Rheology of soft colloids across the onset of rigidity: scaling behavior, thermal, and non-thermal responses†

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We study the rheological behavior of colloidal suspensions composed of soft sub-micron-size hydrogel particles across the liquid–solid transition. The measured stress and strain-rate data, when normalized by thermal stress and time scales, suggest our systems reside in a regime wherein thermal effects are important. In a different vein, critical point scaling predictions for the jamming transition, typical in athermal systems, are tested. Near dynamic arrest, the suspensions exhibit scaling exponents similar to those reported in Nordstrom et al., Phys. Rev. Lett., 2010, 105, 175701. The observation suggests that our system exhibits a glass transition near the onset of rigidity, but it also exhibits a jamming-like scaling further from the transition point. These observations are thought-provoking in light of recent theoretical and simulation findings, which show that suspension rheology across the full range of microgel particle experiments can exhibit both thermal and athermal mechanisms.

1 Introduction

The onset of dynamic arrest associated with the liquid–solid transition is found under a variety of conditions and across a wide range of disordered materials including polymeric glasses, colloidal suspensions, granular media, emulsions and foams. In a suspension of soft colloidal particles under shear, for example, the onset of solidity is readily observed when a critical volume fraction, φc, is approached from below. The precise character of this rigidity-onset transition is a topic of current interest whose physics can depend on parameters such as the strength and character of interparticle interactions and the nature of the spatio-temporal fluctuations in the sample. For colloidal packings with small and soft particles, the effects of random thermal motion are significant. For granular media, on the other hand, the thermal effects are negligible, but non-thermal fluctuations are often present, and the emergence of solidity depends on particle contacts and particle interactions, among other factors. The concepts of jamming theory have been applied to understand this problem, and collectively this work suggests that the behavior of stress and strain-rate near the jamming point should exhibit critical scaling; some of these predictions have been observed in simulation, as well as experimentally in microfluidic rheology measurements of soft colloids. In a different vein, recent simulation work has suggested that two kinds of mechanistic processes can arise in the colloidal rheology experiments; these two types of transitions differ for thermal versus non-thermal systems, and they are most easily distinguished when stress and strain-rate are normalized by specific thermal parameters. Ultimately, a better understanding of the details of these rigidity-onset phenomena will entail careful experiment and attention to details of the colloidal system, including particle size, stiffness, interaction, and more.

In this paper we take new experimental and analytical steps in this direction. Specifically, we employ both steady-state and frequency-dependent macrorheology to study the shear response of monodisperse and bidisperse colloidal suspensions composed of soft, thermoresponsive poly(N-isopropylacrylamide) (PNIPAM) microgel particles. The thermoresponsive microgel particles permit continuous tuning of colloidal volume fraction across φc. Importantly, the size and stiffness of our particular PNIPAM particles are intermediate to those in previous macrorheology work on PNIPAM samples with small particles (≈ 60–200 nm), wherein thermal effects might be expected to dominate, and to those in...
2 Materials and methods

2.1 Hydrogel particles

Poly(N-isopropylacrylamide), PNIPAM, particles of two different diameters ($D_s \approx 500$ nm and $D_l \approx 700$ nm at 296 K, polydispersity < 0.1) were prepared by radical precipitation polymerization of N-isopropylacrylamide, N,N'-methylenebisacrylamide crosslinker and ammonium persulfate initiator.\textsuperscript{32,33} Aqueous suspensions of PNIPAM microspheres with packing fraction $\phi \approx 0.6$ and estimated number density $N \approx 2.6 \times 10^{18} \text{m}^{-3}$ were prepared by centrifugation and subsequent dilution. The samples investigated in this paper include a monodisperse suspension of the larger PNIPAM particles and a bidisperse suspension of both species with an approximately equal number ratio. As shown in previous experiments, such PNIPAM particles most likely interact via a Hertzian potential.\textsuperscript{1,14,15} The elastic moduli of the particles were estimated following a centrifugation procedure introduced previously\textsuperscript{26} (see more details in ESI\textsuperscript{+}). The Young’s moduli, $E$, of our PNIPAM particles were estimated to vary between 8 and 25 kPa for temperatures between 291 and 295 K. We note that the particles used herein are about 50% softer than those used at corresponding temperatures in the microfluidic experiments of Nordstrom et al.\textsuperscript{1} Note that the crosslinking density in PNIPAM particles is not homogeneous, with a high number of cross-links in the center of the particles and much fewer cross-links towards the outer regions.\textsuperscript{26} At the compressions investigated in our study, the interaction of touching particles is dominated by the elastic properties of the soft “shells”.$^{37}$ Note further that in recent publications by Scheffold et al.\textsuperscript{37} and Romeo and Ciamarra,\textsuperscript{38} theoretical models for the concentration-dependence of shear elasticity were developed starting from single particle properties and applied to hydrogel experiments. Future temperature-dependent rheology experiments with PNIPAM particles investigating a much larger range of $\phi-\phi_c$ than this study may be utilized to further test these predictions.

