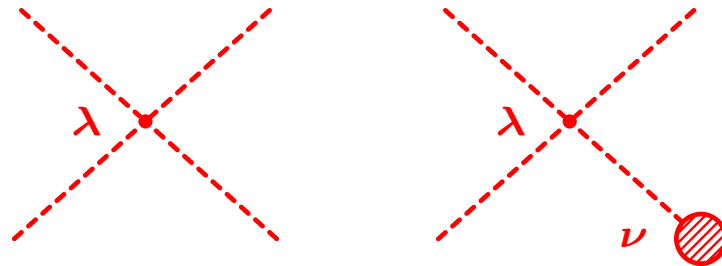
Higgs potential:

$$V(\varphi) = +\mu^2\varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2$$
$$\rightarrow -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda\nu H^3 + \frac{\lambda}{4}H^4$$

Fourth term: Quartic self-interaction

Third: Induced cubic self-interaction

Second: (Tree level) H mass-squared, $M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda\nu}$



No a priori constraint on λ except vacuum stability

($\lambda > 0 \Rightarrow 0 < M_H < \infty$), but

t quark loops destabilize vacuum unless $M_H \gtrsim 130$ GeV

(Depends on scale Λ at which theory breaks down. Doesn't hold in supersymmetry)

Strong coupling for $\lambda \gtrsim 1 \Leftrightarrow M_H \gtrsim 1$ TeV

Triviality: running λ should not diverge below $\Lambda \Rightarrow$

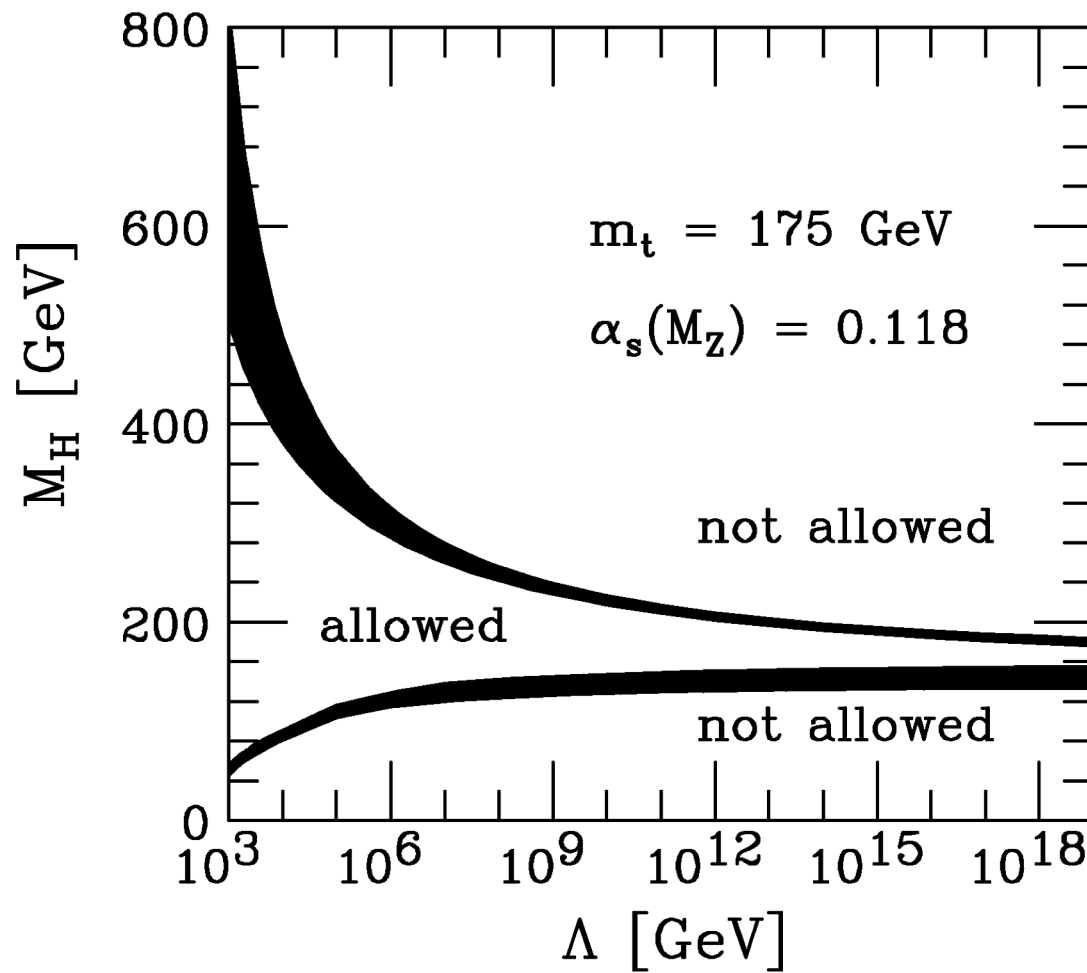
$$M_H < \begin{cases} O(200) \text{ GeV}, & \Lambda \sim M_P = G_N^{-1/2} \sim 10^{19} \text{ GeV} \\ O(750) \text{ GeV}, & \Lambda \sim 2M_H \end{cases}$$

Experimental bound (LEP 2), $e^+e^- \rightarrow Z^* \rightarrow ZH \Rightarrow M_H \gtrsim 114.5$ GeV at 95% cl (can evade with singlet or in MSSM)

