

## Assignment for Physics 295 – Professor Thomson – due May 2 2005

**Note:** The electron-Volt (eV) is a useful unit of energy in particle physics. It is the energy ( $U=qV$ ) acquired by moving the electron charge ( $q=1.6\times 10^{-19}$  Coulombs) through an electric potential of 1 Volt. Therefore  $1\text{ eV} = 1.6\times 10^{-19}$  Joules. Likewise  $\text{eV}/c$ , where  $c$  is the speed of light in vacuum, is a unit of momentum and  $\text{eV}/c^2$  is a unit of mass.

The LEP(LHC) ring is not a perfect circle as claimed by Dan Brown, but consists of eight 2800-metre curved sections or arcs linked by eight 571-metre long straight sections. These two different sections fulfill very different requirements for the accelerator. In the arcs, the direction of the particles is changed by 3392 (1232) **dipole magnets** that bend the particles round round the ring. In the straight sections, the particles gain energy as they pass through **accelerating cavities** containing a **radio-frequency electric field**. In the middle of the straight sections, the beams circulating in opposite directions are brought into collision – this point is surrounded by a large detector in order to learn as much as possible about the particles produced in the collision. The beams of particles are kept focused by other magnets, namely quadrupoles, sextupoles and octupoles.

- 1) **Dipole magnets.** The LEP/LHC bending radius is 2803.95m.
  - i. If electrons and positrons are injected into LEP with an initial momentum of 22 GeV/c, what is the required magnetic field strength to keep them in the LEP ring?
  - ii. After acceleration in LEP, the electrons and positrons each reach a maximum momentum of 104 GeV/c. What is the required magnetic field strength now?
  - iii. If protons are injected into LHC with an initial momentum of 450 GeV/c, what is the required magnetic field strength to keep them in the LHC ring?
  - iv. After acceleration in LHC, the protons reach a maximum momentum of 7000 GeV/c. What is the required magnetic field strength now?

LEP magnet design was fairly straight-forward and used conventional electromagnets. The LHC uses super-conducting electro-magnets to cover the large range of magnetic fields required. Superconducting means zero resistance which means no power loss from resistive heating from the current flowing the magnet coils. However, for the magnet material to remain superconducting, the magnets are cooled with liquid helium to a very low temperature of 1.9K.

- 2) **Radio-frequency cavities.** When a charged particle changes direction, it accelerates. Accelerated charges emit radiation. A particle with energy  $E$  (units: GeV) making one complete revolution around a circle with bending radius  $R$  (units: metres) loses an amount of energy per revolution,  $\Delta U$ , equal on average to

$$\Delta U = c_{\gamma} \frac{E^4}{R} \text{ where } c_{\gamma} = \frac{1.18 \times 10^{-14}}{(mc^2)^3}$$

where  $mc^2$  is the rest-mass energy of the particle (units: GeV). Note that the average energy loss per turn increases as the fourth power of the beam energy!

- i. How much energy is lost per turn of the LEP ring for an electron with energy 22 GeV?
- ii. And for an electron with energy of 104 GeV? What fraction of the electron energy is lost per turn?
- iii. If there are a million million ( $10^{12}$ ) electrons making 4550 turns per second, what is the total energy lost per second from the LEP electron beam for (i) and (ii)?

Note the energy lost per second has to be replaced by the radio-frequency electric field acceleration cavities in the straight sections of the LEP ring. During 1989-1995, LEP collided beams of 45 GeV electrons and 45 GeV positrons to study the  $Z^0$  boson (mass 91 GeV/c<sup>2</sup>). During that time, technology advances were made to develop the higher-power radio-frequency accelerating cavities needed to increase the beam energy beyond the 80 GeV required to produce pairs of  $W^+W^-$  bosons (mass 80 GeV/c<sup>2</sup>) and to search for physics beyond the known Standard Model. Note a 6 GeV increase in electron beam energy from the maximum energy reached of 104 to say 110 GeV would require a factor of  $(110/104)^4 = 1.25$  more energy from the radio-frequency acceleration cavities. That's quite an increase in the CERN electricity bill for a small gain in energy to search for new physics!

- 3) **Protons vs electrons part I.** The formula for energy loss is also inversely proportional to the third power of the rest-mass energy of the particle. That makes the more massive proton a more economic alternative for circular colliders.
  - i. How much energy is lost per turn of the LHC ring for a proton with energy 450 GeV?
  - ii. And for a proton with energy of 7000 GeV? What fraction of the proton energy is lost per turn?
  - iii. If there are 0.4 million million protons ( $0.4 \times 10^{12}$ ) in each of 2808 bunches making 11,200 turns per second, what is the total energy lost from the LHC proton beam each second for (iv) and (v)?
  - iv. How do your answers to 2(iii) and 3(iii) compare?
  
- 4) **Protons vs electrons part II.** How much energy is really available to make new particles? Electrons and positrons are fundamental particles, ie they are not made of anything else. When an electron moving in one direction collides with a positron moving with equal speed in the opposite direction, they completely annihilate and twice the beam-energy is available to make new particles. What about a proton-proton collider? Unlike the electron, the proton is a bean-bag of other particles: namely two up quarks and one down quark held together by numerous gluons. Each of these components (called partons by Feynman) carries some fraction of the proton's momentum. This fraction can actually be a range of values, so we generally talk about the probability to find a parton carrying a fraction  $x$  of the proton's momentum (jargon: parton distribution functions). On average, an up-quark carries about 1/6 of the proton's momentum
  - i. In a proton-proton collision at the LHC, an up quark carrying 1/6 of the proton's momentum collides head-on with an up-quark carrying 1/6 of the

other proton's momentum. What is the "beam-energy" of each up quark? How much energy is available to make new particles?

