Physical Models of Living Systems
Probability, Simulation, Dynamics
Second edition

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Web resources

The book’s Web site (www.physics.upenn.edu/biophys/PMLS2e/) contains links to the following resources in the Student section:

- **Datasets** contains datasets that are used in the problems. In the text, these are cited like this: Dataset 1, with numbers keyed to the list on the Web site.
- **Media** gives links to external media (graphics, audio, and video). In the text, these are cited like this: Media 1, with numbers keyed to the list on the Web site.
- **Errata** will appear as needed.

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