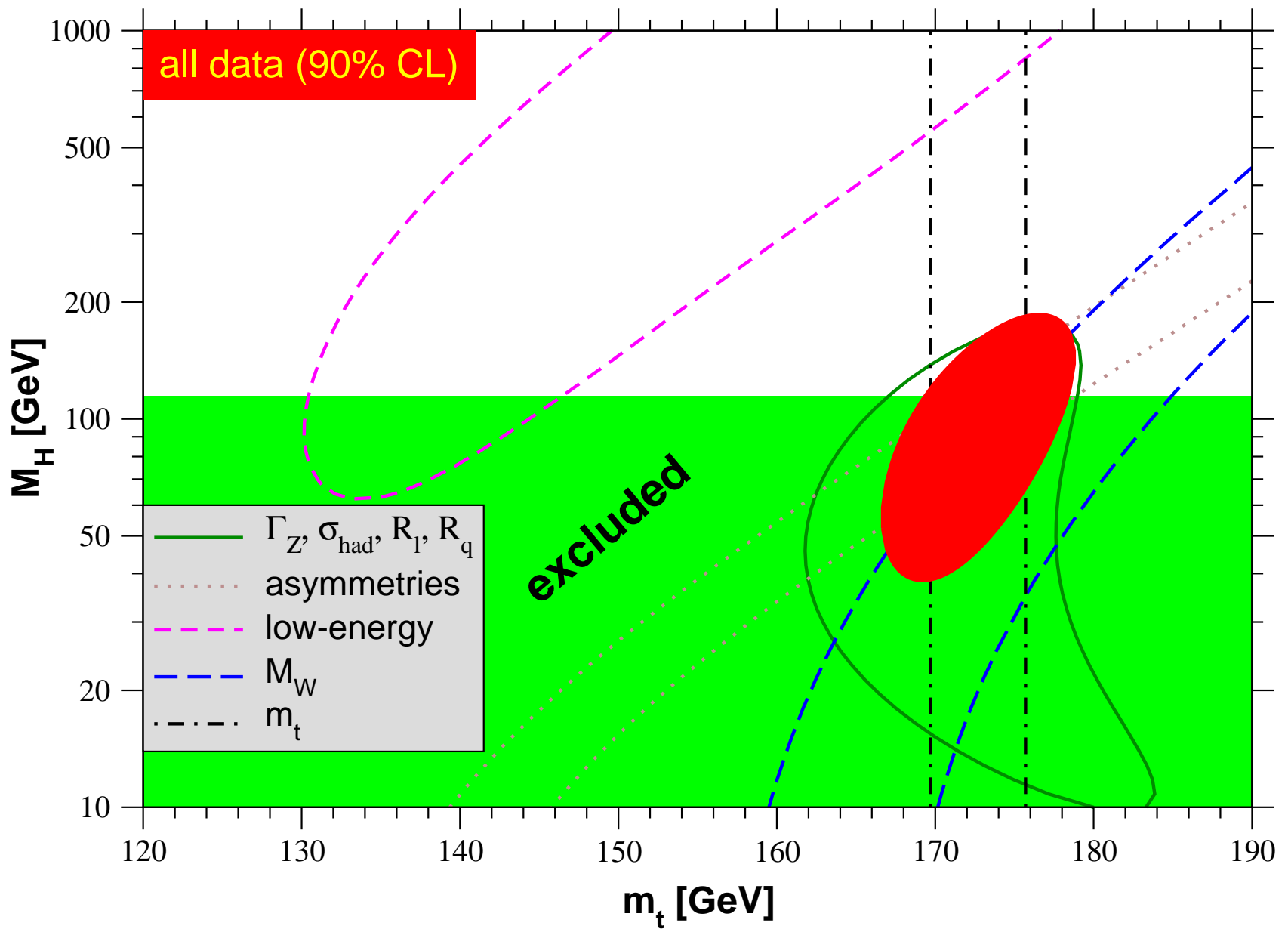
Hint of signal at 115 GeV

Indirect (precision tests): $M_H < 189$ GeV, 95% cl

MSSM: much of parameter space has standard-like Higgs with $M_H < 130$ GeV



Theoretical M_H limits, Hambye and Riesselmann, hep-ph/9708416

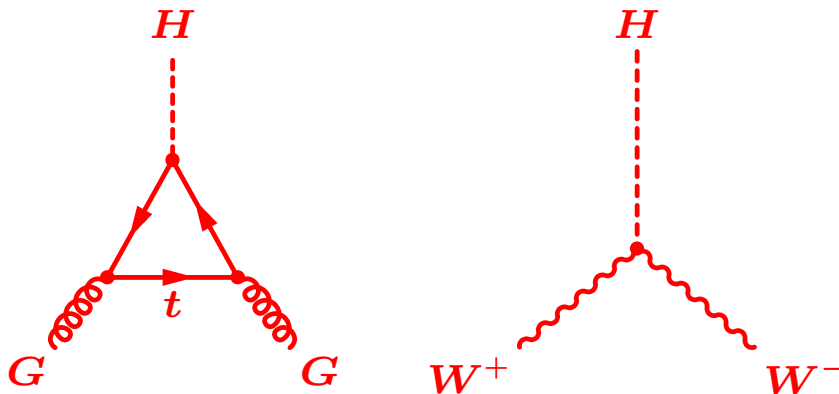


Decays: $H \rightarrow \bar{b}b$ dominates for $M_H \lesssim 2M_W$ ($H \rightarrow W^+W^-$, ZZ dominate when allowed because of larger gauge coupling)

Production:

LEP: Higgstrahlung ($e^+e^- \rightarrow Z^* \rightarrow ZH$)

Tevatron, LHC: $GG \rightarrow H$ via top loop), WW or ZZ fusion ($WW \rightarrow H$), or associated production ($\bar{q}q \rightarrow WH$, ZH , $GG \rightarrow t\bar{t}H$)



First term in V : vacuum energy

$$\langle 0|V|0\rangle = -\mu^4/4\lambda$$

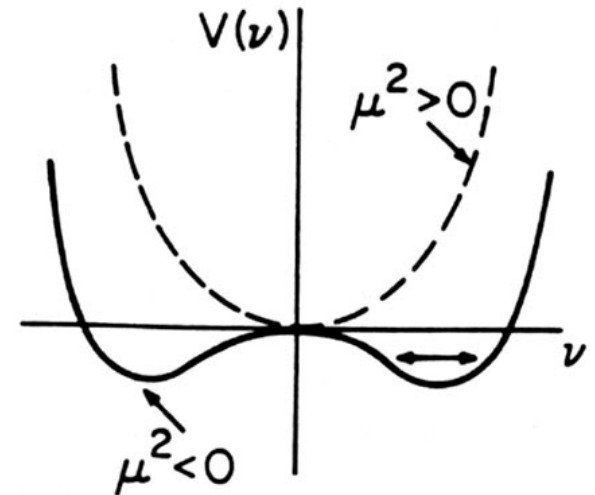
No effect on microscopic interactions, but gives *negative* contribution to cosmological constant

$$|\Lambda_{\text{SSB}}| = 8\pi G_N |\langle 0|V|0\rangle|$$

Require fine-tuned cancellation

$$\Lambda_{\text{cosm}} = \Lambda_{\text{bare}} + \Lambda_{\text{SSB}}$$

Also, QCD contribution from SSB of global chiral symmetry



Yukawa Interactions

$$\begin{aligned} -\mathcal{L}_{\text{Yukawa}} &\rightarrow \sum_{m,n=1}^F \bar{u}_{mL}^0 \Gamma_{mn}^u \left(\frac{\nu + H}{\sqrt{2}} \right) u_{nR}^0 + (d, e) \text{ terms} + \text{H.C.} \\ &= \bar{u}_L^0 (M^u + h^u H) u_R^0 + (d, e, \nu) \text{ terms} + \text{H.C.} \end{aligned}$$

$u_L^0 = (u_{1L}^0 u_{2L}^0 \cdots u_{FL}^0)^T$ is F -component column vector

M^u is $F \times F$ fermion mass matrix $M_{mn}^u = \Gamma_{mn}^u \nu / \sqrt{2}$ (need not be Hermitian, diagonal, symmetric, or even square)

$h^u = M^u / \nu = g M^u / 2M_W$ is the Yukawa coupling matrix

Diagonalize M by separate unitary transformations A_L and A_R
 (($A_L = A_R$) for Hermitian M)

$$A_L^{u\dagger} M^u A_R^u = M_D^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

is diagonal matrix of physical masses of the charge $\frac{2}{3}$ quarks.
 Similarly

$$\begin{aligned} A_L^{d\dagger} M^d A_R^d &= M_D^d \\ A_L^{e\dagger} M^e A_R^e &= M_D^e \\ (A_L^{\nu\dagger} M^\nu A_R^\nu &= M_D^\nu) \end{aligned}$$

(may also be Majorana masses for ν_R)

Find A_L and A_R by diagonalizing Hermitian matrices MM^\dagger and $M^\dagger M$, e.g., $A_L^\dagger MM^\dagger A_L = M_D^2$

Mass eigenstate fields

$$u_L = A_L^{u\dagger} u_L^0 = (u_L \ c_L \ t_L)^T$$

$$u_R = A_R^{u\dagger} u_R^0 = (u_R \ c_R \ t_R)^T$$

$$d_{L,R} = A_{L,R}^{d\dagger} d_{L,R}^0 = (d_{L,R} \ s_{L,R} \ b_{L,R})^T$$

$$e_{L,R} = A_{L,R}^{e\dagger} e_{L,R}^0 = (e_{L,R} \ \mu_{L,R} \ \tau_{L,R})^T$$

$$\nu_{L,R} = A_{L,R}^{\nu\dagger} \nu_{L,R}^0 = (\nu_{1L,R} \ \nu_{2L,R} \ \nu_{3L,R})^T$$

(For $m_\nu = 0$ or negligible, define $\nu_L = A_L^{e\dagger} \nu_L^0$, so that $\nu_i \equiv \nu_e, \nu_\mu, \nu_\tau$ are the weak interaction partners of the $e, \mu,$ and τ .)

Combinations of A_L observable in weak charged current. A_R not observable in standard model.

