PHYSICS 101 QUIZ, FEB 2, 2000    NAME: SOLUTION

\[ x = x_0 + v_0 t + \frac{1}{2}at^2, \quad v = v_0 + at. \]

Since we want the trajectory to pass through a point with given coordinates \( x \) and \( y \), we eliminate the time from

\[
\begin{align*}
  x &= v_x \theta t = v_0 (\cos \phi) t, \\
  y &= v_y \theta t - \frac{1}{2}gt^2 = v_0 (\sin \phi) t - \frac{1}{2}gt^2. 
\end{align*}
\]

Solve the \( x \) equation for \( t \):

\[ t = x / [v_0 \cos \phi] , \]

Insert into the \( y \) equation:

\[ y = \left( \frac{x}{v_0 \cos \phi} \right) v_0 \sin \phi - \frac{1}{2}g \left( \frac{x}{v_0 \cos \phi} \right)^2. \]

Simplify this

\[ y = \left( \frac{x \sin \phi}{\cos \phi} \right) - \frac{1}{2}g \left( \frac{x}{\cos \phi} \right)^2 \left( \frac{1}{v_0^2} \right) . \]

Substitute in \( x = 54 \), \( y = 27 \), \( \cos \phi = 3/5 \), and \( \sin \phi = 4/5 \):

\[
\begin{align*}
  27 &= \left( \frac{54(4/5)}{(3/5)} \right) - \frac{1}{2}(10) \left( \frac{54}{(3/5)} \right)^2 \left( \frac{1}{v_0^2} \right) \\
  &= 72 - 5(90)^2(1/v_0^2),
\end{align*}
\]

or

\[ 45 = 5(90)^2/v_0^2 \]

or

\[ v_0^2 = 5(90)^2/45 = 900. \]

Thus \( v_0 = 30 \) m/s.

As a check, let us find the time to hit the target, from the \( x \) equation:

\[ 54 = v_0 (\cos \phi) t = 30(3/5) t = 18 t , \]
so \( t = 3 \text{ sec} \). Put this into Eqs. (1)

\[
x = v_{x0}t = v_0(\cos \phi)t = 30(3/5)(3) = 54,
\]

\[
y = v_{y0}t - \frac{1}{2}gt^2 = v_0(\sin \phi)t - \frac{1}{2}gt^2 = (30)(0.8)(3) - \frac{1}{2}(10)(3)^2 = 72 - 45 = 27 \text{ m} ,
\]

(9)

So this checks.