Physics 250

Practice Exam II

1. In a Compton scattering event, the scattered photon has an energy of 120 keV and the recoiling electron has an energy of 40 keV. Find (a) the wavelength of the incident photon, (b) the angle $\theta$ at which the photon is scattered, and (c) the recoil angle $\phi$ of the electron.

(a) $E_{\gamma 1} + m_e c^2 = E_{\gamma 2} + Te + m_e c^2$

$E_{\gamma 1} = 120 \text{ keV} + 40 \text{ keV} = 160 \text{ keV}$

$E_{\gamma 1} = h\nu = \frac{hc}{\lambda} = \frac{12,400 \text{ eV} \, \text{Å}}{\lambda}$

$\lambda = \frac{12,400 \text{ eV} \, \text{Å}}{160 \text{ keV}} = 0.0775 \text{Å} \quad (0.00775 \text{nm})$

(b) $\lambda_2 - \lambda_1 = \frac{h}{m_e c} (1 - \cos \theta) = \frac{hc}{m_e c^2} (1 - \cos \theta) = 0.0243 (1 - \cos \theta)$

$\lambda_2 = \frac{hc}{E_{2\gamma}} = \frac{12,400 \text{ eV} \, \text{Å}}{120 \text{ keV}} = 0.104 \text{Å}$

$(1 - \cos \theta) = \frac{(0.103 - 0.0775) \text{Å}}{0.0243} = 1.049$

$\cos \theta = -0.049 \quad \theta = 92.8^\circ$

(c) $p_{\gamma} \sin \theta = p_e \sin \phi \Rightarrow p_{\gamma} c \sin \theta = p_e c \sin \phi$

$p_e c = \sqrt{E^2 - m_0 c^4} = \sqrt{(40 \text{ keV} + 511 \text{ keV})^2 - (511 \text{ keV})^2}$

$= 206 \text{ keV}$

$\sin \theta = \frac{120 \text{ keV} \cdot 0.999}{206 \text{ keV}} = 0.582$

$\phi = 35.6^\circ$
2. The solar constant (energy/sec/m² received from the sun at earth) is

\[ W = 1400 W/m^2 \]. The earth-sun distance is \( R_E = 1.5\times10^{11} m \). The radius of the earth is \( r_E = 2, 400 km \). The radius of Jupiter is \( r_J = 71,400 km \) and its distance from the sun is \( R_J = 5.2R_E = 7.8\times10^{11} m \). Calculate the average temperature of Jupiter. You may assume that Jupiter is a perfect absorber and that it is in equilibrium with respect to energy received from the sun. What is the most probable wavelength of the EM radiation emitted by Jupiter?

Radiation falls off as \( R^2 \Rightarrow S_{\text{jupiter}} = 1400W/m^2 \cdot \frac{R_E^2}{(5.2R_E)^2} = 51.8W/m^2 \)

Total energy hitting Jupiter

\[ P_{j}^{\text{Rec}} = 51.8 \frac{W}{m^2} \cdot \pi r_j^2 \]

\[ P_{j}^{\text{Rad}} = R4\pi r_j^2 = \sigma T^4 \pi r_j^2 \]

\[ 51.8 \cdot \pi r_j^2 = 4\pi r_j^2 \cdot \sigma T^4 \]

\[ T^4 = \frac{51.8}{4\sigma} \]

\[ T = \left( \frac{51.8 (W/m^2)}{4 \cdot 5.67 \times 10^{-8} W/m^2K^4} \right) \]

\[ T = 123^\circ K \]

(b) most probable wavelength

\[ 4.97\lambda_m kT = h\nu \]

\[ \lambda_m = \frac{h\nu}{4.97kT} = \frac{12,400 eV}{4.97 \times 0.025 \times 123 \times 300 eV} \]

\[ = 2.43 \times 10^5 \text{Å} \]

\[ = 24.3 \mu \]
3. Metal A has a work function of \( \phi_A = 4.7 \text{eV} \). When illuminated with light of wavelength \( \lambda \), the maximum kinetic energy of electrons is \( K_A = 0.8 \text{eV} \). Metal B is illuminated with light of the same wavelength and the maximum kinetic energy of ejected electrons is \( K_B = 3.2 \text{eV} \).

