SOLUTIONS TO CHAP 9-10 TEXT PROBLEMS

9-17a We want to evaluate \( \sum_i m_i r_i^2 \), where \( r_i \) is the distance (i.e. the perpendicular distance or shortest distance) between the \( i \)th mass and the axis (line) of rotation. For each mass this distance is the hypotenuse of an isosceles right triangle whose side is 0.2 m. So \( r_i = 0.2\sqrt{2} \) m, and \( m_i = 0.3 \) kg. Thus

\[
I = \sum_i m_i r_i^2 = \sum_i (0.3)(0.2\sqrt{2})^2 = 4(0.3)(0.08) = 0.096 \text{kg m}^2.
\]  

(1)

b) When the axis is along AB, then each \( r_i \) is 0.2 m and

\[
I = \sum_i m_i r_i^2 = \sum_i (0.3)(0.2)^2 = 4(0.3)(0.04) = 0.048 \text{kg m}^2.
\]  

(2)

9-22 From the general formula \( I = \sum_i m_i r_i^2 \), one deduces that if an object consists of several subobjects, \( a, b \), etc., then \( I = I_a + I_b + I_c \ldots \). Here the subobjects are the 4 spokes and the rim, so \( I = 4I_{\text{spoke}} + I_{\text{rim}} \). (NOTE: what they call a ”spoke” is a diameter.) So for \( I_{\text{spoke}} \) we take the moment of inertia of a rod of length \( 2R \) (\( R \) is the radius) ABOUT ITS CENTER. Thus \( I_{\text{spoke}} = \frac{1}{12} m(2R)^2 = \frac{1}{3} mR^2 = \frac{4}{3}(0.25)(0.3)^2 = 0.0075 \text{ kg m}^2 \). Also \( I_{\text{rim}} = mR^2 = (1.20)((0.3)^2 = 0.108 \text{ kg m}^2 \), so \( I = 4(0.0075) + 0.108 = 0.138 \text{ kg m}^2 \).

9-58 At right is shown the situation just before the weight is released. The first question is: where is the angular acceleration zero?? What happens is that after release the weight goes faster and faster until it reaches its lowest point. Then is slows down and for an instant it stops at the same height as it began from. The angular velocity is proportional to the linear speed. So we can say that after release the angular velocity increases until it becomes a maximum (when the weight is at its lowest point) and subsequently decreases to zero when the weight reaches its maximum height (to the left of the support). The weight oscillates back and forth, but the graph is stopped before one can see this. The angular acceleration is the rate at which the angular velocity is changing. In other words the acceleration is the slope of the angular velocity versus time graph at right. The angular acceleration is zero when the angular velocity is a maximum. So we want to get the angular velocity of the weight when it reaches its lowest point. First get the linear velocity by the conservation of energy:

\[
\text{KE}_f + \text{PE}_f = \text{KE}_i + \text{PE}_i
\]
so $\text{KE}_{\text{bot}} = \text{PE}_{\text{top}}$, or $\frac{1}{2}mv_f^2 = mg\Delta h$, or $v_f = \sqrt{2g\Delta h}$, where $\Delta h = L - d = L - L \sin 45^\circ = L - 0.707L = 0.293L = (0.293)(1.6) = 0.47$ m. Thus $v_f = \sqrt{2(10)(0.47)} = 3.1$ m/s and $\omega_f = v_f/R = 3.1/1.6 = 1.9$ rad/sec.

9-60a Use the conservation of energy

$$\text{KE}_f + \text{PE}_f = \text{KE}_i + \text{PE}_i$$

where $\text{KE} = \frac{1}{2}I\omega^2$, where $I$ is the moment of inertia ABOUT ONE END $I = \frac{1}{3}ML^2$. Also $\text{PE}_i = \text{PE}_f = mg\Delta h$, where $\Delta h$ is the height through which the CENTER of the meter stick has dopped, i.e. $\frac{1}{2}L = 0.5$ m. So $\frac{1}{2}(\frac{1}{3}ML^2)\omega_f^2 = Mg(\frac{1}{2}L)$. So $\omega_f^2 = 3g/L$. Thus $\omega_f = \sqrt{3(10)/1} \approx 5.5$ rad/sec.

b) $v = \omega L = 5.5(1) = 5.5$ m/s.

c) The speed of a particle that falls 1 meter is $v = \sqrt{2gh} = \sqrt{2(10)(1)} = 4.5$ m/s.

