# Maximally Informative Input-Output Functions in Biological Networks


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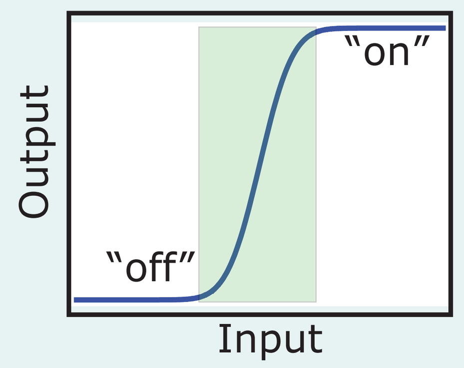
## 1. Characterizing input-output functions with *decision boundaries*

Neurons (or nodes in a network) transform inputs into outputs.

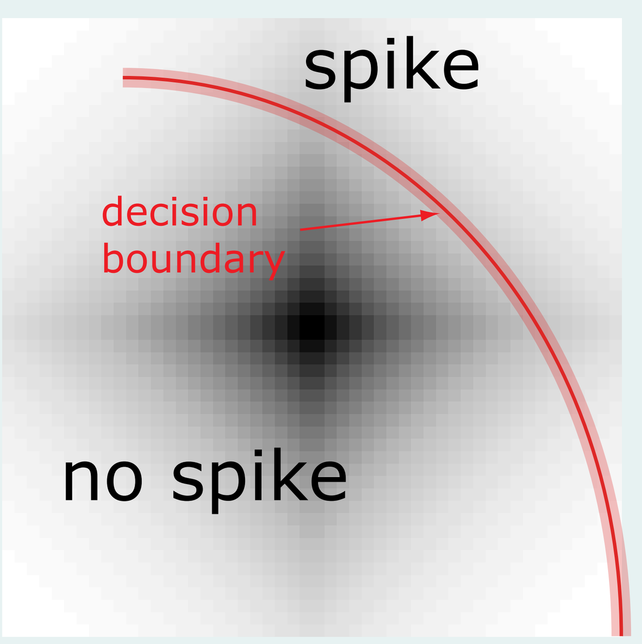
An "input" could be a spike train from a pre-synaptic neuron or a time-varying sensory stimulus, such as light intensity or sound pressure.

**1D input-output functions** are easy (the neural response depends only on one feature of the input signal).

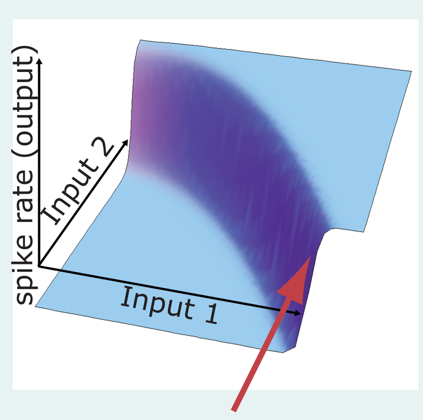



In most cases, the neural responses are modulated by **multiple** stimulus features. Multi-dimensional input-output functions can be described using the concept of a **decision boundary** that separates stimuli that elicit spikes from stimuli that do not elicit spikes.

*T Sharpee, W Bialek, PLoS ONE (2007).*




gray scale represents the input distribution




Stimuli near the decision boundary produce most variable responses.

### What network configurations of decision boundaries can convey the maximal information about stimuli?

Information theory in a nutshell:

Consider whether a neuron spikes or remains silent...

Before Observation: ? Uncertain

After Observation: spike or silent Certain

Observing the output $O$ reduces your uncertainty, giving you *information*

$$H_r = -\sum_O P(O)\ln P(O)$$

Knowing the input reduces your uncertainty about the output, so the "information" the output gives about the input is less than $H_r$:

$$\boxed{Information = H_r - H_n}$$

Information is lost due to noise (transition region of I-O function)

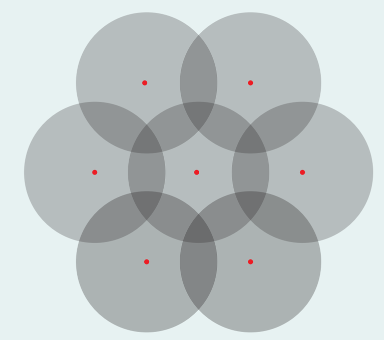
$$H_n = -\left\langle \sum_O P(O|I)\ln P(O|I) \right\rangle_I$$

## 2. Optimal Decision Boundaries

- maximize response entropy by making probabilities of various network responses more equal.
- minimize noise entropy by passing through less likely stimuli and having shorter overall length.
- maintain average spike probability in a network.

