

Line junctions in the quantum Hall effect

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A long narrow gate across a fractional quantum Hall fluid at filling $\nu=1/m$ with odd integer m , creates a one-dimensional (1D) system that is isomorphic to a disordered 1D electron gas with *attractive* interactions. By varying the gate potential along such a line junction, it should be possible to tune through the 1D localization transition, predicted for an attractively interacting electron gas. The key signature of this 1D metal-insulator transition is the temperature dependence of the conductivity, which diverges as a power of temperature in the metallic phase, and vanishes rapidly in the insulator. We show that the 1D conductivity can be extracted from a standard Hall transport measurement, in the regime where the Hall conductance is close to its quantized value. A line junction in a $\nu=2/3$ quantized Hall fluid is predicted to exhibit a similar localization transition. [S0163-1829(97)02544-7]

I. INTRODUCTION

Edge states in the quantum Hall effect offer a highly controlled laboratory for the experimental study of quantum transport in one dimension. The right and left moving edge modes, which reside on the opposite edges of a quantum Hall bar form an ideal one-dimensional electron gas. Since the edges are spatially separated from one other, backscattering due to impurities, which usually localizes electrons in one dimension, may effectively be eliminated.

Following Wen's suggestion that the edge states in the fractional quantum Hall effect are chiral Luttinger liquids,¹ there has been considerable interest in the experimental implications of Luttinger liquid theory on edge state transport. Much of the focus has been on the nature of point contact tunneling. Specifically, pinching a quantum Hall bar at a point using a patterned gate electrode introduces local and controllable backscattering between oppositely moving edge modes. This is analogous to a single impurity in an otherwise clean one-dimensional electron gas.² Luttinger liquid theory predicts that the tunneling conductance through the point contact vanishes as a power of temperature with a universal exponent, which depends on the structure of the bulk quantum Hall fluid. Milliken, Umbach, and Webb have observed a temperature dependence consistent with the predicted T^4 behavior for tunneling between two $\nu=1/3$ fluids.³ More recently, Chang, Pfeiffer, and West have measured the tunneling conductance between a Fermi liquid and a $\nu=1/3$ edge state, and found behavior consistent with the predicted T^2 temperature dependence.⁴

A different and perhaps more interesting way of introducing intermode backscattering is depicted schematically in Fig. 1. A bulk quantum Hall fluid is divided into two pieces by depleting the electron gas along a narrow line, using a long "skinny" gate. Such a "line junction" creates "internal" edge states that propagate in opposite directions on either side of the gate. Together, these two modes constitute a novel (nonchiral) one-dimensional system — a "quantum

antiwire." As the gate potential is varied, the degree of backscattering between the two counterpropagating modes can be varied. For strong depletion under the gate, all backscattering can be effectively eliminated, and the source to drain conductance vanishes. In the opposite limit, the gate potential can be turned off, and the (two-terminal) source-to-drain conductance is quantized. But what happens in between? For intermediate values of V_G , intermode backscattering will be mediated by inhomogeneities, either of the gate itself or due to nearby impurities in the electron gas.

Since the line junction is effectively a disordered one-dimensional electron system, one might expect that electron localization is inevitable. For the integer quantum Hall effect, this expectation is valid. However, Renn and Arovas⁵ have recently shown that for a fractional quantum Hall fluid at filling $\nu=1/3$, the "antiwire" line junction is formally equivalent to a 1D electron gas with *attractive* electron interactions. As shown some years back by Giamarchi and Schulz,⁶ a disordered 1D electron gas becomes metallic for sufficiently strong attractive interactions — that is, all states are *not* localized in 1D. Upon varying the strength of the

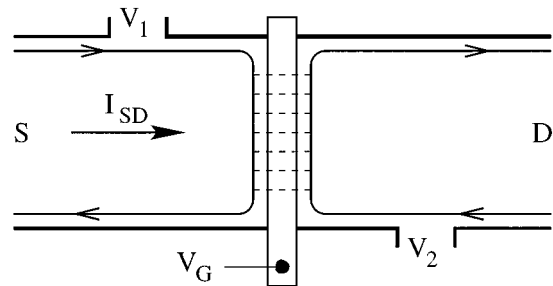


FIG. 1. A long narrow gate across a quantum Hall bar creates a line junction, with oppositely moving edge modes (lines with arrows) on either side of the gate. The intermode backscattering rate (dotted lines) can be varied by changing the gate potential V_G , which drives a one-dimensional metal-insulator transition.

attractive interaction, a disorder-driven metal-insulator transition was predicted. This metal-insulator transition should be directly observable in such a fractional quantum Hall effect line contact.

In this paper we describe in detail the experimental signature of a 1D metal-insulator transition for a quantum Hall line junction. The transition is conveniently characterized by the temperature dependence of a one-dimensional *conductivity*, σ — an intensive quantity. For an infinitely long system, Giamarchi and Schulz⁶ argue that the conductivity vanishes at $T=0$ in the insulating phase, but diverges as $T \rightarrow 0$ in the metallic phase. However, the most accessible experimental quantity is the source-to-drain *conductance*, for a Hall bar with a finite width, L . Nevertheless, by tuning the gate potential into a regime where the Hall conductance is close to its quantized value, it is possible to extract the “antiwire” conductivity, as we discuss in detail below.

We begin in Sec. II with a review of the Luttinger-liquid model for a line junction and show, following Renn and Arovas, that for fractional quantum Hall effect (FQHE) states in the Laughlin sequence, $\nu=1/m$ with odd m , a 1D metal-insulator transition should be accessible. We describe the temperature dependence of the conductivity in the metal and insulating phases as well as near the transition, in Sec. III. In Sec. IV we show how the conductivity can be extracted from a Hall conductance measurement. Finally, in Sec. V we consider the line contact for a hierarchical FQHE state at filling $\nu=2/3$, and argue that a similar metal-insulator transition should occur there as well.

