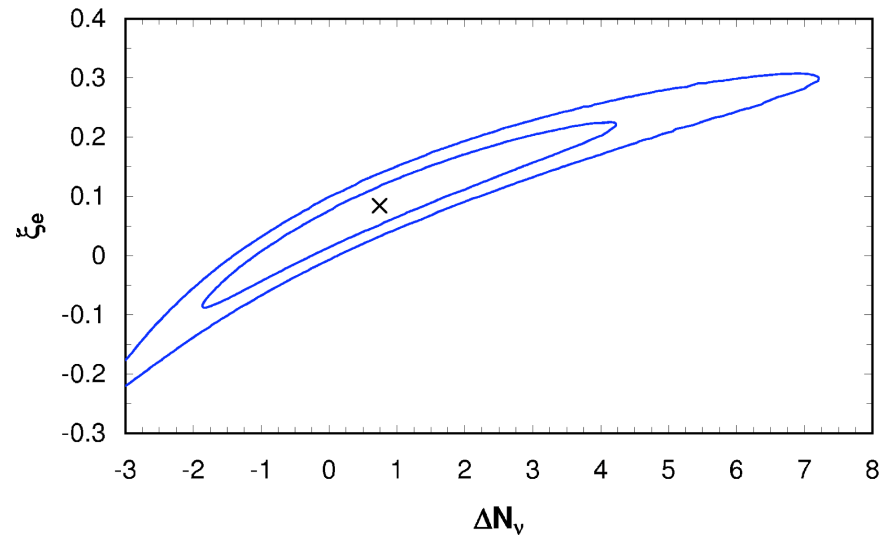


Neutrinos and Strings



- Introduction
- Neutrino preliminaries
- Models
- String embeddings
 - Intersecting brane
 - The Z_3 heterotic orbifold
 - Embedding the Higgs triplet
- Outlook



SteigmanFest (May 18, 2006)

Paul Langacker (Penn)

Neutrino mass

- Nonzero mass may be first break with standard model
- Enormous theoretical effort: GUT, family symmetries, bottom up
 - Majorana masses may be favored because not forbidden by SM gauge symmetries
 - GUT seesaw (heavy Majorana singlet). Usually ordinary hierarchy.
 - Higgs triplets (“type II seesaw”), often assuming GUT, Left-Right relations

The minimal seesaw

- Active (sterile) neutrinos ν_L (N_R) (3 flavors each)

$$L = \frac{1}{2} (\bar{\nu}_L \quad \bar{N}_L^c) \begin{pmatrix} m_T & m_D \\ m_D^T & m_S \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{hc}$$

- $m_T = m_T^T =$ triplet Majorana mass matrix (Higgs triplet)
- $m_D =$ Dirac mass matrix (Higgs doublet)
- $m_S = m_S^T =$ singlet Majorana mass matrix (Higgs singlet)

- Ordinary (type I) seesaw: $m_T = 0$ and (eigenvalues) $m_S \gg m_D$:

$$m_\nu^{\text{eff}} = -m_D m_S^{-1} m_D^T$$

diagonalized by U_ν , with

$$U_{PMNS} = U_e^\dagger U_\nu$$

- To achieve large mixings, most models assume either
 - $U_e \sim I$ in basis with manifest symmetries for $m_{D,S} \Rightarrow$ need large mixings in U_ν (requires clever m_D, m_S collaboration)
 - Large U_e mixings from lopsided m_e in basis with $m_{D,S} \sim$ diagonal (harder to achieve in $SO(10)$ than $SU(5)$)
- $SO(10)$ models, combined with family symmetries, often large Higgs representations (e.g., 126-plet); typically, $m_S \sim 10^{14}$ GeV

Extended (TeV) Seesaw

- $m_\nu \sim m^{p+1}/m_S^p$, $p > 1$ (e.g., $m \sim 100$ MeV, $m_S \sim 1$ TeV for $p = 2$)
- ν_L, N_R, N'_R (3 flavors each)

$$L = \frac{1}{2} (\bar{\nu}_L \quad \bar{N}_L^c \quad \bar{N}'_L{}^c) \begin{pmatrix} 0 & m_D & m_{D'} \\ m_D^T & 0 & m_{SS'} \\ m_{D'}^T & m_{SS'}^T & 0 \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \\ N'_R \end{pmatrix} + \text{hc}$$

or

$$L = \frac{1}{2} (\bar{\nu}_L \quad \bar{N}_L^c \quad \bar{N}'_L{}^c) \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & m_{SS'} \\ 0 & m_{SS'}^T & m_{S'} \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \\ N'_R \end{pmatrix} + \text{hc}$$