Typical estimates: $m_u = 1.5 - 4 \text{ MeV}$, $m_d = 4 - 8 \text{ MeV}$, $m_s = 80 - 130 \text{ MeV}$, $m_c \sim 1.3 \text{ GeV}$, $m_b \sim 4.2 \text{ GeV}$, $m_t = 170.9 \pm 1.8 \text{ GeV}$

Implications for global $SU(3)_L \times SU(3)_R$ of QCD

These are current quark masses. $M_i = m_i + M_{dyn}$, $M_{dyn} \sim \Lambda_{\overline{MS}} \sim 300 \text{ MeV}$ from chiral condensate $\langle 0 | \bar{q}q | 0 \rangle \neq 0$

m_t is pole mass; others, running masses at m or at 2 GeV^2

Yukawa couplings of Higgs to fermions

$$L_{\text{Yukawa}} = \sum_i \bar{\psi}_i \left(-m_i - \frac{gm_i}{2M_W} H \right) \psi_i$$

Coupling $gm_i/2M_W$ is flavor diagonal and small except t quark

$H \rightarrow \bar{b}b$ dominates for $M_H \lesssim 2M_W$ ($H \rightarrow W^+W^-$, ZZ dominate when allowed because of larger gauge coupling)

Flavor diagonal because only one doublet couples to fermions \Rightarrow fermion mass and Yukawa matrices proportional

Often flavor changing Higgs couplings in extended models with two doublets coupling to same kind of fermion (*not* MSSM)

Stringent limits, e.g., tree-level Higgs contribution to $K_L - K_S$ mixing (loop in standard model) $\Rightarrow h_{\bar{d}s}/M_H < 10^{-6} \text{ GeV}^{-1}$

The Weak Charged Current

Fermi Theory incorporated in SM and made renormalizable

W -fermion interaction

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} \left(J_W^\mu W_\mu^- + J_W^{\mu\dagger} W_\mu^+ \right)$$

Charge-raising current (ignoring ν masses)

$$\begin{aligned} J_W^{\mu\dagger} &= \sum_{m=1}^F \left[\bar{\nu}_m^0 \gamma^\mu (1 - \gamma^5) e_m^0 + \bar{u}_m^0 \gamma^\mu (1 - \gamma^5) d_m^0 \right] \\ &= (\bar{\nu}_e \bar{\nu}_\mu \bar{\nu}_\tau) \gamma^\mu (1 - \gamma^5) \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} + (\bar{u} \ \bar{c} \ \bar{t}) \gamma^\mu (1 - \gamma^5) V \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \end{aligned}$$

Ignore ν masses for now

Pure $V - A \Rightarrow$ maximal P and C violation; CP conserved except for phases in V

$V = A_L^{u\dagger} A_L^d$ is $F \times F$ unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix from mismatch between weak and Yukawa interactions

Cabibbo matrix for $F = 2$

$$V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}$$

$\sin \theta_c \simeq 0.22 \equiv$ Cabibbo angle

Good zeroth-order description since third family almost decouples

CKM matrix for $F = 3$ involves 3 angles and 1 CP -violating phase (after removing unobservable q_L phases) (new interactions involving q_R could make observable)

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{td} & V_{td} \end{pmatrix}$$

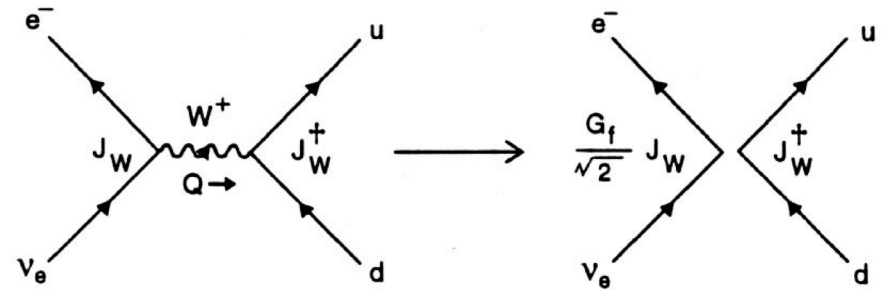
Extensive studies, especially in B decays, to test unitarity of V as probe of new physics and test origin of CP violation

Need additional source of CP breaking for baryogenesis

Effective zero- range 4-fermi interaction (Fermi theory)

For $|Q| \ll M_W$, neglect Q^2 in W propagator

$$-L_{\text{eff}}^{cc} = \frac{G_F}{\sqrt{2}} J_W^\mu J_{W\mu}^\dagger$$



Fermi constant

$$\frac{G_F}{\sqrt{2}} \simeq \frac{g^2}{8M_W^2} = \frac{1}{2\nu^2}$$

Muon lifetime $\tau^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} \Rightarrow G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$

Weak scale $\nu = \sqrt{2} \langle 0 | \varphi^0 | 0 \rangle \simeq 246 \text{ GeV}$

Excellent description of β , K , hyperon, heavy quark, μ , and τ decays, $\nu_\mu e \rightarrow \mu^- \nu_e$, $\nu_\mu n \rightarrow \mu^- p$, $\nu_\mu N \rightarrow \mu^- X$

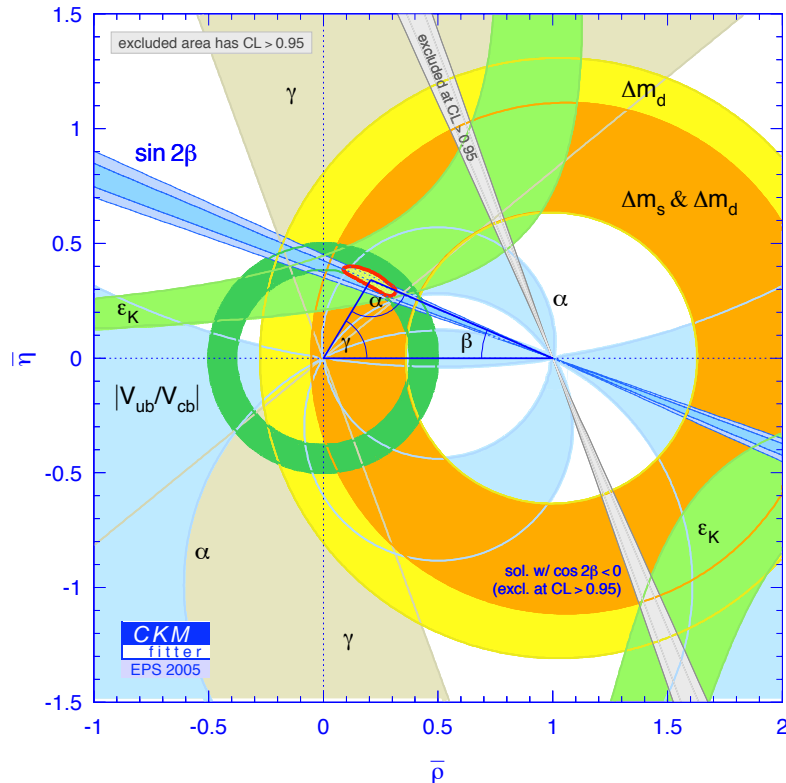
Full theory probed:

$e^\pm p \rightarrow \nu_e X$ at high energy (HERA)

Electroweak radiative corrections (loop level)

(Very important. Only calculable in full theory.)