**Uniform stimuli:**

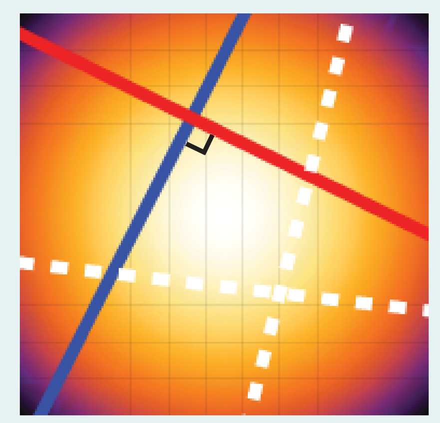
Decision boundaries are circles both for independent neurons and networks.



**Gaussian inputs:**

Optimal boundaries are planar, both independent neurons or networks, BUT in a network they must be orthogonal.

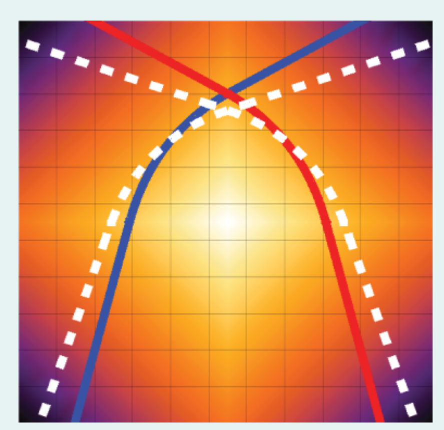
Optimal boundaries: red, blue
Independent: white



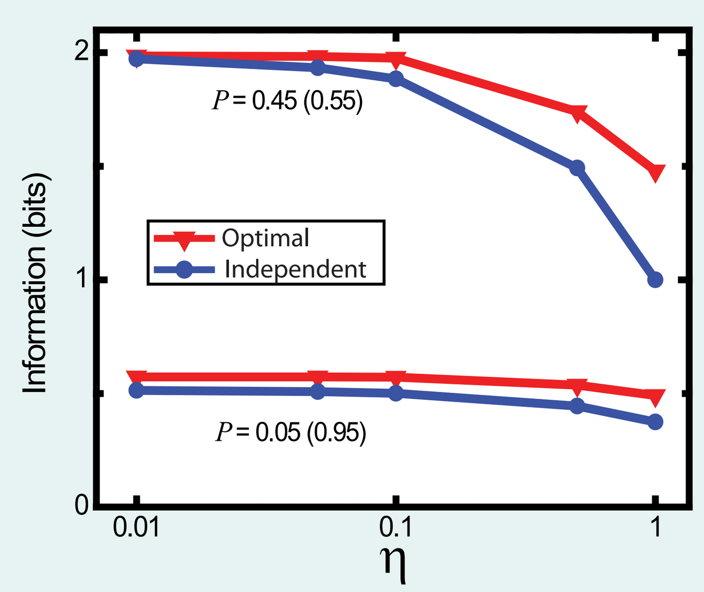
**Naturalistic inputs:**

Optimal boundaries depend on noise level & neural coupling.

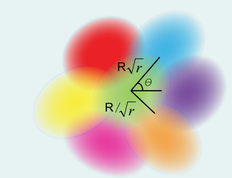
Neural coupling leads to a "kink" at the intersection.



- Interactions with unmeasured parts of the network can be inferred from input-output functions for the measured neurons.
- Neural coupling increases with noise.
- Coupling between neurons **reduces** the effect of noise:

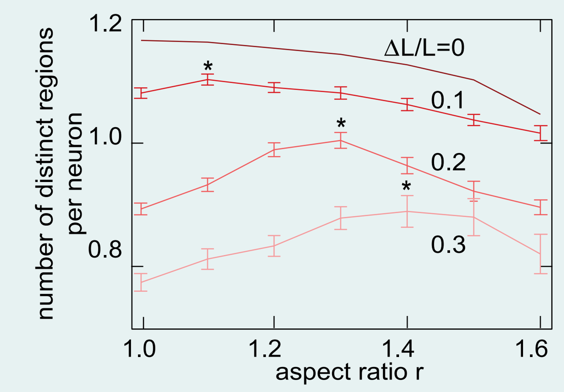



## 3. Optimization with scatter in receptive field center positions

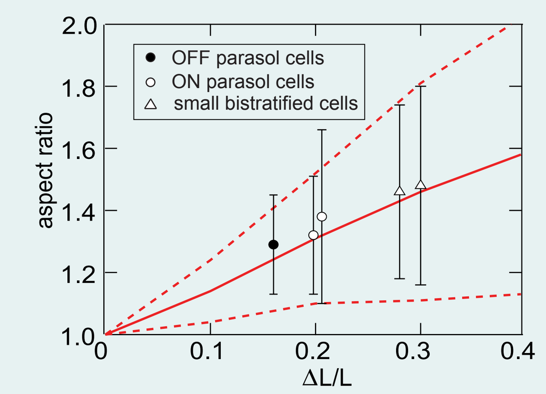


**irregular** receptive fields are optimal!

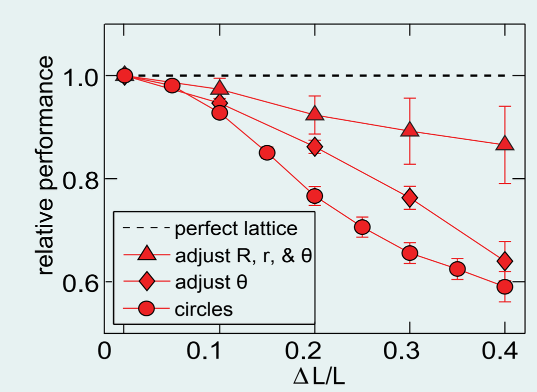
Optimal eccentricity **increases** with scatter in RF center postions.




Optimal **variance** in eccentricity among cells increases with scatter in RF center postions.



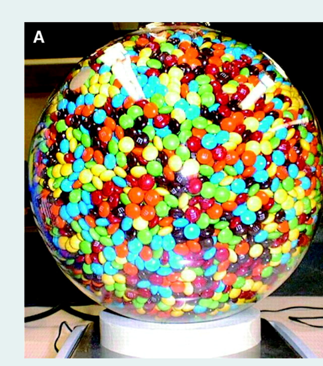

Adjusting orientation, aspect ratio, and size for individual cells in a mosaic **restores** performance.

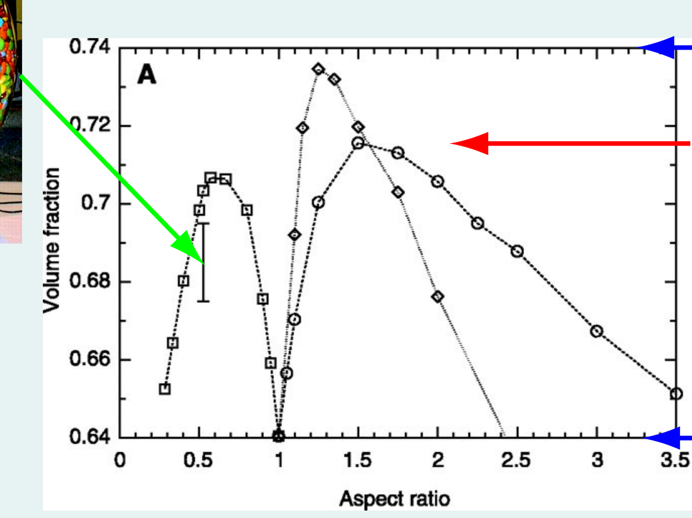



## 4. What M&M's and retina have in common?

Ellipsoids provide **better** random packings than shperes!