Triplet models

- Introduce Higgs triplet $T = (T^{++} T^+ T^0)^T$ with weak hypercharge $Y = 1$
- Majorana masses m_T generated from $L_\nu = \lambda_{ij}^T L_i T L_j$ if $\langle T^0 \rangle \neq 0$
- Old Gelmini-Roncadelli model: $\langle T^0 \rangle \ll$ EW scale with spontaneous L violation
 - Excluded by $Z \rightarrow$ Majoron + scalar (equivalent to $\Delta N_\nu = 2$)
- Modern triplet models (type II seesaw) break L explicitly by THH couplings, giving large Majoron mass (Lazarides, Shafi, Wetterich, Mohapatra, Senjanovic, Schechter, Valle, Ma, Hambye, Sarkar, Rossi, ...)
- Often considered in $SO(10)$ or LR context, with both ordinary and triplet mechanisms competing and with related parameters, but can consider independently.

Dirac Masses

- Can achieve small Dirac masses (neutrino or other) by higher dimensional operators

$$L_\nu \sim \left(\frac{S}{M_{Pl}} \right)^p L N_L^c H_2, \quad \langle S \rangle \ll M_{Pl}$$

$$\Rightarrow m_D \sim \left(\frac{\langle S \rangle}{M_{Pl}} \right)^p \langle H_2 \rangle$$

- Large $p \Rightarrow \langle S \rangle$ close to M_{Pl} (e.g., anomalous $U(1)_A$)
- Small $p \Rightarrow$ intermediate scale $\ll M_{Pl}$
- Similar HDO may give light steriles and ordinary/sterile mixing

Other Models

- Large extra dimensions (suppressed Dirac Yukawa couplings)
- R -parity violation in supersymmetry
- TeV scale loops with new ad hoc scalars
- Ad hoc flavor symmetries, textures, anarchic models
- Anthropic considerations (string landscape)

