SOLUTIONS TO PROBLEMS IN CHAP 21 OF THE TEXT

P2 Use \( c = f\lambda \).
   a) \( f = 1240\text{kHz} = 1.24 \times 10^6\text{Hz} \). So \( \lambda = c/f = 3 \times 10^8/1.24 \times 10^6 = 242 \text{ meters} \).
   b) \( f = 90.9\text{MHz} = 9.09 \times 10^7\text{Hz} \). So \( \lambda = c/f = 3 \times 10^8/9.09 \times 10^7 = 3.30 \text{ meters} \).

P5 a) \( c = f\lambda \). It takes 8 sec to go down and back up to complete one cycle. So \( f = 1/T = 1/8\text{Hz} \) and \( \lambda = 8\text{m} \) is given. So \( c = (1/8)8 = 1 \text{ m/s} \) is the wave velocity.
   b) To go from top to bottom means to go from hieght \( +A \) to height \( -A \). So set \( 2A = 3 \text{ meters} \) and \( A = 1.5 \text{ meters} \).

P6 \( c = \sqrt{T/\mu} \), where \( \mu \) is the mass per unit length of the wire. Here \( \lambda = m/L = 0.06/4 = 0.015 \text{ kg/m} \) and \( T = 1000 \text{ Nt} \). So \( c = \sqrt{1000/0.015} = [6.67 \times 10^4]^{1/2} = 2.58 \times 10^2 \text{ m/s} \).

P7 Use \( c = f\lambda \): \( c = (60.0)(0.800) = 48 \text{ m/s} \). We want the tension to be such that \( 48 = \sqrt{T/\mu} \), where \( \mu = m/L = 0.160/5 = 0.032 \text{ kg/m} \). Thus \( 48^2 = T/\mu = T/0.032 \). This gives \( T = (2304)(0.032) = 73.7 \text{ Nt} \).

P8 \( C = \sqrt{T/\mu} \). Here \( T = mg = (5)(9.8) = 49 \text{ Nt} \) and \( \mu = 0.012 \text{ kg/m} \), so \( c = \sqrt{49/0.012} = \sqrt{4.08 \times 10^4} \), so that \( c \approx 64 \text{ m/s} \).
   b) \( \lambda = c/f = 64/240 = 0.267 \text{ meters} \).

P16a The amplitude is the prefactor to the trig function. So \( A = 2.50 \text{ cm} \).
   b) The wavelength appears inside the trig function as \( 2\pi x/\lambda \). Here we identify this as \( 2\pi x/30\text{cm} \), so that \( \lambda = 30 \text{ cm} \).
   c) The period appears inside the trig function as \( 2\pi t/T \). Here we identify this as \( 2\pi t/0.01 \text{ sec} \), so that \( T = 0.01 \text{ sec} \). Then \( f = 1/T = 100 \text{ Hz} \).
   d) \( c = f\lambda = 100(0.30) = 30 \text{ m/s} \).

P17a \( f = c/\lambda = 15/0.6 = 25\text{Hz} \). Also \( T = 1/f = 0.04 \text{ sec} \). I will not worry about the wave number - we didn’t talk about this.
   b) The string lies along the \( x \)-axis and its transverse displacement is along the \( y \)-direction. So we want an equation for \( y \) in terms of \( x \) and \( t \). Since \( y \) is zero when \( x \) and \( t \) are zero, we choose a sine function:

\[
y(x, t) = \pm A \sin \left( \frac{2\pi x}{\lambda} - \frac{2\pi t}{T} \right) .
\]

Now we have to choose the sign plus or minus. It says that the string at \( x = 0 \) is moving downward. At \( x = 0 \) the above equation says that
\[ y(0, t) = \pm \sin \left( -\frac{2\pi t}{T} \right). \] (2)

We see that as \( t \) is nonzero but very small, the argument of the sine function is negative and small. To get \( y \) negative (moving downward at \( t = 0 \)) take the plus sign:

\[ y(0, t) = \sin \left( -\frac{2\pi t}{T} \right) \] (3)

and thus

\[ y(x, t) = A \sin \left( \frac{2\pi x}{\lambda} - \frac{2\pi t}{T} \right). \] (4)

Now we put in the constants:

\[ y(x, t) = (0.08\text{m}) \sin \left( \frac{2\pi x}{0.6\text{m}} - \frac{2\pi t}{0.04\text{sec}} \right). \] (5)

c) To find the displacement at \( x = 0.4 \) m and at time \( t = 0.2 \) sec, substitute these into Eq. (5):

\[
y(0.4\text{m}, t = 0.2\text{sec}) = (0.08\text{m}) \sin \left( \frac{2\pi \cdot 0.4}{0.6} - \frac{2\pi \cdot 0.2}{0.04} \right)
= (0.08\text{m}) \sin[(4\pi/3) - (10\pi)] = (0.08\text{m}) \sin[-(4\pi/3)]
= -(0.08\text{m}) \sin[\pi/3] = -0.08(\sqrt{3}/2) = -0.0693\text{m}.
\] (6)

P24a Given

\[ y(x, t) = (1.75\text{cm}) \sin \left( 250\pi t\text{sec}^{-1} + 0.04\pi x\text{cm}^{-1} \right). \] (7)

The amplitude is [\( A = 1.75 \text{ cm} \)]. To get the wave length set

\[ 0.4\pi x\text{cm}^{-1} = 2\pi x/\lambda. \] (8)

Thus \( 0.4 = 2/\lambda \) (in cm), so [\( \lambda = 5 \text{ cm} \)].

To get the period set

\[ 250\pi t\text{sec}^{-1} = 2\pi t/T. \] (9)

Thus \( 250 = 2/T \) (in sec), or [\( T = 0.008 \text{ sec} \)].

Then \( f = 1/T = 1/0.008 \), so that [\( f = 125 \text{ Hz} \)] and the speed of propagation is \( c = f\lambda = 125(5) = 625 \text{ cm/sec or } c = 6.25 \text{ m/sec} \).

b) At the times asked for, we have

\[
y(x, t = 0) = (1.75\text{cm}) \sin(0.4\pi x)
\]

\[
y(x, t = 0.02\text{sec}) = (1.75\text{cm}) \sin(0.5\pi + 0.4\pi x)
\]

\[
y(x, t = 0.04\text{sec}) = (1.75\text{cm}) \sin(\pi + 0.4\pi x). \] (10)
Thus

\[ y(x, t = 0) = (1.75\text{cm}) \sin(0.4\pi x) \]
\[ y(x, t = 0.02\text{sec}) = (1.75\text{cm}) \cos(0.4\pi x) \]
\[ y(x, t = 0.04\text{sec}) = -(1.75\text{cm}) \sin(0.4\pi x) . \]

(11)

I plot these below:

Curve A is for \( t = 0 \), B is for \( t = 0.02 \text{ sec} \), and C is for \( t = 0.04 \text{ sec} \). You can see from these curves that the wave is moving to the left. (Well, to be more certain of that one ought to do if for times separated not by 0.02 sec, but by 0.01 sec. Why?)

c) The wave is travelling in the negative \( x \)-direction. The argument of the trig function is of the form \( ax + bt \) (with \( a \) and \( b \) positive). If we want to follow a crest, we would set \( ax + bt = \pi /2 \), which would give \( x = (\frac{1}{2}\pi - bt)/2 \), so that the crest is moving in the negative \( x \)-direction.

d) \( c = \sqrt{T/\mu} \). So \( T = c^2 \mu = (6.25)^2(0.5) = 19.5 \text{ Nt} \).