Chirality

Lack of mirror symmetry.

Tetrahedral bonding of 4 different "things" to a carbon atom makes it chiral.

Elastic free energy: try adding terms linear in the curvature, and see that they are eliminated by symmetry. Without mirror symmetry we find a term linear in twist \((n \cdot \mathbf{x} \times \mathbf{y})\) is allowed.

Nematic:

\[
f = \frac{1}{2} K_{11} \langle \mathbf{n} \cdot \mathbf{n} \rangle^2 + \frac{1}{2} K_{22} \langle n \cdot (\mathbf{x} \times \mathbf{y} + t_0) \rangle^2 + \frac{1}{2} K_{33} \langle (\mathbf{x} \times \mathbf{y})^2 \rangle^2
\]

\[
\begin{align*}
\mathbf{n}_x &= \cos t \mathbf{z} \\
\mathbf{n}_y &= \sin t \mathbf{z} \\
\mathbf{n}_z &= 0
\end{align*}
\]

\(\mathbf{n} \cdot \mathbf{x} \times \mathbf{y} = -\frac{\partial \psi}{\partial t} = -t\)

\[
\frac{1}{2} K_{22} (t_0 - t)^2 \text{ minimal when } t = t_0.
\]

Smectic C: effects on elasticity + polarity

1. Helix: tilt direction \(\mathbf{C}\) precesses around the layer normal, making a helix, like the cholesteric.

2. In a smectic layer: \((\mathbf{C} \times \mathbf{C})^2\) term (spontaneous bend) is allowed in the free energy:

\[
f_c = \frac{1}{2} K_5 (\nabla \cdot \mathbf{C})^2 + \frac{1}{2} K_8 [ (\nabla \cdot \mathbf{C})^2 - b_0 ]^2
\]
$f_c$ is minimized for $(Qx\ell)_c = b_0$. Can this be done uniformly, as the cholesteric helix does for $z$??

No! - not in the plane of the smectic layer. The helix (precession of $\hat{\mathbf{c}}$ or $\hat{\mathbf{n}}$ from layer to layer) involves twist and bend of $\hat{\mathbf{n}}$, but the bending of $\hat{\mathbf{c}}$ in a single layer is a separate effect!

3 Chiral smectic C is ferroelectric.

Problems involving chirality

0 Unwind helix with an external field.

\[ H = 0 \quad E = 0 \]
\[ 0 < H < H_c \quad 0 < E < E_c \]
\[ H > H_c \rightarrow \hat{\mathbf{c}} \]
\[ E > E_c \rightarrow \hat{\mathbf{c}} = \hat{\mathbf{p}} \]
Blue phases  "Modulated Phases"

Double twist cylinders: For the right sign of the saddle-splay elastic energy, this cylinder has lower free energy than the simple cholesteric helix. But this is only true within a small radius around the tube axis, up to \( \phi = \pi/4 \).

Fill space with tubes packed together.

2D hex pattern:

This looks good since tubes join with a continuous vector field (no defects). But for \( \pi/2 < \phi < \pi \), saddle-splay reverses sign, so Saffdy (tube) \( \chi_{24}^{(---)} = 0 \). Without the negative saddle-splay term, all the distortions in this texture just cost energy, compared to the cholesteric helix.

Cubic patterns: Pack cylinders with \( \phi = \pi/4 \) at \( r_0 \), with \( \mathbf{n} \) continuous at contact. Then build 3D network, with arrays of disclination points and/or lines. These internal "surfaces" break the SS \( \chi_{24}^{(---)} \) rule!!
Sm C* 2D layer: Modulated Phase again!

Try to construct texture with $\nabla \times \hat{c} = 0$.

Try $\phi = x$.

- $r$ = right turn in $\hat{c}$ direction
- $l$ = left turn in $\hat{c}$ direction

$r$ = good, $l$ = bad for lowering energy.

$\int \nabla \times \hat{c} \, dx = 0$

Next: try to "shrink" bad parts, and expand good parts.

Solution: Shrink "bad" parts until they are so small that non-linear terms lower their "bad" energy. Then the "good" parts can win! Final domain width is a balance between negative chiral energy and positive wall energy, and positive (quadratic) curvature energy.

$$ f = -\frac{\text{chiral}}{d} + \frac{\text{wall}}{d^2} + \frac{\text{curvature}}{d^2} $$

if $-$, then domains