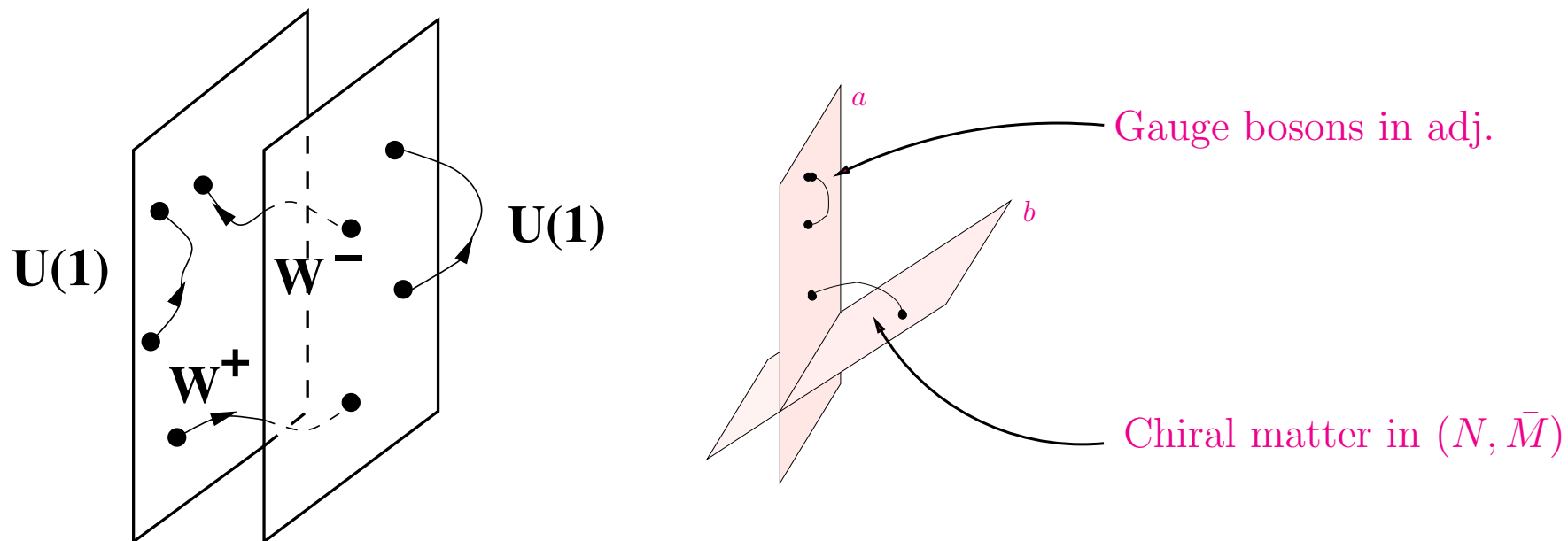
Neutrino Mass in Strings

- Very little work from string constructions, even though may be Planck scale effect
- Key ingredients of most bottom up models forbidden in known constructions (heterotic or intersecting brane)
(Due to string symmetries or constraints, not simplicity or elegance)
 - “Right-handed” neutrinos may not be gauge singlets
 - Large representations difficult to achieve (bifundamentals, singlets, or adjoints)
 - GUT Yukawa relations broken
 - String symmetries/constraints severely restrict couplings, e.g., Majorana masses, or simultaneous Dirac and Majorana masses

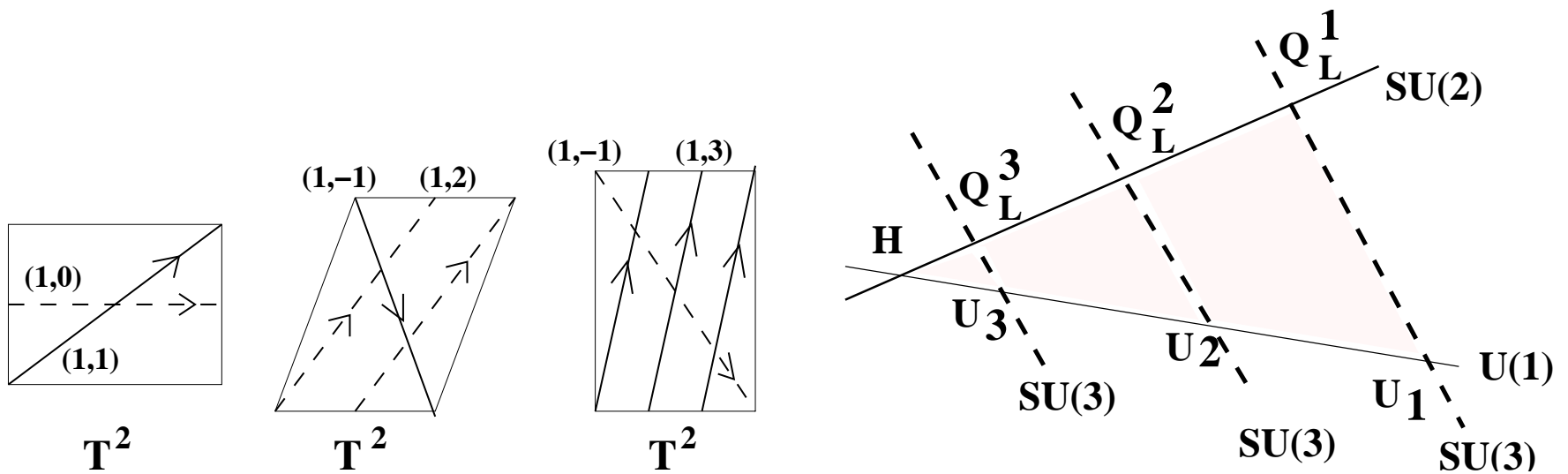
Quasi-realistic string constructions

- Two classes of quasi-realistic: intersecting D-brane, heterotic
- Intersecting D-brane (review: R. Blumenhagen, M. Cvetič, P.L., G. Shiu, hep-th/0502005)
 - Closed strings (gravitons) and open strings ending on D-branes
 - D6-branes: fill ordinary space and 3 of the 6 extra dimensions
 - Stringy implementation of “brane world” ideas