*A. Donev et al. Science 2004*






**regular** packings with spheres

random packings with ellipsoids

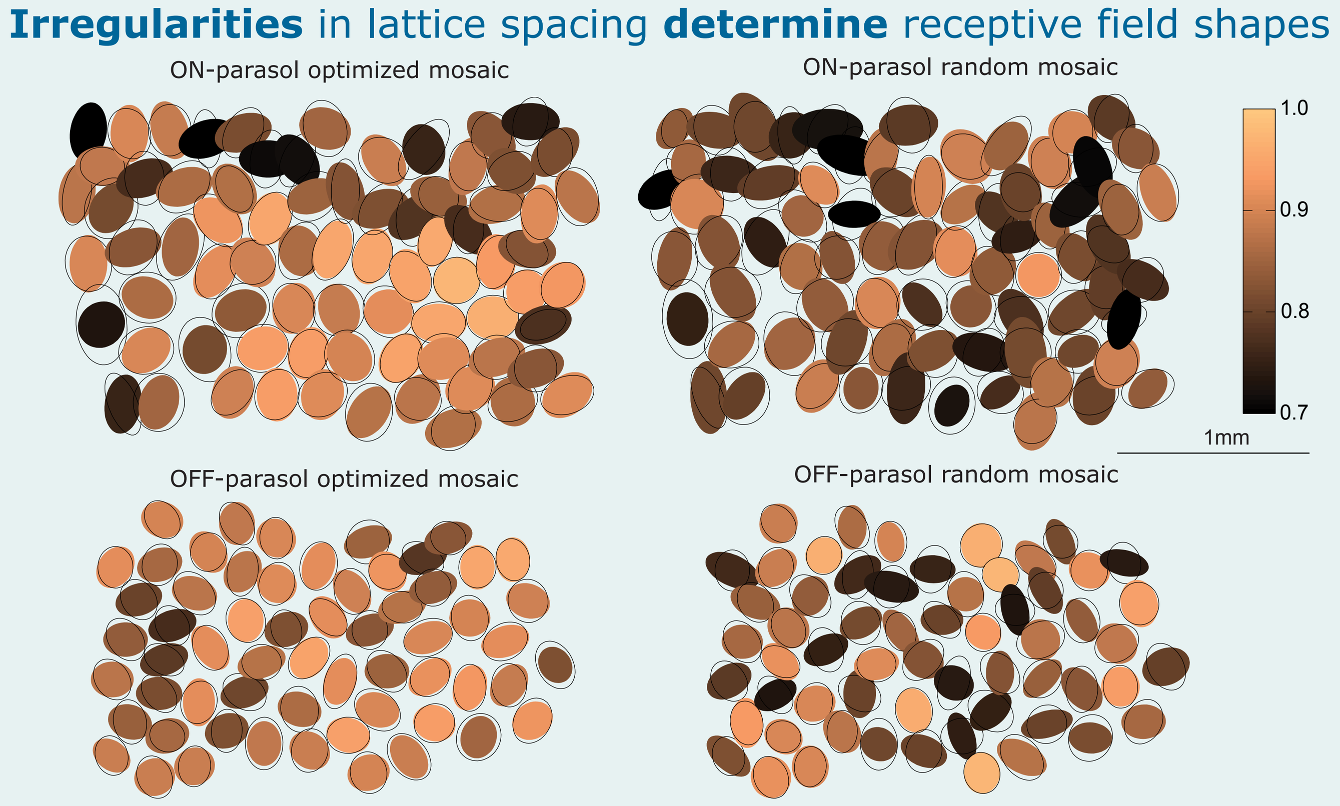
**random** packings with spheres

## Error-correction!

Random packings can be used to design error-corrective codes. In retina, it arises from overlaping irregular receptive fields.

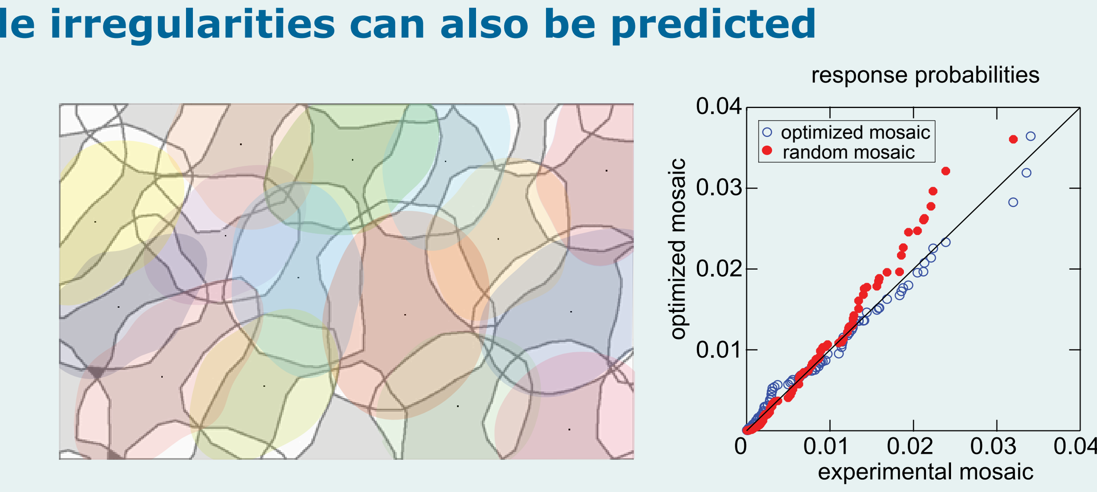
## 5. Predictable Irregularities in Retinal Receptive Fields

**Irregularities** in lattice spacing **determine** receptive field shapes




empty contours: data from G. Field & E.J. Chichilnisky, Annu Rev Neurosci (2007); color: optimized mosaic; p<0.002 for improvement compared to random elliptical mosaics

**Fine-scale irregularities can also be predicted**




color: optimized mosaic; gray: data from J. Gauthier et al PLoS Biology (2009).

## 6. SUMMARY


For naturalistic stimuli, input-output functions should be

- muli-dimensional;
- kinks in decision boundaries indicate interations with other (possibly unmeasured) neurons;
- interactions between neurons increase with noise.

For retinal circuits, with scatter in receptive field center positions:

- optimal receptive fields are irregular;
- irregularities in lattice spacing determine receptive field shapes.


## Acknowledgments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For more information, please see the papers:


- Fitzgerald and Sharpee, Phys. Rev. E (2009);
- Liu, Stevens, & Sharpee, PNAS (2009).