2.2 Dynamic light scattering (DLS)

Particle diameter, $D_s$, as a function of temperature, $T$, was measured by dynamic light scattering (Brookhaven Instruments, $\lambda_0 = 632.8$ nm, 15 mV, $\theta = 60^\circ$). Fig. 1a shows the measured particle diameters in the temperature range between 291 K and 309 K. $D(T)$ is essentially linear in this regime.$^4$ The slopes of these lines were obtained from linear fits to the DLS data for smaller and larger PNIPAM particles and were $dD_1/dT = -10.3 \pm 0.8 \text{nm K}^{-1}$ and $dD_2/dT = -22.0 \pm 1.4 \text{nm K}^{-1}$, respectively. This information enables calculation of the packing fraction of the investigated dense suspensions as a function of $T$ using the following relations:

$$\phi_{\text{mono}}(T) = \phi_c \left( \frac{D_c + \frac{dD}{dT}(T - T_c)}{D_c} \right)^3$$

with critical temperature, $T_c$, corresponding particle diameter at the critical temperature, $D_c$, and corresponding critical volume fraction, $\phi_c$. For the bidisperse case:

$$\phi_B(T) = \phi_{c,s} \left( \frac{D_{s} + \frac{dD_1}{dT}(T - T_c)}{D_{s}} \right)^3 + \phi_{c,l} \left( \frac{D_{l} + \frac{dD_2}{dT}(T - T_c)}{D_{l}} \right)^3$$

with $\phi_{c,s} = \phi_c n_s D_{s}^3/(n_s D_{s}^3 + (1 - n_s) D_{c}^3)$ and $\phi_{c,l} = \phi_c - \phi_{c,s}$ being the portions of the critical packing fraction occupied by the small or large particles, respectively. Here $n_s$ is the number fraction of small particles in the sample. In our case, $n_s = 0.5$. In our rheology experiments, temperature is the only control parameter, and the critical temperature, $T_c$, is readily identified as the temperature at which the yield stress becomes finite;
again, \( D_c = D(T_c) \) is the particle diameter measured at the critical temperature (Fig. 1a).

Therefore, the only unknown in eqn (1) and (2) is the critical packing fraction, \( \phi_c \), which we define empirically as the packing fraction when solidification occurs. Note that for different physical scenarios, different \( \phi_c \) are expected. For example, hard thermal particles undergo a colloidal glass transition at \( \phi_c \approx 0.58 \), whereas the athermal jamming transition occurs at random close packing, i.e., \( \phi_j \approx 0.64 \). Soft particles at finite temperature, such as those investigated in this study, are expected to undergo a liquid-to-solid transition at packing fractions between these limiting values.