- Gauge interactions from strings beginning/ending on stack of parallel branes (one for each group factor)
- Chiral matter: strings at intersection of branes, e.g., $SU(N) \times SU(M) \rightarrow$ bifundamental (N, \bar{M})



- Family replication from multiple intersections on compactified geometry
- Yukawa interactions $\sim \exp(-A_{ijk}) \rightarrow$ hierarchies
- Existing models: conserved L ; no diagonal (Majorana) triangles
- However, no realistic model with large enough A for small Dirac neutrino masses (more generic geometries?)



The $E_8 \times E_8$ Heterotic String

- Dirac masses

- Can achieve small Dirac masses (neutrino or other) by higher dimensional operators

$$L_\nu \sim \left(\frac{S}{M_{Pl}} \right)^p L N_L^c H_2, \quad \langle S \rangle \ll M_{Pl}$$

$$\Rightarrow m_D \sim \left(\frac{\langle S \rangle}{M_{Pl}} \right)^p \langle H_2 \rangle$$

- Recent variant: N_L^c is a modulus (Bouchard, Cvetič, Donagi, hep-th/0602096)

- Majorana masses

- Can one generate large effective m_S from

$$W_\nu \sim c_{ij} \frac{S^{q+1}}{M_{Pl}^q} N_i N_j \quad \Rightarrow \quad (m_S)_{ij} \sim c_{ij} \frac{\langle S \rangle^{q+1}}{M_{Pl}^q},$$

consistent with D and F flatness?

- Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints?
- Are bottom-up model assumptions for relations to quark, charged lepton masses maintained?

- **The Z_3 Heterotic Orbifold** (J. Giedt, G. Kane, P.L., B. Nelson, hep-th/0502032)
 - Systematically studied large class of vacua
 - * Is minimal seesaw common?
 - * If rare, possibly guidance to model building
 - * Clues to textures, etc.
 - Several models from each of 20 patterns; W through degree 9; huge number of D flat directions reduced greatly by F -flatness
 - Only two patterns had Majorana mass operators $\langle S_1 \cdots S_{n-2} \rangle NN / M_{\text{PL}}^{n-3}$
 - **None** had simultaneous Dirac operators $\langle S'_1 \cdots S'_{d-3} \rangle NLH_u / M_{\text{PL}}^{d-3}$ leading to $\Delta m^2 > 10^{-10} \text{ eV}^2$ (one apparent model ruined by off-diagonal Majorana)
 - Feature of Z_3 orbifold? Or more general?

- **Systematic searches in other constructions important** (Is seesaw generic? Rare? Alternatives?)
- **Consider alternatives seriously**
 - **Small Dirac masses from high degree terms** (very common in constructions) (could also give light sterile ν 's and mixing)
 - **Extended seesaws**, $m_\nu \sim m_D^{2+k}/M^{1+k}$, with $k \geq 1$ and low (e.g., TeV) scale M
 - **Higgs triplet models**: non-trivial to embed in strings (higher level), but very predictive (e.g., inverted hierarchy with nearly bi-maximal mixing) (B. Nelson, PL, hep-ph/0507063)

Triplet models

- Introduce Higgs triplet $T = (T^{++} T^+ T^0)^T$ with weak $Y = 1$
- Majorana masses m_T generated from $L_\nu = \lambda_{ij}^T L_i T L_j$ if $\langle T^0 \rangle \neq 0$
- General SUSY case

$$W_\nu = \lambda_{ij}^T L_i T L_j + \underbrace{\lambda_1 H_1 T H_1 + \lambda_2 H_2 \bar{T} H_2}_{\text{needed to avoid Majoron}} + M_T T \bar{T} + \mu H_1 H_2$$

T, \bar{T} are triplets with $Y = \pm 1$, $M_T \sim 10^{12} - 10^{14}$ GeV. Typically,

$$\langle T^0 \rangle \sim -\lambda_2 \langle H_2^0 \rangle^2 / M_T \Rightarrow m_{ij}^\nu = -\lambda_{ij}^T \lambda_2 \frac{v_2^2}{M_T}$$