$M_{K_S} - M_{K_L}$, kaon CP violation, $B \leftrightarrow \bar{B}$ mixing (loop level)



(CKMFITTER group:

<http://ckmfitter.in2p3.fr/>)

Quantum Electrodynamics (QED)

Incorporated into standard model

Lagrangian:

$$\mathcal{L} = -\frac{gg'}{\sqrt{g^2 + g'^2}} J_Q^\mu (\cos \theta_W B_\mu + \sin \theta_W W_\mu^3)$$

Photon field:

$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3$$

Positron electric charge: $e = g \sin \theta_W$, where $\tan \theta_W \equiv g'/g$

Electromagnetic current:

$$\begin{aligned} J_Q^\mu &= \sum_{m=1}^F \left[\frac{2}{3} \bar{u}_m^0 \gamma^\mu u_m^0 - \frac{1}{3} \bar{d}_m^0 \gamma^\mu d_m^0 - \bar{e}_m^0 \gamma^\mu e_m^0 \right] \\ &= \sum_{m=1}^F \left[\frac{2}{3} \bar{u}_m \gamma^\mu u_m - \frac{1}{3} \bar{d}_m \gamma^\mu d_m - \bar{e}_m \gamma^\mu e_m \right] \end{aligned}$$

Electric charge: $Q = T^3 + Y$, where $Y =$ weak hypercharge
(coefficient of $ig' B_\mu$ in covariant derivatives)

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

Purely vector (parity conserving): L and R fields have same charge
($q_i = t_i^3 + y_i$ is the same for L and R fields, even though t_i^3 and y_i are not)

| Experiment | Value of α^{-1} | | Difference from $\alpha^{-1}(a_e)$ |
|--|------------------------|------------------------|-------------------------------------|
| Deviation from gyromagnetic ratio, $a_e = (g - 2)/2$ for e^- | 137.035 999 58 (52) | $[3.8 \times 10^{-9}]$ | – |
| ac Josephson effect | 137.035 988 0 (51) | $[3.7 \times 10^{-8}]$ | $(0.116 \pm 0.051) \times 10^{-4}$ |
| h/m_n (m_n is the neutron mass) from n beam | 137.036 011 9 (51) | $[3.7 \times 10^{-8}]$ | $(-0.123 \pm 0.051) \times 10^{-4}$ |
| Hyperfine structure in muonium, μ^+e^- | 137.035 993 2 (83) | $[6.0 \times 10^{-8}]$ | $(0.064 \pm 0.083) \times 10^{-4}$ |
| Cesium D_1 line | 137.035 992 4 (41) | $[3.0 \times 10^{-8}]$ | $(0.072 \pm 0.041) \times 10^{-4}$ |

Spectacularly successful:

Most precise: e anomalous magnetic moment $\rightarrow \alpha$

Many low energy tests to few $\times 10^{-8}$

$$m_\gamma < 6 \times 10^{-17} \text{ eV}$$

$$q_\gamma < 5 \times 10^{-30} |e|$$

Running $\alpha(Q^2)$ observed

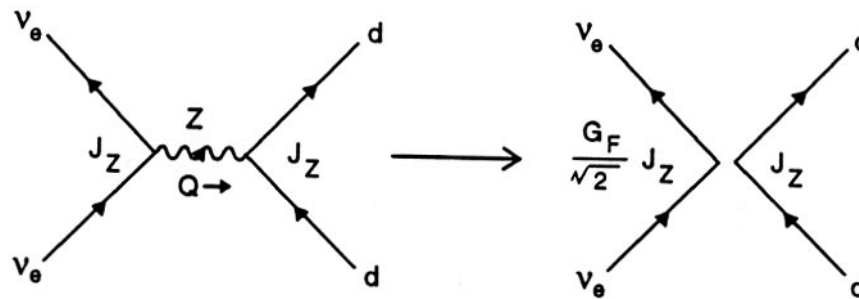
High energy well-measured (PEP, PETRA, TRISTAN, LEP)

Muon $g - 2$ sensitive to new physics. Anomaly?

The Weak Neutral Current

Prediction of $SU(2) \times U(1)$

$$\begin{aligned}\mathcal{L} &= -\frac{\sqrt{g^2 + g'^2}}{2} J_Z^\mu \left(-\sin \theta_W B_\mu + \cos \theta_W W_\mu^3 \right) \\ &= -\frac{g}{2 \cos \theta_W} J_Z^\mu Z_\mu\end{aligned}$$



Neutral current process and effective 4-fermi interaction for $|Q| \ll M_Z$

Neutral current:

$$\begin{aligned} J_Z^\mu &= \sum_m [\bar{u}_{mL}^0 \gamma^\mu u_{mL}^0 - \bar{d}_{mL}^0 \gamma^\mu d_{mL}^0 + \bar{\nu}_{mL}^0 \gamma^\mu \nu_{mL}^0 - \bar{e}_{mL}^0 \gamma^\mu e_{mL}^0] \\ &\quad - 2 \sin^2 \theta_W J_Q^\mu \\ &= \sum_m [\bar{u}_{mL} \gamma^\mu u_{mL} - \bar{d}_{mL} \gamma^\mu d_{mL} + \bar{\nu}_{mL} \gamma^\mu \nu_{mL} - \bar{e}_{mL} \gamma^\mu e_{mL}] \\ &\quad - 2 \sin^2 \theta_W J_Q^\mu \end{aligned}$$

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

GIM mechanism: c quark predicted so that s_L could be in doublet to avoid unwanted flavor changing neutral currents (FCNC) at tree and loop level

Parity and charge conjugation violated but not maximally: first term is pure $V - A$, second is V

Effective 4-fermi interaction for $|Q^2| \ll M_Z^2$:

$$-\mathcal{L}_{\text{eff}}^{NC} = \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}$$

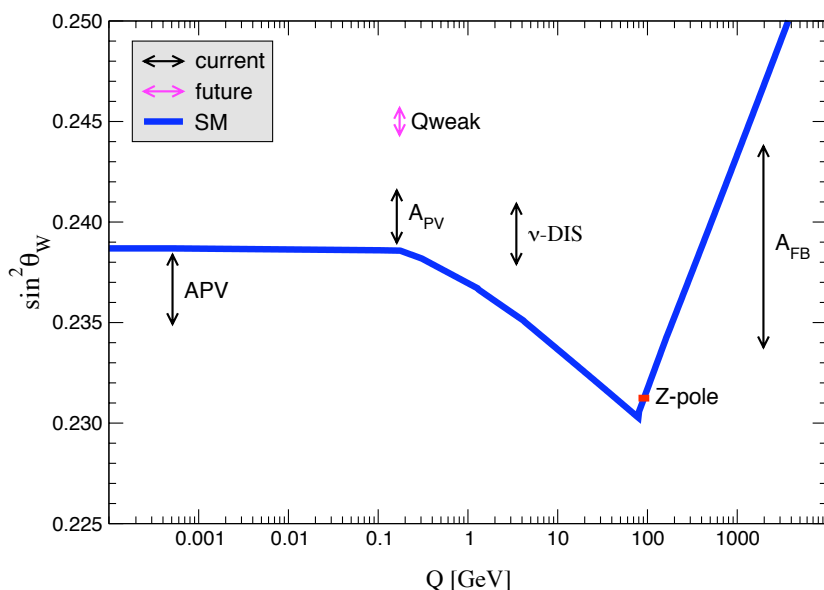
Coefficient same as WCC because

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2 + g'^2}{8M_Z^2}$$

The Z , the W , and the Weak Neutral Current

- Primary prediction and test of electroweak unification
- WNC discovered 1973 (Gargamelle at CERN, HPW at FNAL)
- 70's, 80's: weak neutral current experiments (few %)
 - Pure weak: νN , νe scattering
 - Weak-elm interference in eD , e^+e^- , atomic parity violation
 - $SU(2) \times U(1)$ group/representations; t and ν_τ exist; m_t limit; hint for SUSY unification; limits on TeV scale physics
- W , Z discovered directly 1983 (UA1, UA2)

