Physics 198: Problem on Dark Matter

Due 5:00 pm Wednesday February 14 in DRL Room 2E5.

This is about the mass of the Milky Way. As we discussed in class, the mass of the Milky Way is dominated by its dark halo. The dark halo extends much further from the center of the Milky Way than the visible disk of stars, so estimating its mass is challenging.

Recall the discussion in class about the rotation curve of the disk of the Milky Way. Analysis of the rotation curve led to the conclusion that the mass \( M(< R) \) interior to a radius \( R \), is given by the expression:

\[
M(< R) = R v_{rot}^2 / G
\]

where \( v_{rot} \) is the rotation speed in the disk and \( G \) is Newton’s constant of gravitation. This analysis assumed that the dark halo is spherically symmetric.

1. The density of matter in the dark halo.

By considering the difference in the mass \( M(< R) \) for a very small increase in \( R \) to \( R + \delta R \), derive an expression for the density of mass \( \rho \). Evaluate \( \rho \) in two different systems of units: SI (kilograms per cubic meter) and astronomical, solar masses per cubic parsec.

2. The outer limit of the dark halo.

The expression for \( M(< R) \) clearly applies only for some range of radius \( R < R_{max} \). The dark halo must have an outer edge, and let’s pretend that this edge is sharp, in the sense that the density falls abruptly to zero for \( R > R_{max} \), and is given by the above expression for all \( R < R_{max} \).

There is a small galaxy that appears to orbit the Milky Way at a great distance. It is called Leo I, and all we know about it is its distance from the center of the Milky Way (7 \( \times \) 10²⁰ meters), and one vector component of its velocity, the radial velocity, which is 1.8 \( \times \) 10⁵ meters per second. Assume that Leo I is bound to the Milky Way, and use this to estimate that total mass and \( M(< R_{max}) \) and the outer radius \( R_{max} \). Express your answers in SI units and astronomical units.

Solar mass: \( M_\odot = 2 \times 10^{30} \) kilograms
One parsec = 3.1 \( \times \) 10¹⁸ meters
Newton’s constant: \( G = 6.7 \times 10^{-11} \) SI units
Rotation speed: \( v_{rot} = 2 \times 10^3 \) meters per second.