Importantly, many of the major conclusions in this paper are based on scaling laws that depend on the difference, \( \phi - \phi_j \) or \( \phi - \phi_c \). Note, \( \phi_c \) and \( \phi_j \) need not be the same. For soft particles, a colloidal glass transition can be responsible for the onset of rigidity, and for our particular particles, \( \phi_c \) is expected to 0.61 ± 0.02. Therefore, when plotting \( \phi(T) - \phi_c \) in Fig. 1b, we adopt the reasonable assumption that \( \phi_c \approx 0.61 + \phi_j - 0.03 \), slightly below the value of random close packing fraction in three-dimension. It is also evident from eqn (1) that \( \phi - \phi_c \approx \phi_c \), therefore, small uncertainties in the absolute value of \( \phi_c \) should lead only to small errors in the calculated \( \phi(T) - \phi_c \). In order to demonstrate the weak dependency of \( \phi(T) - \phi_c \) from the actual value of \( \phi_c \), we compute the deviations that occur if we assume \( \phi_c \) to be 0.58 or 0.64, respectively; these deviations correspond to the width of error bars in Fig. 1b. Thus, the uncertainty in \( \phi(T) - \phi_c \) is small, even when we do not know the exact value of \( \phi_c \).

2.3 Rheology

The mechanical responses of these suspensions were measured in an AR-G2 rheometer capable of independent stress and strain measurements (TA Instruments), with 4°/40 mm cone-and-plate geometry. Sample temperature was controlled and measured by a Peltier unit and a thermocouple built into the rheometer. A solvent trap was used to prevent sample evaporation during the experiment. The experiments were performed under steady as well as oscillatory shear conditions in order to study both steady-state and frequency-dependent responses. For the range of stresses measured in these experiments, the material density of the PNIPAM microgel particles remains constant at any given temperature, even though their polymer network structure may become deformed. Under steady shear, shear stress (\( \sigma \)) versus strain-rate (\( \dot{\gamma} \)) data were obtained as a function of \( \phi - \phi_c \). Similarly, shear elastic (\( G' \)) and viscous (\( G'' \)) moduli were recorded as a function of oscillatory frequency (\( \omega \)) across the liquid-solid transition. All oscillatory measurements were carried out in the linear elastic strain regime, wherein the maximum strain amplitude is 2% or less. Special care was taken to restrict all data-sets to low Reynolds numbers (\( \text{Re} = 0.5 \)). By restricting \( \dot{\gamma} \) and \( \omega \) to low values, we ensured that laminar flow conditions were maintained. We note that \( \text{Re} < 0.5 \) is approached at maximum shear rate (\( \dot{\gamma} = 10 \text{ s}^{-1} \)) for liquid-like samples. However, Re is still far less than 0.5 for solid-like samples due to their higher effective viscosity.

We explored the possibility that wall-slip could have a significant effect for the samples and flow regimes studied. These test measurements involved a set of control experiments performed using the same instrument with identical plate geometry. In particular, the effect of rheometer surface roughness was checked by performing control experiments with and without roughening the rheometer cone and plate for PNIPAM suspensions with \( \phi - \phi_c \) ranging between −0.19 and 0.23. The control experiments suggest that, for the flow regimes we use, surface roughness has small effect on the resulting steady-state rheology data.

Another potential systematic error that we attempted to avert concerns the effects of shear history. Because PNIPAM particles deswell isotropically with increasing temperature, the samples were always tested starting from low temperature and working to high temperature; in this way any shear history between different volume fractions was effectively erased. Further, for measurements at the same temperature, we performed a set of control experiments with and without pre-shear. The results suggest that the same steady-state is reached for each strain-rate (for shearing times of \( \approx 25-30 \text{ s} \) used in our experiment), independent of the shear history (see ESI†).

3 Results and discussion

3.1 Steady shear data

Stress (\( \sigma \)) versus strain rate (\( \dot{\gamma} \)) data curves for monodisperse and bidisperse samples are plotted across the liquid-to-solid transition in Fig. 2. The onset of a finite yield stress can be readily identified at \( T_c = 295 \pm 1 \text{ K} \) and \( T_c = 297 \pm 1 \text{ K} \) for monodisperse and bidisperse samples, respectively. Rheological data in the solid regime can be fit (dashed lines) to the well-known Herschel–Bulkey (HB) phenomenological model: \( \sigma = \sigma_s + k\dot{\gamma}^\beta \). Here \( \sigma_s \) is the yield stress, \( k \) is a material-dependent constant, and \( n \) is the HB scaling exponent. The HB model is commonly used for capturing the non-Newtonian behaviors of colloidal suspensions with strain-rate-dependent viscosity.