- Most previous models: GUT/LR symmetry, ordinary hierarchy

String constructions

- Expect $\lambda_{ij}^T = 0$ for $i = j$ (off-diagonal) $\Rightarrow m_{ii}^\nu = 0$
- Also, need multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal
- Partial explanation: $SU(2)$ triplet with $Y \neq 0$ requires higher level embedding, e.g., of $SU(2) \subset SU(2) \times SU(2)$ (Have Z_3 constructions with some but not all of the features, B. Nelson, PL, hep-ph/0507063)

$$W \sim \lambda_{1j}^T L_1(2, 1) T(2, 2) L_j(1, 2), \quad j = 2, 3$$

yields

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

- Typical string case: $|a| = |b|$

- HDO (or $SU(2) \subset SU(2) \times SU(2) \times SU(2)$) can give $m_{23}^\nu \neq 0$

- For

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

can take a, b, c real w.l.o.g. by redefinition of fields (not true for general m^ν)

- $\text{Tr } m^\nu = 0$ and $m^\nu = m^{\nu\dagger} \Rightarrow m_1 + m_2 + m_3 = 0$

- $|\Delta m_{\text{Atm}}^2| \sim 2 \times 10^{-3} \text{ eV}^2$, $\Delta m_{\odot}^2 \sim 8 \times 10^{-5} \text{ eV}^2 \Rightarrow$ two solutions

- For $\Delta m_{\odot}^2 = 0$

- (a) $m_i \propto 1, -\frac{1}{2}, -\frac{1}{2}$ (ordinary, with shifted masses)

- (b) $m_i \propto 1, -1, 0$ (inverted)

- With $\Delta m_{\odot}^2 \neq 0$

- (a) $m_i = 0.054, -0.026, -0.026 \text{ eV}$ ($\sum |m_i| = 0.107 \text{ eV}$ (cosmology))

- (b) $m_i = 0.046, -0.045, -0.001 \text{ eV}$ ($\sum |m_i| = 0.092 \text{ eV}$ (cosmology))

$$m_a^\nu \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad m_b^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

- (a) leads to unrealistic mixing matrix \Rightarrow consider (b)

A special texture

- The $L_e - L_\mu - L_\tau$ conserving texture

$$m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

has been considered phenomenologically by *many authors* (Zee; Barbieri, Hall, Smith, Strumia, Weiner; King, Singh; Ohlsson; Barbieri, Hambye, Romanino; Lebed, Martin; Babu, Mohapatra; Lavignac, Masina, Savoy; Feruglio, Strumia, Vissani; Altarelli, Feruglio, Masina)

$$m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

- **New aspects**
 - **Strong string motivation**
 - **Motivation for special case $|a| = |b|$**
 - **Can perturb by HOT**
 - **No reason for $U_e = I$ in this basis**
- **Yields inverted hierarchy, with eigenvalues $0, \pm\sqrt{|a|^2 + |b|^2}$**
- **Diagonalization: $\tan \theta_{\text{Atm}} = b/a \Rightarrow$ need $|b| = |a|$ for maximal**

- If $U_e = I$: $\theta_{\odot} = \frac{\pi}{4}$ (maximal) (experiment: $\frac{\pi}{4} - \theta_{\odot} = 0.19_{-0.06}^{+0.05}$, 2σ)
 - Comparable to Cabibbo angle, $\theta_C \sim 0.23$
- Perturbations on m^ν cannot give both Δm_{\odot}^2 and $\frac{\pi}{4} - \theta_{\odot} \sim 0.19$ (cf $\theta_C \sim 0.23$) without fine-tuning between terms, e.g.,

$$\frac{\Delta m_{\odot}^2}{\sqrt{2}|\Delta m_{\text{Atm}}^2|} \sim (m_{23}^\nu + m_{11}^\nu) \sim \frac{1}{43}$$

$$\frac{\pi}{4} - \theta_{\odot} \sim \frac{1}{4}(m_{23}^\nu - m_{11}^\nu) \sim 0.19$$