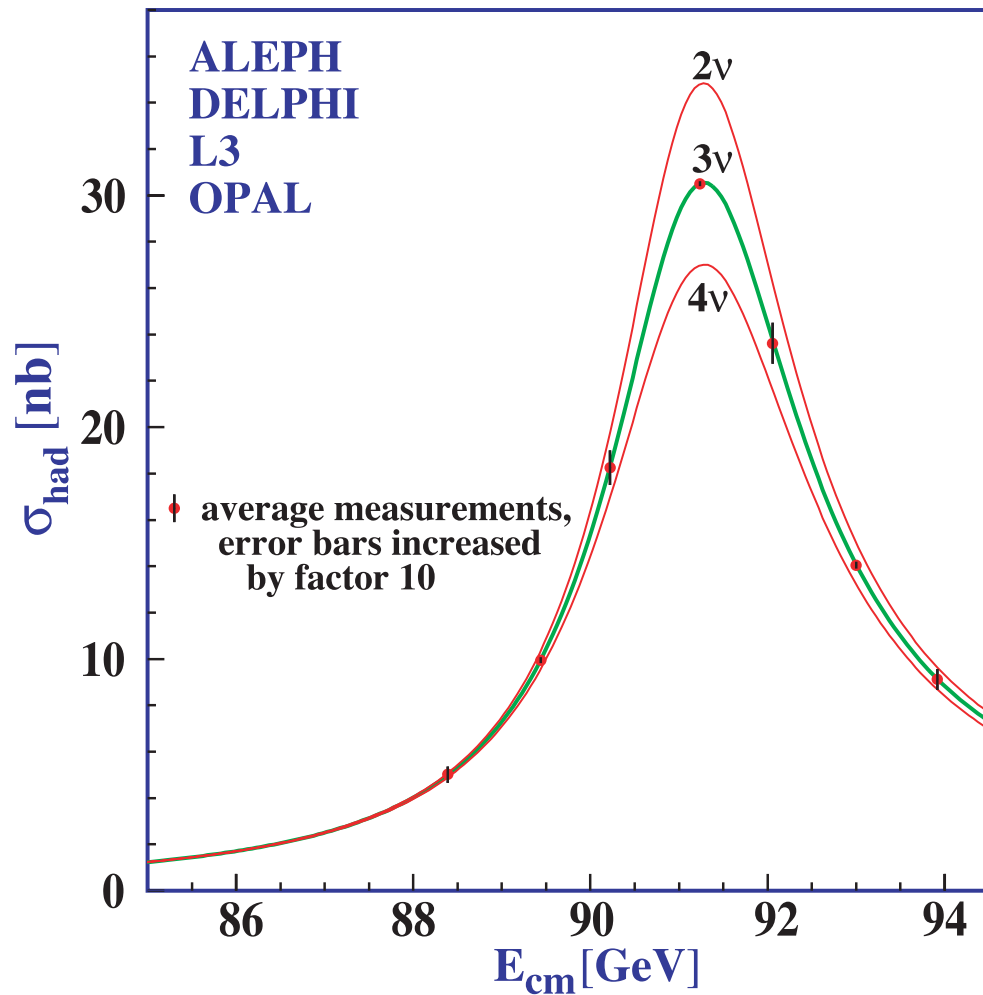
- 90's: Z pole (LEP, SLD), 0.1%; lineshape, modes, asymmetries
- LEP 2: M_W , Higgs search , gauge self-interactions
- Tevatron: m_t , M_W , M_H search
- 4th generation weak neutral current experiments (atomic parity (Boulder); νe ; νN (NuTeV); polarized Møller asymmetry (SLAC))



Running \hat{s}_Z^2 in $\overline{\text{MS}}$ scheme

The LEP/SLC Era

- **Z Pole:** $e^+e^- \rightarrow Z \rightarrow \ell^+\ell^-, q\bar{q}, \nu\bar{\nu}$
 - LEP (CERN), 2×10^7 Z's, unpolarized (ALEPH, DELPHI, L3, OPAL);
SLC (SLAC), 5×10^5 , $P_{e^-} \sim 75\%$ (SLD)
- **Z pole observables**
 - lineshape: M_Z, Γ_Z, σ
 - branching ratios
 - * $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$
 - * $q\bar{q}, c\bar{c}, b\bar{b}, s\bar{s}$
 - * $\nu\bar{\nu} \Rightarrow N_\nu = 2.984 \pm 0.009$ if $m_\nu < M_Z/2$
 - asymmetries: FB, polarization, P_τ , mixed
 - lepton family universality



- $N_\nu = 3 + \Delta N_\nu = 2.984 \pm 0.009$
- $\Delta N_\nu = 1$ for fourth family ν with $m_\nu \lesssim M_Z/2$
- $\Delta N_\nu = \frac{1}{2}$, light $\tilde{\nu}$ in super-symmetry
- $\Delta N_\nu = 2$, Majoron + scalar in triplet model of m_ν with spontaneous L violation

The Z Pole Observables: LEP and SLC (09/05)

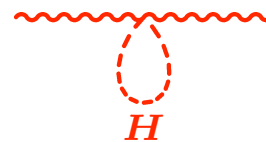
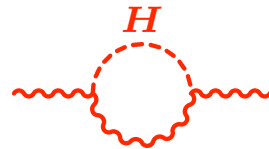
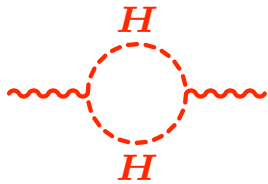
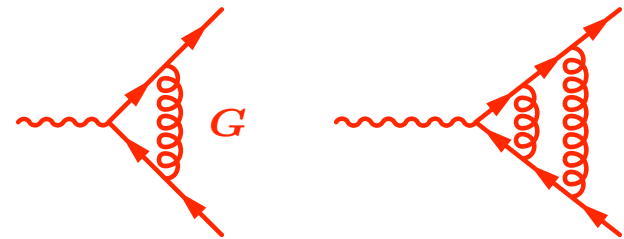
| Quantity | Group(s) | Value | Standard Model | pull |
|------------------------------|----------|----------------------|-----------------------|------------|
| M_Z [GeV] | LEP | 91.1876 ± 0.0021 | 91.1874 ± 0.0021 | 0.1 |
| Γ_Z [GeV] | LEP | 2.4952 ± 0.0023 | 2.4968 ± 0.0011 | -0.7 |
| $\Gamma(\text{had})$ [GeV] | LEP | 1.7444 ± 0.0020 | 1.7434 ± 0.0010 | — |
| $\Gamma(\text{inv})$ [MeV] | LEP | 499.0 ± 1.5 | 501.65 ± 0.11 | — |
| $\Gamma(\ell^+\ell^-)$ [MeV] | LEP | 83.984 ± 0.086 | 83.996 ± 0.021 | — |
| σ_{had} [nb] | LEP | 41.541 ± 0.037 | 41.467 ± 0.009 | 2.0 |
| R_e | LEP | 20.804 ± 0.050 | 20.756 ± 0.011 | 1.0 |
| R_μ | LEP | 20.785 ± 0.033 | 20.756 ± 0.011 | 0.9 |
| R_τ | LEP | 20.764 ± 0.045 | 20.801 ± 0.011 | -0.8 |
| $A_{FB}(e)$ | LEP | 0.0145 ± 0.0025 | 0.01622 ± 0.00025 | -0.7 |
| $A_{FB}(\mu)$ | LEP | 0.0169 ± 0.0013 | | 0.5 |
| $A_{FB}(\tau)$ | LEP | 0.0188 ± 0.0017 | | 1.5 |