The exponent, \( n \) is approximately 1/2 for our solid data: \( n = 0.50 \pm 0.02 \) for the monodisperse PNIPAM system, and \( n = 0.48 \pm 0.01 \) for the bidisperse system.

The stress vs. strain-rate curves in Fig. 2 exhibit general features that are similar to those measured in previous microfluidic rheology experiments. Therefore, we first fit our rheological data to predicted critical scaling functions that were employed in Nordstrom et al. Specifically, the dimensionless stress, \( \sigma E \), and strain rate, \( \dot{\gamma} E \), when scaled as \( \sigma E[\phi - \phi_f]^{\Delta} \) and \( \dot{\gamma} E[\phi - \phi_f]^\Gamma \), were predicted to collapse onto two distinct curves, one above and one below the liquid-solid transition. Here, \( E \) is the Young’s modulus of PNIPAM particles, \( \eta_s \) is the viscosity of the solvent, and \( \Delta \) and \( \Gamma \) are scaling parameters. The value of \( \Delta \) is related to the interparticle potentials and is predict as \( \Delta = \alpha - 1/2 \), where \( \alpha = 5/2 \) for particles with Hertzian potentials. It is also predicted that \( \beta = \Delta/\Gamma \) should recover the value of HB scaling exponent, \( n \). The differences \( \left| \phi - \phi_f \right| \) were calculated from eqn (1) and (2) with \( T_c = 295 \pm 1 \text{ K} \) (monodisperse) and \( T_c = 297 \pm 1 \text{ K} \) (bidisperse), respectively. (We demonstrate in the ESI† that these same values for \( T_c \), taken
here from the onset of a finite yield stress, are also obtained as fit parameters from a more comprehensive fitting procedure.

The best-fit scaling exponents, \( \Delta \) and \( \Gamma \), are deduced by finding the best collapse of the data onto the two separate branches above and below the transition. To this end we employed a minimum mean square error (\( \chi^2 \)) fit procedure, analyzing data in discrete steps of 0.1 in \( \Delta \) and \( \Gamma \). The best-fit values of the scaling exponents \( \Delta \) and \( \Gamma \) obtained in this manner for both the monodisperse and bidisperse PNIPAM systems are summarized in Table 1. Uncertainties in fit parameters are determined conservatively for each of the three scaling parameters by the difference between their values at the minimum and their values when \( \chi^2 \) is increased by a multiplicative factor of two. We see that the fits capture scaling collapse of the experimental data over many orders of magnitude around the liquid-to-solid transition in both the monodisperse and bidisperse suspensions. These high quality fits of solid-like and liquid-like branches are shown in Fig. 3.

We note that the collapsed stress vs. strain-rate plots closely resemble the plots in Nordstrom et al. (Fig. 5)\(^1\) however, the liquid-like branches of our data show log–log plot slope values that are close to unity at low shear rates for samples with \(-0.19 < \phi - \phi_c < 0 \) (dashed lines in Fig. 3), and therefore the present samples exhibit more Newtonian-like behavior in the liquid region than the samples in Nordstrom et al.,\(^7\) where the log–log slope was approximately 0.5. Finally, we note that values of \( \beta \equiv \Delta/\Gamma \sim 0.5 \) are obtained using the \( \chi^2 \)-minimization method and are consistent with the Herschel–Bulkley fitting exponent, \( n \approx 0.5 \), obtained from fitting the rheology data of all PNIPAM suspensions in the solid region (see details in the ESI†).