- However, $U_e \neq I$ with small angles (comparable to CKM) can give agreement with experiment (Frampton, Petcov, Rodejohann; Romanino; Altarelli, Feruglio, Masina)

$$U_e^\dagger \sim \begin{pmatrix} 1 & -s_{12}^e & 0 \\ s_{12}^e & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

yields

$$\frac{\pi}{4} - \theta_\odot \sim \frac{s_{12}^e}{\sqrt{2}} \Rightarrow s_{12}^e \sim 0.27_{-0.08}^{+0.07}$$

$$|U_{e3}|^2 \sim \frac{(s_{12}^e)^2}{2} \sim (0.017 - 0.059), \quad 2\sigma \text{ (exp : } < 0.032)$$

$$m_{\beta\beta} \sim m_2(\cos^2 \theta_\odot - \sin^2 \theta_\odot) \sim 0.018 \text{ eV}$$

Outlook

- Neutrino mass likely due to large or Planck scale effects, but little work in string context
- Specific orbifold string constructions (heterotic, intersecting brane) not consistent with common GUT and bottom up assumptions for m_ν
- No examples of minimal seesaw in large class of heterotic Z_3 orbifold vacua
- Small Dirac, extended seesaw, Higgs triplet (inverted hierarchy in string context) should be seriously considered

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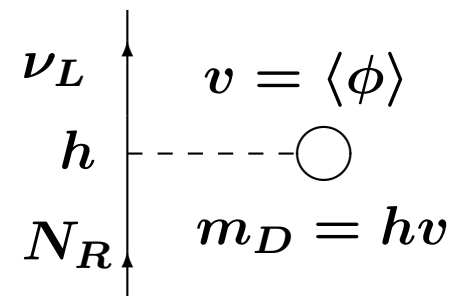
Happy Birthday Gary!

Neutrino Preliminaries

- **Weyl fermion**
 - Minimal (two-component) fermionic degree of freedom
 - $\psi_L \leftrightarrow \psi_R^c$ by CPT
- **Active Neutrino (a.k.a. ordinary, doublet)**
 - in $SU(2)$ doublet with charged lepton \rightarrow normal weak interactions
 - $\nu_L \leftrightarrow \nu_R^c$ by CPT
- **Sterile Neutrino (a.k.a. singlet, right-handed)**
 - $SU(2)$ singlet; no interactions except by mixing, Higgs, or BSM
 - $N_R \leftrightarrow N_L^c$ by CPT
 - **Almost always present: Are they light? Do they mix?**

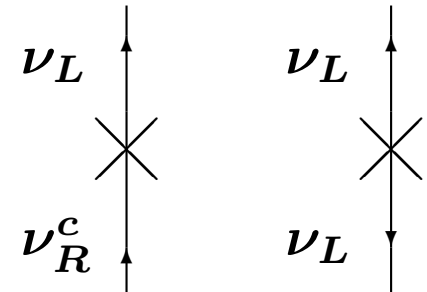
- Dirac Mass

- Connects distinct Weyl spinors (usually active to sterile):
($m_D \bar{\nu}_L N_R + h.c.$)
- 4 components, $\Delta L = 0$
- $\Delta I = \frac{1}{2} \rightarrow$ Higgs doublet
- Why small? HDO? LED?
- Variant: couple active to anti-active, e.g., $m_D \bar{\nu}_{eL} \nu_{\mu R}^c \Rightarrow L_e - L_\mu$ conserved; $\Delta I = 1$



- Majorana Mass

- Connects Weyl spinor with itself:
 - $\frac{1}{2}(m_T \bar{\nu}_L \nu_R^c + h.c.)$ (active);
 - $\frac{1}{2}(m_S \bar{N}_L^c N_R + h.c.)$ (sterile)
- 2 components, $\Delta L = \pm 2$
- Active: $\Delta I = 1 \rightarrow$ triplet or seesaw
- Sterile: $\Delta I = 0 \rightarrow$ singlet or bare mass



- Mixed Masses

- Majorana and Dirac mass terms
- Seesaw for $m_S \gg m_D$
- Ordinary-sterile mixing for m_S and m_D both small and comparable (or $m_S \ll m_d$ (pseudo-Dirac))