Figure 1. Newton’s diagram of projectile motion from his popular *On the System of the World*. From [1, vol II, p 551], reproduced by permission of the University of California Press. This is an engraved representation of the original sketch; see figure 4 for the original.

(written approximately between late 1684 to mid 1685) that originally was intended to be part of the *Principia*. One goal in this paper is to analyse the diagram (figure 1) using a simple descriptive mathematical model. By doing this we will come to appreciate the ambiguity present in Newton’s writings on this type of projectile motion, an ambiguity that led several well known scholars astray.

2. The mathematical description

As far as we know, no modern mathematical analysis of the mountain diagram exists. Hence we present a simple description of the projectile motion that is relevant to Newton’s diagram. We set up our model such that the projectiles are fired from a raised position corresponding to Newton’s mountain (point V in his diagram). The inverse square law of gravity demands that the projectiles follow an elliptical path with the Earth’s centre at one of the foci [3]. In terms of standard two-dimensional polar coordinates we have the following equation describing the projectile’s motion:

\[ r(\theta) = \frac{(R_e + h)(1 - e \cos(\theta_0))}{1 - e \sin(\theta + \theta_0)}. \]  

In this equation \( r(\theta) \) is the radial displacement of the projectile, \( \theta \) is the polar angle measured from the horizontal axis, \( R_e \) is the radius of the Earth, \( h \) is the height of the mountain, \( e \) is the eccentricity, and \( \theta_0 \) represents the angle by which the entire ellipse, being constrained to always pass through the top of the mountain, is rotated if the projectile is not sent off horizontally. Using conservation of angular momentum, the eccentricity can be written in terms of the ratio of the magnitudes of the initial projectile velocity and the velocity necessary to put the projectile into a circular orbit [4]. To model Newton’s diagram it is only necessary to consider velocities less than or equal to the velocity required to achieve a fully circular orbit. Newton