In short, our rheological data collapses well onto two branches, and good agreement is found between our values for the scaling exponents and those measured for PNIPAM particles\(^2\) and for emulsions\(^2\) undergoing a jamming transition. In particular, for jamming, scaling arguments predict that \( \Delta = -1/2 \) with \( \alpha = 5/2 \) for Hertzian interparticle potentials.\(^1,2,4\) We find that \( \Delta \approx 2.5 \) and \( \Gamma \approx 5.0 \) for both monodisperse and bidisperse PNIPAM systems. Although the values of \( \Delta \) and \( \Gamma \) are slightly larger than predicted, within the error bars they are the same of those reported for jammed systems. This scaling could be the signature of a jamming-like transition.\(^4\) Note also, our values for \( |\phi - \phi_c| \) are mostly of order of 0.1 and are therefore relatively far from the critical point. Many simulation studies of athermal jammed systems have found good fits to single power-law scaling for \( |\phi - \phi_c| \) up to order of 0.1\(^,29-31\). On the other hand, Olsson and Teitel\(^39\) specifically explored the corrections to the scaling and found the range of \( |\phi - \phi_c| \) for critical scaling to be smaller. Our experimental scaling exponents suggest the interpretation of jamming-like scaling, but the limited range of \( |\phi - \phi_c| \) precludes an unambiguous conclusion.

**Table 1** Critical scaling exponents for viscometry shear data around the fluid–solid transition. Scaling exponents around jamming transition from Nordstrom et al.\(^5\) are given for comparison.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( \Delta )</th>
<th>( \Gamma )</th>
<th>( \beta = \Delta/\Gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monodisperse</td>
<td>2.6 ± 0.7</td>
<td>5.0 ± 1.0</td>
<td>0.52 ± 0.16</td>
</tr>
<tr>
<td>Bidisperse</td>
<td>2.6 ± 0.8</td>
<td>5.6 ± 1.0</td>
<td>0.46 ± 0.17</td>
</tr>
<tr>
<td>Jamming(^1)</td>
<td>2.1 ± 0.4</td>
<td>4.1 ± 0.6</td>
<td>0.48 ± 0.03</td>
</tr>
</tbody>
</table>
Although the scaling of our rheometer data closely resembles the scaling behaviors observed in the microfluidic rheological measurements, the measured yield stresses, $\sigma_y$, near the transition in Fig. 2a are of order 0.1 Pa, about one order of magnitude lower than those measured with the microfluidic rheological setup. Moreover, the strain rate, $\dot{\gamma}_c$, at which the curves just below and just above $\phi_c$ collapse, is approximately two orders of magnitude smaller in the macrorheology experiment ($\dot{\gamma}_c \approx 0.1 \text{ s}^{-1}$) than in the microfluidic experiment ($\dot{\gamma}_c \approx 10 \text{ s}^{-1}$).

To ascertain the possible origin of these differences, we re-analyze these and other microgel-particle rheology data in the context of recent theory and simulation work that explore thermal and non-thermal contributions to the rheological properties of colloidal suspensions.\textsuperscript{16-18} The first change of variable of this approach normalizes the measured stress by the thermal stress of the concentrated suspension, i.e., by $\sigma_T = k_B T D^{1/3}$. The second change of variable replaces the strain rate with dimensionless Péclet number, $Pe = \dot{\gamma} T = \dot{\gamma} \times 3 \pi n a^3 / k_B T$, where $k_B$ is Boltzmann’s constant, and $n$ is the viscosity of the solvent. Similar normalizations were applied in earlier studies investigating the colloidal glass transition.\textsuperscript{19} Note that for both normalizations, the particle size contributes as $D^{1/3}$. Therefore, the measurements might be expected to be very sensitive to the particle size: a slight difference in particle size can produce significantly different rheological behaviors. The resultant normalized data curves for both the rheometric experiment and the microfluidic experiment are shown in Fig. 4a and b, respectively. Notice, the normalized shear stresses near the liquid–solid transition in the present experiments are close to unity, while those values in the experiments of Nordstrom et al.\textsuperscript{1} are at the order of $10^3$. Evidently, the two experiments probe very different regions of the rescaled stress/strain-rate diagram, although the overall features look very similar.