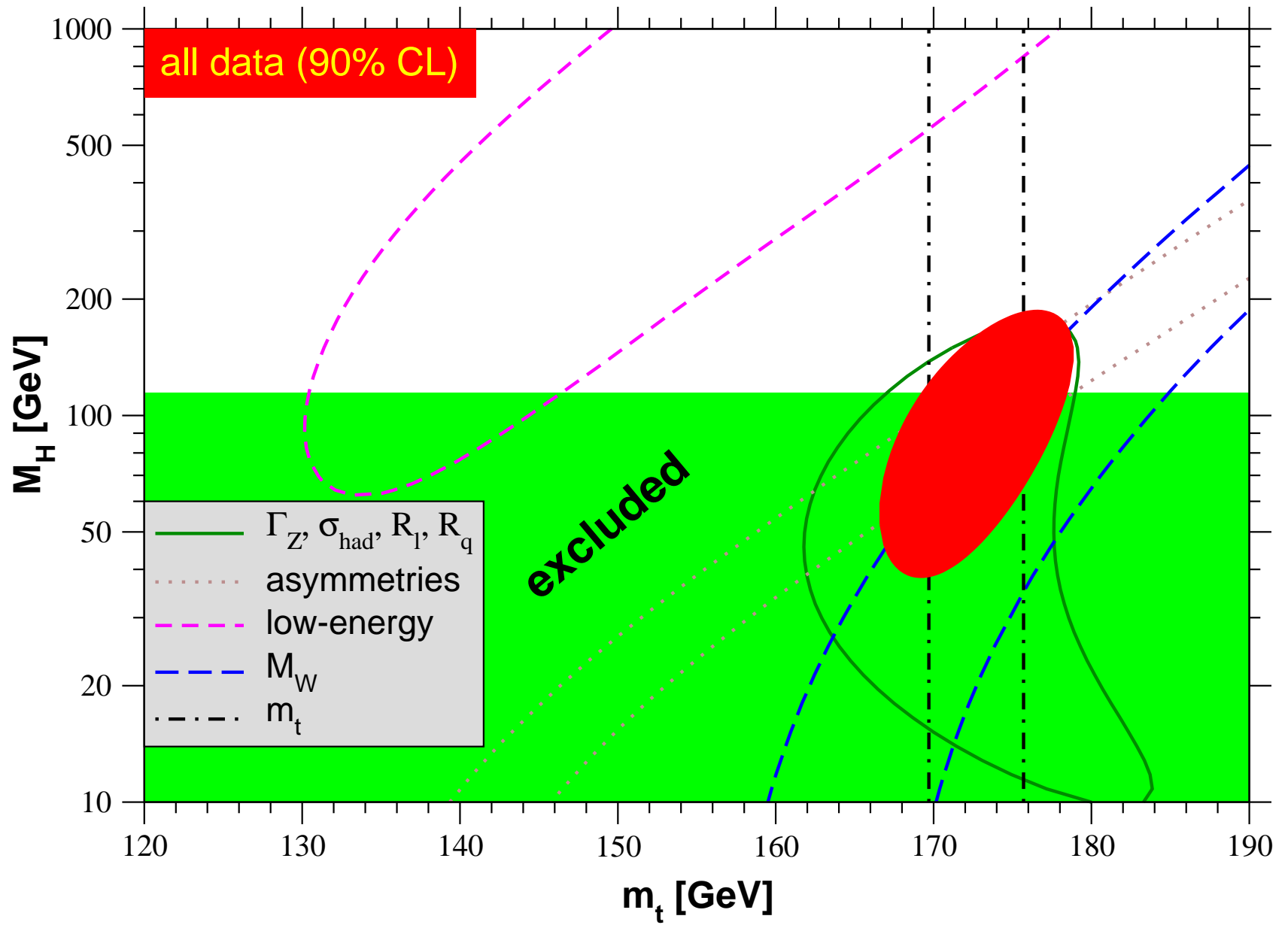
| Quantity | Group(s) | Value | Standard Model | pull |
|-----------------------------|-------------|-----------------------|-----------------------|------|
| R_b | LEP/SLD | 0.21629 ± 0.00066 | 0.21578 ± 0.00010 | 0.8 |
| R_c | LEP/SLD | 0.1721 ± 0.0030 | 0.17230 ± 0.00004 | -0.1 |
| $A_{FB}(b)$ | LEP | 0.0992 ± 0.0016 | 0.1031 ± 0.0008 | -2.4 |
| $A_{FB}(c)$ | LEP | 0.0707 ± 0.0035 | 0.0737 ± 0.0006 | -0.8 |
| $A_{FB}(s)$ | DELPHI/OPAL | 0.0976 ± 0.0114 | 0.1032 ± 0.0008 | -0.5 |
| A_b | SLD | 0.923 ± 0.020 | 0.9347 ± 0.0001 | -0.6 |
| A_c | SLD | 0.670 ± 0.027 | 0.6678 ± 0.0005 | 0.1 |
| A_s | SLD | 0.895 ± 0.091 | 0.9356 ± 0.0001 | -0.4 |
| A_{LR} (hadrons) | SLD | 0.15138 ± 0.00216 | 0.1471 ± 0.0011 | 2.0 |
| A_{LR} (leptons) | SLD | 0.1544 ± 0.0060 | | 1.2 |
| A_μ | SLD | 0.142 ± 0.015 | | -0.3 |
| A_τ | SLD | 0.136 ± 0.015 | | -0.7 |
| $A_\tau(\mathcal{P}_\tau)$ | LEP | 0.1439 ± 0.0043 | | -0.7 |
| $A_e(\mathcal{P}_\tau)$ | LEP | 0.1498 ± 0.0049 | | 0.6 |
| $\bar{s}_\ell^2(Q_{FB})$ | LEP | 0.2324 ± 0.0012 | 0.23152 ± 0.00014 | 0.7 |
| $\bar{s}_\ell^2(A_{FB}(q))$ | CDF | 0.2238 ± 0.0050 | | -1.5 |

- LEP 2
 - M_W, Γ_W, B (also hadron colliders)
 - M_H limits (hint?)
 - WW production (triple gauge vertex)
 - Quartic vertex
 - SUSY/exotics searches

Electroweak Theory Probed at Loop Level

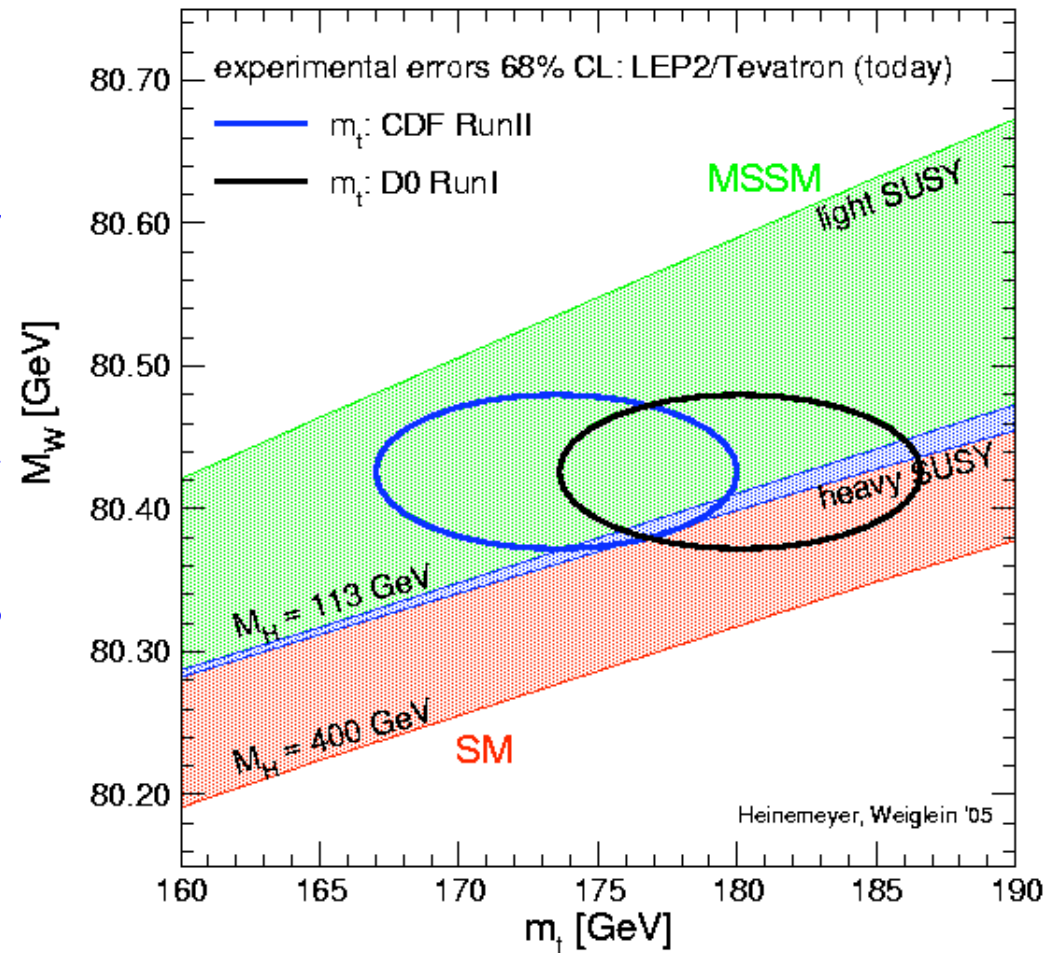
- Higher order corrections essential for agreement
- Depend on m_t , α_s , and M_H
- Good agreement with direct m_t and other α_s measurements; M_H prediction