- ii. In a proton-proton collision, two partons with exactly equal and opposite momentum collide (a) always (b) frequently (c) rarely (d) never?
- iii. Compare this to the energy available to make new particles at a linear collider with 1000 GeV electron and positron beams. Do you now see why the reach of a linear electron-positron collider with beam energy of 1000 GeV is comparable to the reach of a circular proton-proton collider with 7x higher beam energies?

5) **Top mass reconstruction (simple)?** You all know  $E=mc^2$ . This is true for a particle at rest. (Aside: But maybe you don't know this was an addendum to Einstein's paper on special relativity. He didn't notice it straight away and thought it was curious! Note also that Einstein's original paper on special relativity has no bibliography – he did not refer back to the work of a single previous physicist, mathematician or scientist. And that's because nobody had ever postulated anything as daring or as simple: the speed of light is always constant in any frame of reference.) The more general formula for a particle in motion is  $E^2 - \mathbf{p}^2 c^2 = m^2 c^4$ , where  $\mathbf{p}$  is the momentum of the particle.

- i. A top quark and an anti-top quark are pair-produced at the LHC. If the top quark mass is  $175 \text{ GeV}/c^2$  and the energy of the top quark is 200 GeV, what is the momentum of the top quark?
- ii. The top (anti-top) quark decays in  $10^{-25}$  s and is not seen directly in the detector. However, one electron and 4 jets of particles are seen in the detector with energy and momentum given by  $(E_e, \mathbf{p}_e)$ ,  $(E_1, \mathbf{p}_1)$ ,  $(E_2, \mathbf{p}_2)$ ,  $(E_3, \mathbf{p}_3)$  and  $(E_4, \mathbf{p}_4)$ , where bold font indicates a vector. In this case, the top quark ( $q=+2/3$ ) decayed into a bottom-quark ( $q=-1/3$ ) and a  $W^+$  boson that subsequently decayed into a quark-anti-quark pair, while the anti-top quark decayed into an anti-bottom quark ( $q=1/3$ ) and a  $W^-$  boson that subsequently decayed into an electron and an electron anti-neutrino. The invariant mass of two of the jets should be equal to the W boson mass.
  - i. Adding the "four-vectors" of two of the jets together, we find a total "four-vector" of  $(E_3+E_4, \mathbf{p}_2 + \mathbf{p}_3)$ . Show that the approximate invariant mass of this object is  $m^2 c^4 = 2E_3 E_4 (1 - \cos\theta_{34})$ . You may assume that invariant mass of a jet is zero so that  $E_3 = |\mathbf{p}_3|$  and similarly for jet 4.
  - ii. The third and fourth jets have an invariant mass close to the W boson mass of  $80 \text{ GeV}/c^2$ . If the third jet energy  $E_2 = 60 \text{ GeV}$  and the angle between the third and fourth jets is  $\theta_{34} = 120$  degrees, what is the energy of the fourth jet?
  - iii. Write down the four-vector of the W boson reconstructed by jets three and four: What is the total energy of these two jets? What is the magnitude of the total momentum of these two jets?
  - iv. From the four jets, how many unique combinations of pairs of jets can you make?
  - v. The first jet, with energy  $E_1 = 104 \text{ GeV}$ , makes an angle of 122 degrees with the direction of the W boson reconstructed from jets three and

- four. What is the invariant mass of the sum of the first jet and the W boson?
- vi. You have one other option: you could reconstruct the invariant mass of the second jet and the W boson reconstructed from jets three and four, where the second jet has an energy of  $E_2=70\text{GeV}$ , and an angle of 30 degrees with the direction of the W boson. What is the invariant mass? Assuming we are trying to reconstruct the top quark mass, do you think this combination is more likely to be correct than (v)?
- vii. Take a look at the CDF Top physics web page for the best current world measurement of the top quark mass:  
<http://www-cdf.fnal.gov/physics/new/top/top.html>
1. It's a bit more sophisticated than our discussion...since jet energies are typically not well-measured by particle detectors, an assumption that two of the jets come from a W boson decay can actually reduce the uncertainty on the jet energies and thus on the top quark mass. The world's best measurement from CDF goes a step further and uses the hadronic W decays in top pair production events di-jet mass distribution to reduce the systematic uncertainty on the jet energy calibration.
  2. More familiar examples of a systematic uncertainty are using an incorrectly calibrated ruler (eg neglecting thermal expansion of ruler on a hot day) - so all your measurements of the length of this sheet of paper are 5% too long. Checking the ruler against a well-known length (US letter size paper) would allow you to notice this mistake and correct for it. The W boson mass here is the well-known quantity.

If you're interested in particle physics research, the Penn high energy group usually has funding for several undergraduates to work over the summer. Contact me for more details at [thomsone@fnal.gov](mailto:thomsone@fnal.gov).