Next we consider the strength of the particle interactions compared to thermal energies. Recent simulation work by Ikeda et al.\textsuperscript{16,17} suggests that under shear, the nature of the liquid–solid transition depends strongly on temperature and particle softness.\textsuperscript{16,17} To be precise, it depends on the so-called reduced temperature $k_B T / \epsilon$, where $\epsilon$ corresponds to the stiffness of the short-range repulsive interaction potential between two particles. For example, in our systems the interaction potential for contacting frictionless spheres is typically assumed to have the form $V(r_{ij}) = (e/\alpha)(1 - r_{ij}/\sigma)^6$ for $r_{ij} < a$, where $r_{ij}$ is the inter-particle distance; $\alpha = 5/2$ for Hertzian interactions.

If we assume our particles interact via Hertzian interactions, then it can be readily shown that $\epsilon = ED^3/\sqrt{3(1 - v^2)}$, where $E$ and $v$ are the Young’s modulus and Poisson’s ratio of individual PNIPAM particles, respectively.\textsuperscript{16,21} For the 500 nm PNIPAM particles used in this paper, we measured $E \approx 5 - 25$ kPa and $d = 0.4 - 0.6 \mu m$ for the particles used in Nordstrom et al.,\textsuperscript{1} the measured $E \approx 10 - 50$ kPa and $d = 1.3 - 1.8 \mu m$. Taking $v = 0.5$, at the temperatures close to $\phi_c$, the particles used in this paper have $k_B T / \epsilon \approx 5 \times 10^{-6}$, but those used in Nordstrom et al.\textsuperscript{1} have $k_B T / \epsilon \approx 10^{-7}$, much closer to the athermal limit where $k_B T / \epsilon \rightarrow 0$. In addition, the stress/strain-rate data in Fig. 4a for the sample with $k_B T / \epsilon \approx 5 \times 10^{-6}$, probes a region wherein $\sigma_T / \epsilon \approx 1$ and $g_c T \approx 1$, and thus it should be significantly influenced by thermal fluctuations. Therefore, the transition observed in the present experiment is akin to a rheological glass transition. By contrast, the suspensions in the microfluidic experiment, with $k_B T / \epsilon \approx 10^{-7}$ and whose data is shown in Fig. 4b, probes a substantial region wherein $\sigma_T / \epsilon \gg 1$ and $g_c T / \epsilon > 1$; thus the corresponding sample behaviors are strongly athermal, and the transition is akin to a jamming transition. We note that this is in contrast to the conclusion in Ikeda et al.,\textsuperscript{17} where it was suggested that the thermal effect was significant in particle suspensions in Nordstrom et al.;\textsuperscript{1} this conclusion, however, relied on a different value of $\epsilon$ which was taken from Chen et al.\textsuperscript{4} We surmise that its high yield stress (in absolute terms) at the liquid–solid transition arises from strong athermally-driven mechanical interactions between colloidal particles, as suggested by this theoretical framework.

To summarize, we test our rheological data against critical scaling models characteristics of jamming theory,\textsuperscript{19,21-23} and we find that our systems exhibit some of the signatures of jamming-like scaling. However, mainly as a result of differences in elastic modulus, $E$, and particle size ($\epsilon \propto D^3$), the absolute stresses and corresponding strain-rates at $\phi_c$ are different for the different soft PNIPAM particle suspensions, and these differences appear to be due to the thermal versus non-thermal character of the suspensions.\textsuperscript{16,17,48}

In other words, the suspensions appear to undergo a glass transition, and, within experimental signal-to-noise, they exhibit jamming-like scaling for $|\phi - \phi_c| \gg 0$. We note that these two conclusions are not necessarily mutually exclusive. Wang and Xu\textsuperscript{48} have shown in simulations, for example, that

![Fig. 4](image-url) Rescaled stress/strain-rate data sets reproduced from (a) Fig. 2a and (b) Nordstrom et al. (Fig. 3), respectively. Stress is rescaled by the thermal stress scale $D^3 k_B T$, and strain-rate is rescaled using the thermal time scale $[\tau_T]$ as described in the main text.
soft packings at low finite temperature can undergo a colloidal
glass transition, leading to the onset of rigidity as a result of
kinetic arrest; upon further increase of \( \phi \), above the isostatic
point, they found that jamming-like scaling can be recovered.
Unfortunately, because we probe a wide range of packing frac-
tions with minimum \( \Delta \phi = 0.05 \), the present experiments are
not optimally sensitized to detect scaling very near the glass
transition; nevertheless, a jamming-like scaling fits the experi-
mental data.