- **Supersymmetry**

- decoupling limit ($M_{new} \gtrsim 200 - 300 \text{ GeV}$): only precision effect is light SM-like Higgs
- little improvement on SM fit
- Supersymmetry parameters constrained

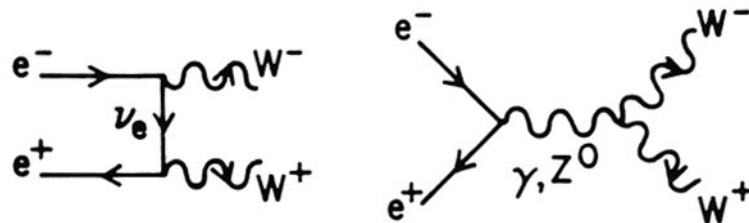


Gauge Self-Interactions

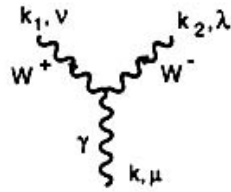
Three and four-point interactions predicted by gauge invariance

Indirectly verified by radiative corrections, α_s running in QCD, etc.

Strong cancellations in high energy amplitudes would be upset by anomalous couplings

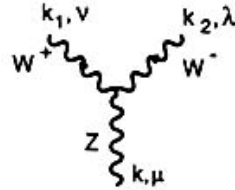


Tree-level diagrams contributing to $e^+e^- \rightarrow W^+W^-$

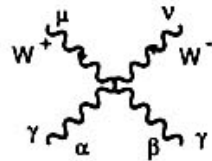


$$ieF_{\nu\lambda\mu}$$

$$F_{\nu\lambda\mu} = g_{\nu\lambda}(k_1 - k_2)_\mu + g_{\lambda\mu}(k_2 - k)_\nu + g_{\mu\nu}(k - k_1)_\lambda$$

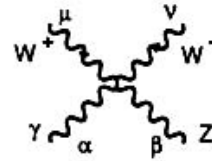


$$\frac{ie}{\tan\theta_W} F_{\nu\lambda\mu}$$

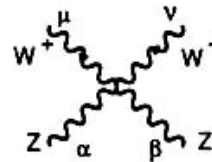


$$-ie^2 G_{\alpha\beta\mu\nu}$$

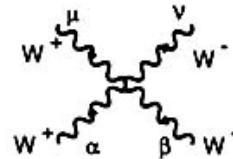
$$G_{\alpha\beta\mu\nu} = 2g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}$$



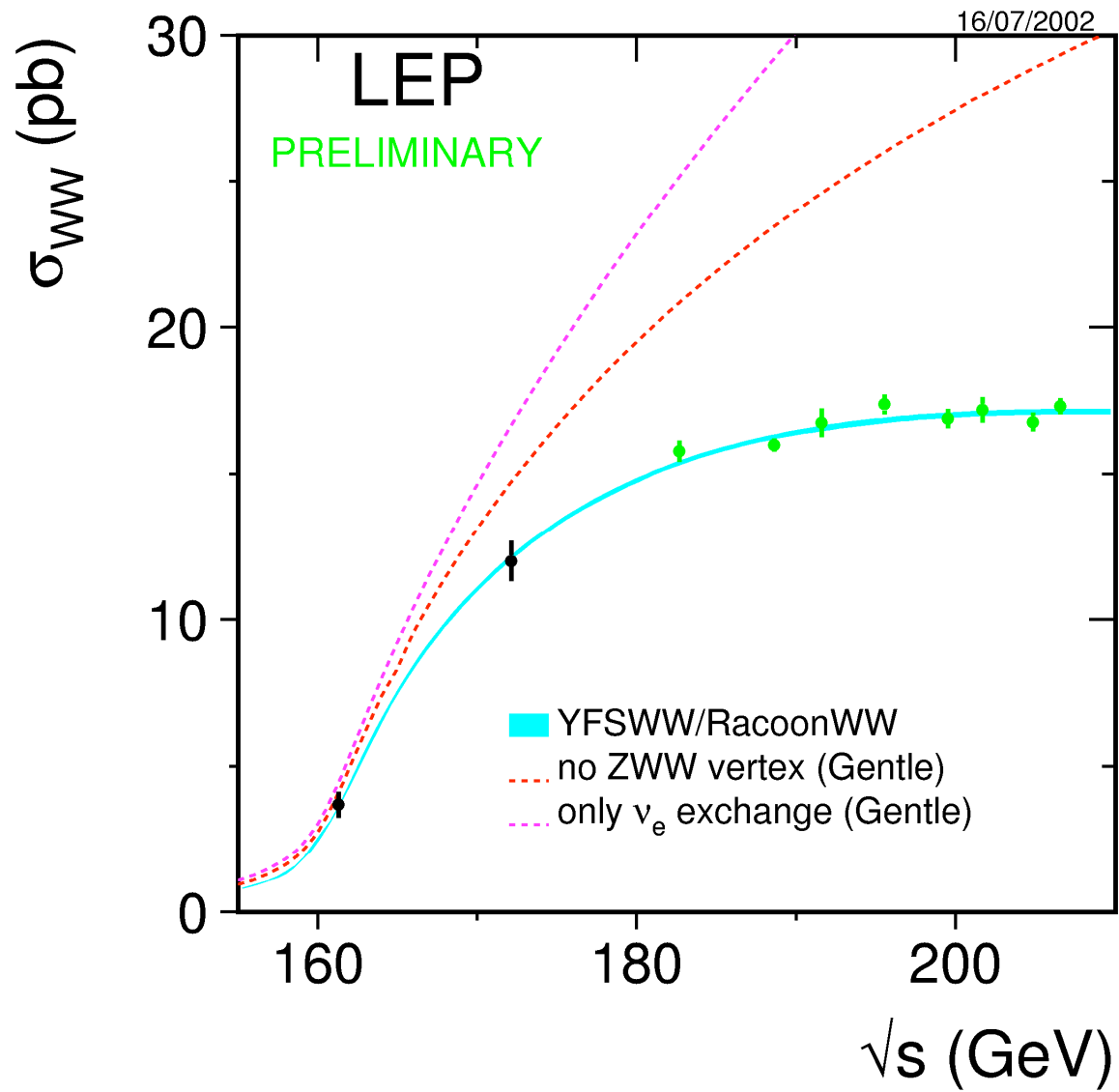
$$\frac{-ie^2}{\tan\theta_W} G_{\alpha\beta\mu\nu}$$