(a) What is the wavelength \( \lambda \) of the incident light?

\[
h\nu = \phi_A + 0.8 \text{eV} = 4.7 + 0.8 = 5.5 \text{eV}
\]
\[
h\nu = \phi_B + 3.2 \text{eV}
\]
\[
h\nu = \frac{hc}{\lambda} = 5.5 \text{eV}
\]
\[
\lambda = \frac{hc}{5.5 \text{eV}} = \frac{12,400 \text{eVÅ}}{5.5 \text{eV}} = 2250 \text{Å}
\]

(b) What is the work function of metal B?

\[
5.5 \text{eV} = \phi_B + 3.2 \text{eV}
\]
\[
\phi_B = 2.3 \text{eV}
\]
4. In semiconductors, electrons with charge \(-e\) and effective mass \(m_e^*\) can carry current. In addition, there are excitations, called holes that behave exactly as if they were positively charged electrons with effective mass \(m_h^*\). In addition semi-conductors have a large dielectric constant \(\kappa\), so the Coulomb force between the electron and the hole is \(-ke^2/(\kappa r^2)\). Electrons and holes can form a “hydrogenic atom” called an exciton. In Germanium, \(m_e^* = 0.082m_e\), \(m_h^* = 0.042m_e\), and \(\kappa = 16.0\). Calculate the ground state energy and wavelength of the first Balmer line of an exciton in Germanium.

\[
\mu = \frac{m_h^*m_e^*}{m_h^* + m_e^*} = \frac{(0.042)(0.082)}{0.042 + 0.082} m_e = 0.0278m_e
\]

\[
E_n = \frac{\mu k^2 Z^2 e^4}{\kappa^2 \cdot 2n^2 \hbar^2} = -\frac{\mu}{m_e \kappa^2} \frac{1}{n^2} E_B \quad \text{since} \quad k \to \frac{k}{\kappa}
\]

\[
E_1 = \frac{0.0278}{16} \times 13.6 \cdot V = -1.48 \times 10^{-3} eV
\]

\[
\frac{hc}{\lambda_B} = E_3 - E_2 = |E| \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{5}{36} E_1 \Rightarrow \lambda_B = \frac{hc}{\frac{5}{36} E_1}
\]

\[
\lambda_B = \frac{1240}{(5/36) \times 1.48 \times 10^{-3}} = 6.03 \times 10^6 nm = 6.03 mm
\]
5. Helium atoms of mass \( M = 3.72 \text{ GeV}/c^2 \) and kinetic energy \( K = 1.2 \times 10^{-3} \text{ eV} \) are scattered off cubic crystalline material. The smallest scattering at which a Bragg peak is observed is at \( \Theta = 140^\circ \). What is the lattice spacing of the crystal?

\[
\Theta = 2\theta \Rightarrow \theta = 70^\circ
\]

\[
\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240}{\sqrt{[2 \times 3.72 \times 10^9 \times 1.2 \times 10^{-3}]^{1/2}}} = 0.415 \text{ nm}
\]

Bragg scattering: \( 2d \sin \theta = n \lambda \rightarrow \lambda \)

\[
d = \frac{\lambda}{2 \sin \theta} = \frac{0.415}{2 \sin 70^\circ} = 0.22 \text{ nm} = d
\]
6. Estimate the temperature below which you would not expect to see the Balmer series in the absorption spectrum of hydrogen. (use as a guideline that only about 1% of the electrons are in the correct energy level for the Balmer series to be observed).

Balmer series in emission: from level \( n > 2 \) to \( n = 2 \)

Balmer series in absorption: from level \( 2 \) to \( n > 2 \)

\( E_2 \rightarrow n = 2 \) must be occupied before Balmer can be seen.

\[
\frac{P_2}{P_1} = e^{-\frac{(E_2-E_1)}{k_BT}} = 0.01
\]

\( E_1 \rightarrow \) Need \( \Delta E = E_2 - E_1 = \frac{3}{4} \times 13.6eV \sim 4.6k_B T \Rightarrow \frac{T}{300} = \frac{2.217}{1/40}, \quad T = 26,604^\circ\)
7. Calculate the wavelength of light emitted when an electron in a one-dimensional square well of length 3 nm undergoes a transition from level 4 to level 2.

\[
E_n = \frac{n^2 \frac{\hbar^2}{8mL^2}}{8mc^2} = n^2 \frac{(hc)^2}{8mc^2L^2} = \frac{n^2(1240)^2}{8 \times 0.511 \times 10^6 \times 3^2} = 0.042 \text{eV}n^n
\]

\[
\frac{hc}{\lambda} = \Delta E = (4^2 - 2^2)E_1 = 12 \times 0.042 \text{eV}
\]

\[
\lambda = \left( \frac{\Delta E}{hc} \right)^{-1} = \left( \frac{12 \times 0.042}{1240} \right)^{-1} = 2460 \text{nm}
\]