These findings can be rationalized when we consider the
expected limits of application of jamming-like scaling. Jamming
theory is valid for systems at finite temperature \( T \), as long as
\( T < T^* \), where \( T^* \) is a critical temperature. For
Hertzian particles, one expects \( k_B T^*/\varepsilon_{\text{eff}} \propto \alpha (\phi - \phi_1)^{3/2} \), where
\( \varepsilon_{\text{eff}} = \varepsilon (\phi - \phi_1)^{1/2} \). The prefactor \( \alpha \) is unknown for Hertzian
particles, but should be of the same order of magnitude as for
harmonic particles, i.e., \( \alpha \approx 0.1 \). For our particles,
\( \varepsilon = 10^3 k_B T \), and we find \( T/T^* \approx 10^3 (\phi - \phi_1)^{3/2} \). In other words,
\( T < T^* \) for \( \phi - \phi_1 \geq 0.03 \), i.e., all of our data (except maybe one)
fulfill the condition for jamming-like scaling.

Interestingly, with respect to the signature of glass and
jamming transitions, the findings of the simulation work,\(^{16,47}\)
suggest that it may be possible to observe both thermal and
non-thermal transitions in the same experimental system if the
factor \( k_B T \varepsilon_{\text{eff}} \) can be appropriately tuned and if a wide range of \( \gamma \)
can be experimentally accessed. Future work is needed to
further explore these fascinating questions.

### 3.2 Frequency-dependent rheology

An advantage of our conventional rheometry approach
compared to microfluidic measurements is the possibility to
test the elastic response in frequency-dependent experiments.
In particular, we measure the storage (\( G' \)) and loss (\( G'' \)) moduli
of the monodisperse and bidisperse PNIPAM systems as a
function of oscillation frequency, \( \omega \), across the liquid-to-solid
transition. We then compare the scaling of the static shear
modulus, \( G_0 \), with packing fraction, \( \phi - \phi_1 \), against the scaling
predicted for jammed packings of Hertzian spheres.\(^{16,54}\) As
before, the temperature of these systems is systematically varied
to change volume fraction, \( \phi_1 \); thus we obtain \( G' \) and \( G'' \) as a
function of \( \phi - \phi_1 \) and \( \omega \). Fig. 5 shows \( G' \) and \( G'' \) of mono-
disperse (a), and bidisperse (b) PNIPAM suspensions as a
function of \( \omega \), and for a range of \( \phi - \phi_1 > 0 \), i.e., in the solid regime.
(Note, the maximum applied strain amplitude was \( \gamma \approx 0.01 \),
wherein the response to oscillatory shear is strictly linear, and
all measurements are restricted to the laminar flow regime (i.e.,
Re < 0.1).)

The static shear modulus, \( G_0 \), may be extracted from a fitting
of the frequency-dependent function \( G' = G_0 (1 + \sqrt{\omega/\omega_n}) \),\(^{14,54}\)
where \( G' = G' + i G'' \), and where \( G_0 \) and characteristic frequency,
\( \omega_n \), are fitting parameters. In particular, \( G' \) and \( G'' \) fit to the
following functional forms: \( G' = G_0 (1 + \sqrt{\omega/\omega_n}) \), and
\( G'' = G_0 \sqrt{\omega/2\omega_n} \). These fits are shown in Fig. 5, where the
solid lines indicate \( G' \) fits and the dashed lines indicate the fits
to \( G'' \). The quality of the \( G' \) fits are high for both monodisperse
and bidisperse data-sets over the entire range of frequencies
shown in the figure. \( G'' \), by contrast, is well-approximated by
the equation only for \( \omega \geq 0.4 \) rad s\(^{-1} \). The upturn in \( G'' \) for
\( \omega < 0.4 \) rad s\(^{-1} \), is indicative of relaxation\(^{84,55,56} \) and is not well
captured by the fitting form used here.