$$\frac{-ie^2}{\tan^2\theta_W} G_{\alpha\beta\mu\nu}$$

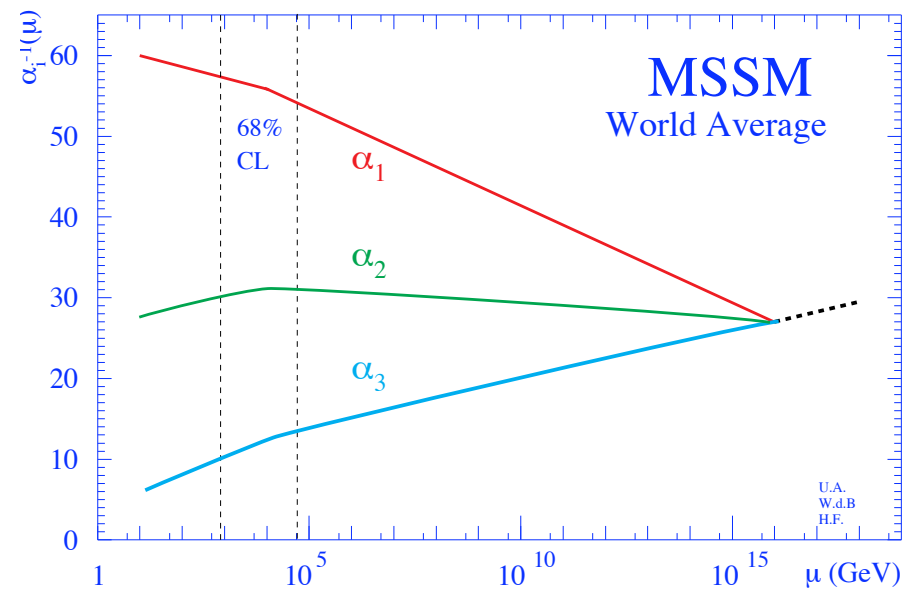
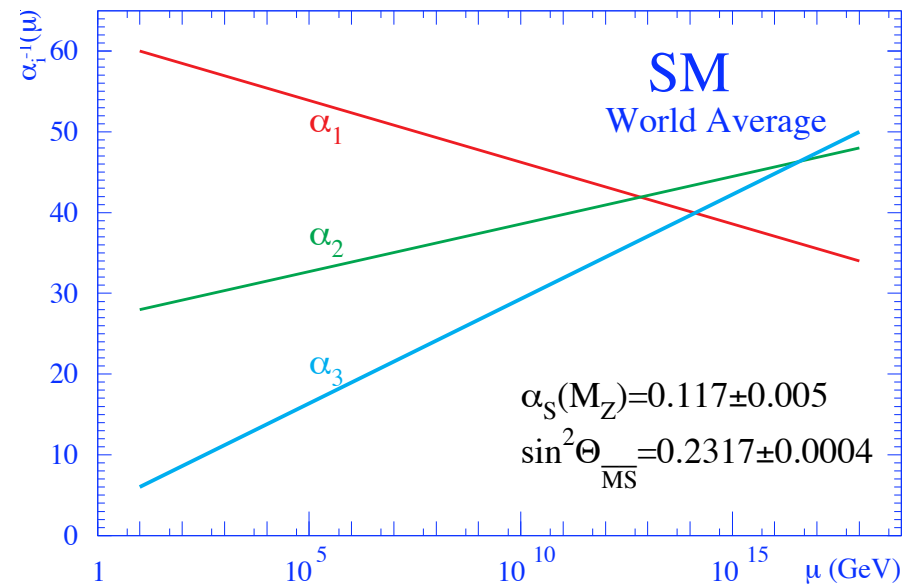


$$\frac{ie^2}{\sin^2\theta_W} G_{\alpha\beta\mu\nu}$$



- Gauge unification: GUTs, string theories

- $\alpha + \hat{s}_Z^2 \rightarrow \alpha_s = 0.130 \pm 0.010$ (MSSM) (non-SUSY: 0.073(1))
- $M_G \sim 3 \times 10^{16}$ GeV
- Perturbative string: $\sim 5 \times 10^{17}$ GeV (10% in $\ln M_G$). Exotics: $O(1)$ corrections.



Implications of Precision Electroweak

- WNC, Z , W are primary predictions and test of electroweak unification
- SM correct and unique to first approx. (gauge principle, group, representations)
- SM correct at loop level (renorm gauge theory; m_t , α_s , M_H)
- Watershed: TeV physics severely constrained (unification vs compositeness)
 - unification (decoupling): expect 0.1%
 - TeV compositeness/dynamics: several % unless decoupling
 - Z' , W_R ; 4-Fermi; exotic fermions/mixings; extended Higgs; ...
- Precise gauge couplings (gauge unification)

Problems with the Standard Model

Lagrangian after symmetry breaking:

$$\mathcal{L} = L_{\text{gauge}} + L_{\text{Higgs}} + \sum_i \bar{\psi}_i \left(i \not{\partial} - m_i - \frac{m_i H}{\nu} \right) \psi_i - \frac{g}{2\sqrt{2}} \left(J_W^\mu W_\mu^- + J_W^{\mu\dagger} W_\mu^+ \right) - e J_Q^\mu A_\mu - \frac{g}{2 \cos \theta_W} J_Z^\mu Z_\mu$$

Standard model: $SU(2) \times U(1)$ (extended to include ν masses) + QCD + general relativity

Mathematically consistent, renormalizable theory

Correct to 10^{-16} cm

However, too much arbitrariness and fine-tuning: $O(27)$ parameters (+ 2 for Majorana ν) and electric charges

- Gauge Problem

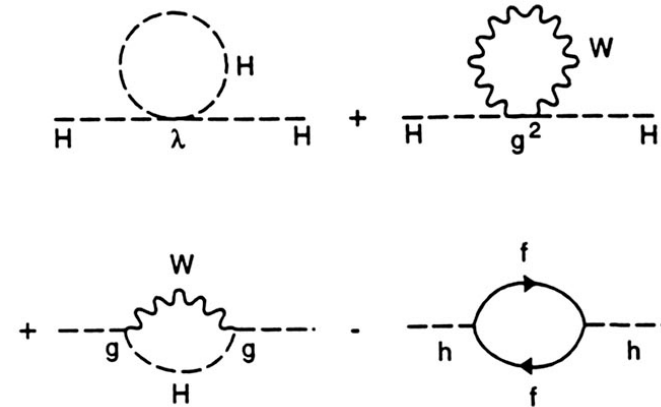
- complicated gauge group with 3 couplings
- charge quantization ($|q_e| = |q_p|$) unexplained
- Possible solutions: strings; grand unification; magnetic monopoles (partial); anomaly constraints (partial)

- Fermion problem

- Fermion masses, mixings, families unexplained
- Neutrino masses, nature? Probe of Planck/GUT scale?
- CP violation inadequate to explain baryon asymmetry
- Possible solutions: strings; brane worlds; family symmetries; compositeness; radiative hierarchies. New sources of CP violation.

- Higgs/hierarchy problem

- Expect $M_H^2 = O(M_W^2)$
- higher order corrections:
 $\delta M_H^2 / M_W^2 \sim 10^{34}$



Possible solutions: supersymmetry; dynamical symmetry breaking; large extra dimensions; Little Higgs; anthropically motivated fine-tuning (split supersymmetry) (landscape)

- Strong CP problem

- Can add $\frac{\theta}{32\pi^2} g_s^2 F \tilde{F}$ to QCD (breaks, P, T, CP)
- $d_N \Rightarrow \theta < 10^{-9}$, but $\delta\theta|_{\text{weak}} \sim 10^{-3}$
- Possible solutions: spontaneously broken global $U(1)$ (Peccei-Quinn) \Rightarrow axion; unbroken global $U(1)$ (massless u quark); spontaneously broken CP + other symmetries

- Graviton problem

- gravity not unified
- quantum gravity not renormalizable
- cosmological constant: $\Lambda_{\text{SSB}} = 8\pi G_N \langle V \rangle > 10^{50} \Lambda_{\text{obs}}$
(10^{124} for GUTs, strings)

Possible solutions:

- supergravity and Kaluza Klein unify
- strings yield finite gravity
- Λ ? Anthropically motivated fine-tuning (landscape)?

- Necessary new ingredients
 - Mechanism for small neutrino masses
 - * Planck/GUT scale? Small Dirac (intermediate scale)?
 - Mechanism for baryon asymmetry?
 - * Electroweak transition (Z' or extended Higgs?)
 - * Heavy Majorana neutrino decay (seesaw)?
 - * Decay of coherent field? CPT violation?
 - What is the dark energy?
 - * Cosmological Constant? Quintessence?
 - * Related to inflation? Time variation of couplings?
 - What is the dark matter?
 - * Lightest supersymmetric particle? Axion?
 - Suppression of flavor changing neutral currents? Proton decay? Electric dipole moments?
 - * *Automatic* in standard model, but not in extensions

Conclusions

- The standard model is spectacularly successful, but is incomplete
- Promising theoretical ideas at Planck and TeV scale
- Eagerly anticipate guidance from LHC