10-14a Let’s enumerate the forces which act ON the boom. There is its weight $w$ which acts at the center of the boom. At the lower end there are horizontal ($H$) and vertical ($V$) forces at the support. At the top end there is the left tension $T_L$ to the left and the right tension $T_R$ downward. At first we do not know whether or not the tension in the rope is the same on both sides of the boom? We start by considering the torque equation for equili-rium, To eliminate the effect of the forces $H$ and $V$ we take torques about the bottom end of the boom. We will use the formula for the torque that

$$\Gamma = Fr_{\perp} , \quad (3)$$

where $r_{\perp}$ is the distance from the fulcrum to the line of the force. The torque from $T_L$ is therefore

$$\Gamma(T_L) = T_Lr ,$$

where, from the diagram, we see that $r = L \sin \theta$, where $\theta = 60^\circ$. Since the tension in the guy wire tends to make the boom rotate CCW (counterclockwise), this torque is CCW. The torque due to $T_r = W = 5000$ Nt is

$$\Gamma(T_R) = Ws ,$$

where, from the diagram, we see that $s = L \cos \theta$, CW. To get the torque due to the weight $w$ of the boom, note that the line of this force is half as far from the fulcrum as the line of force of $W$. So
\[ \Gamma(w) = \frac{1}{2}ws . \]

The total CW torque is thus
\[ WL \cos \theta + \frac{1}{2}wL \cos \theta - T_L L \sin \theta . \]

For this to be zero we must have
\[ T_L = (W + \frac{1}{2}w) \cos \theta \ \sin \theta = [5000 + 1900](1/2)/0.866 \approx 3980 \text{NT}. \]

To find \( H \) and \( V \) apply \( \sum F = 0 \). \( \sum F_x = H - T_L \), so \( T_L = H = 3980 \text{Nt}. \) Also \( V = W + w = 5000 + 3800 = 8800 \text{NT.} \)

b) To find the direction of the force exerted on the boom at its lower end use \( \tan \phi = \frac{F_y}{F_x} = \frac{V}{H} = 8800/3980 = 2.2 \), so \( \phi \approx 65.5^\circ \) above the horizontal.

**10-25a** We will assume that the stick is in equilibrium. We will calculate \( H, V \) and \( T \). The stick will actually stay in place provided
\[ V \leq H . \]

The sum of the vector forces is zero if
\[ V + T \sin \theta - W = 0 , \quad H - T \cos \theta = 0 . \]

Take torques about the left end of the stick. Use
\[ \Gamma = F \cos \theta , \]

So \( \Gamma(W) = W(L/2) \) and \( \Gamma(T) = T_L L = T \sin \theta L \). Thus the total torque is zero if
\[ W(L/2) - T \sin \theta(L) = 0 . \]

The torque equation gives
\[ T = W/(2 \sin \theta) . \]

The the force equations give
\[ V = W/2 , \quad H = W \cos \theta/(2 \sin \theta) . \]

This will work if if Eq. (4) is satisfied. So the meter stick will just begin to slip when
\[ (W/2) = \mu W \cos \theta/[2 \sin \theta] , \]
or \( \tan \theta = \mu = 0.4 \), i.e. \( \theta = 22^\circ \).

**10-26** In the diagram at right we enumerate the forces acting on the massless bar. (I drew the directions of the forces by noting that their vector sum should be zero.) Since the floor is frictionless, the floor can only exert a normal force \( N \) upward. The joint at the wall exerts horizontal (\( H \)) and vertical (\( V \)) forces. The condition that there be zero net force acting on the bar is

\[
\begin{align*}
F - H &= 0, \\
V - N &= 0.
\end{align*}
\]

There are three unknowns and only two equations. For the torque equation we use \( \Gamma = Fr_\perp \), where \( r_\perp \) is the distance from the fulcrum to the line of the force. I have taken the fulcrum at the bottom end of the bar (but it would be perfectly OK to take torques about the top of the bar). Anyway, from the diagram you can see that for \( V \) the perpendicular distance is \( X \) and for \( H \) it is \( Y \). So with CW positive

\[ VX - HY = 0. \]

From the force equation (4a) we get \( H = F = 90Nt \) and from the torque equation we get

\[ V = HY/X = FY/X = 90(4/3) = 120Nt. \]