O’Hern et al.\(^{39,53} \) predicted that \( G_0 \) of systems above the
jamming transition should scale with \( |\phi - \phi_1| \), according to
the following relation: \( G_0 \propto |\phi - \phi_1|^{(3/2)/2} \), where \( \alpha = 5/2 \) for
particles with Hertzian interactions. That is, we expect a linear
scaling with \( |\phi - \phi_1| \) for Hertzian particles. The experimental
\( G_0 \), normalized by \( E_0 \), are plotted as a function of \( \phi - \phi_1 \) in Fig. 6
for the monodisperse and bidisperse samples. Solid lines are linear fits to the data, showing that, once again, the experimental
results in our disordered colloidal packings are in
reasonable agreement with the scaling predictions made for
jammed systems.

Recent calculations by Tighe\(^{39} \) predict critical scaling
behavior of \( G' \) and \( G'' \) as a function of \( \omega \) for the jammed
systems. In particular, for Hertzian particles, a scaling collapse
of \( G' \) and \( G'' \) versus \( \omega \) is predicted when the moduli are scaled by
\( |Z - Z_c|^2 \) and \( \omega \) is scaled by \( |Z - Z_c|^2 \) or \( |Z - Z_c|^3 \), depending on
whether the damping mechanism is dominated by the drag
force of the solvent or the viscoelastic Hertzian contacts of
particles. Here \( Z \) is the average coordination number of the
interacting particles, and \( Z_c \) is the critical co-ordination number
at the jamming transition, when isostaticity is just reached. We
experimentally explored the scaling collapse of the oscillatory
data using glassy PNIPAM suspensions. Because \( |Z - Z_c| \) cannot
be measured directly in our experiment, however, we use the
relation, \( |Z - Z_c| \sim |\phi - \phi_1|^{1/2} \); this relation was first shown in
simulations with 2D harmonically repulsive disks near the
jamming transition.\(^{49,58} \) It was also observed in simulations\(^{33} \)
in 3D systems with Hertzian potentials and theoretically studied
by Wyart et al.\(^{16} \). Later experiments with 2D photelastic
we derive evidence for Tighe’s collapse of monodisperse and bidisperse PNIPAM microgel spheres at of PNIPAM particles. The error bars for are very small effects already seen in Fig. 5). Through this data collapse, we derive evidence for Tighe’s prediction that the scaling of holds, not just for the quasi-static limit, but for a broad range of frequencies. We note that similarly good scaling collapse was found when was scaled by (see details in ESI†). Unfortunately, we cannot unambiguously determine which damping mechanism plays the dominant role in our system.

Finally, we measure the cross-over frequency in the liquid-like suspensions, i.e., for . The corresponding time, , indicates the characteristic relaxation time of the system. Fig. 8 plots the , versus for the bidisperse PNIPAM system (see ESI† for plots of at different ). , and can both be fit well by exponential functions of as indicated by the black dashed lines. At , for example, , or, alternatively, . Interestingly, this time-scale is of the same order of magnitude as the -relaxation time reported in a 2D PNIPAM system. Cross-over frequencies measured in the monodisperse PNIPAM system had similar values (data not shown), albeit for the more limited range of investigated.

4 Conclusions

We have investigated both steady-state and frequency-dependent rheological behaviors of 3D monodisperse and bidisperse soft particle colloidal suspensions across the liquid-to-solid transition using macro-rheology. The shear stress versus strain-rate curves, for samples far from the liquid–solid transition, exhibit scaling features similar to the critical scaling predictions for jammed athermal systems near the critical point, e.g. as found in suspension experiments employing a micro-fluidic apparatus. However, the magnitude of the observed stresses were lower for the smaller softer microgel particles, and the strain-rates at rigidity onset differed substantially, too. The size and stiffness of the individual particles are expected to modulate the importance of thermal fluctuations in rheology experiments. For example, while the thermal effects are negligible in suspensions of large/hard particles, thermal contributions can be significant in systems with relatively smaller and softer particles. Our observations of thermal glass transitions and jamming-like scaling is quantitatively consistent with recent simulation results, but a full understanding of the
underlying mechanisms will require further theoretical and experimental investigation.

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**References**