II. MODEL AND TRANSITION

The bosonized Hamiltonian density for a clean line junction can be written in terms of right and left moving electron densities, $n_{R/L}$:

$$\mathcal{H}_0 = \frac{\pi v_0}{\nu} (n_R^2 + n_L^2 + 2\lambda n_R n_L). \quad (2.1)$$

These densities satisfy Kac-Moody commutation relations:¹

$$[n_{R/L}(x), n_{R/L}(x')] = \pm (i\nu/2\pi) \partial_x \delta(x-x'). \quad (2.2)$$

When $\lambda=0$ this Hamiltonian describes decoupled right and left moving modes, which propagate at a velocity v_0 . The term proportional to λ represents a screened Coulomb interaction between the right and left moving modes. We have assumed that the long-ranged piece of the Coulomb interaction is screened by a ground plane, or the line junction gate itself.

When the gate potential is large, there is a large barrier between the quantum Hall fluids. The two modes are then well separated spatially, and the interaction λ is small. As the gate potential is decreased, the modes move closer together, increasing the repulsive interaction λ . But in addition, tunneling of electrons between the right and left modes *under* the gate becomes possible. To incorporate these processes we add an additional term to the Hamiltonian:

$$\mathcal{H}_1 = \xi(x) \psi_R^\dagger(x) \psi_L(x) + \text{H.c.}, \quad (2.3)$$

where ψ_R is an electron destruction operator in the right moving mode. These operators can be reexpressed in terms of boson fields $\phi_{R/L}$, which are proportional to the electron densities:

$$n_{R/L} = \pm \frac{1}{2\pi} \partial_x \phi_{R/L}. \quad (2.4)$$

Specifically,

$$\psi_R \sim e^{i\phi_R/\nu}, \quad (2.5)$$

and similarly for the left moving mode.

The electron tunneling amplitude $\xi(x)$ is generally complex. For a perfectly clean line junction one expects $\xi(x) \sim e^{i\delta k x}$ where δk is a gauge-invariant momentum difference between the right and left moving modes. If the edge modes are separated by a distance d then $\delta k = 2\pi B d / \Phi_0$, where B is the applied magnetic field, and $\Phi_0 = hc/e$ is the magnetic flux quantum. However, in any real device one expects the presence of impurities near the line junction, which will effect also the magnitude of the tunneling strength, $|\xi|$. We thus assume that $\xi(x)$ is a random complex variable, uncorrelated on length scales long compared to the interimpurity spacing “ a .” In practice one expects a to be comparable to or smaller than the distance to the line-junction gate. For further simplicity, we take $\xi(x)$ to have a Gaussian distribution,

$$[\xi(x)\xi^*(x')]_{\text{ens}} = \Delta_W \delta(x-x'), \quad (2.6)$$

where the square brackets denote an ensemble average over impurity configurations. For later convenience we define a dimensionless impurity strength W :

$$W = \frac{a}{\omega_c} \Delta_W. \quad (2.7)$$

Here the cutoff frequency ω_c is set by the bulk quantum Hall gap — the cyclotron frequency when $\nu=1$. Upon decreasing the gate potential, which brings the edge modes closer together enabling tunneling, one expects that the effective disorder strength W increases in magnitude.

As emphasized by Renn and Arovas⁵ the above model for a line junction is mathematically equivalent to a model of a one-dimensional interacting electron gas with impurity scattering present. For the integer quantum Hall effect (IQHE) at $\nu=1$ the electron gas is repulsively interacting, and one anticipates that the line junction will be insulating with all states localized. But most remarkably, when λ is small the electron gas isomorphic to the FQHE line junction has *attractive* interactions. To see this we define new nonchiral boson fields,

$$\phi_{R/L} = \sqrt{\pi} (\phi \pm \nu \theta), \quad (2.8)$$

which are canonically conjugate variables,

$$[\theta(x), \partial_x \phi(x')] = i \delta(x-x'). \quad (2.9)$$

In terms of these fields the pure Hamiltonian becomes

$$\mathcal{H}_0 = \frac{v}{2} \left[K (\partial_x \phi)^2 + \frac{1}{K} (\partial_x \theta)^2 \right], \quad (2.10)$$

with a renormalized velocity

$$v = v_0(1 + \lambda^2)^{1/2}, \quad (2.11)$$

and a dimensionless ‘‘stiffness’’

$$K = \frac{1}{\nu} \left[\frac{1 - \lambda}{1 + \lambda} \right]^{1/2}. \quad (2.12)$$

The random piece of the Hamiltonian involves only the field θ :

$$\mathcal{H}_1 = \xi(x) e^{i2\sqrt{\pi}\theta(x)} + \text{H.c.} \quad (2.13)$$

The model is equivalent to a bosonized representation of an interacting Luttinger liquid with impurity scattering. The stiffness K is equal to the dimensionless conductance g for the Luttinger liquid. Thus, $K < 1$ describes a repulsively interacting electron gas, whereas $K > 1$ an attractively interacting gas.

Remarkably, for a $\nu = 1/3$ line junction with well separated modes (small λ), the equivalent electron gas is strongly attractive, $K = 1/\nu$. This should be contrasted to a very narrow quantum Hall bar which also has right and left moving modes. In this case, the dominant intermode tunneling process is a fractionally charged Laughlin quasiparticle. The system is isomorphic to a repulsively interacting electron gas with $K = \nu$, rather than $K = 1/\nu$ as above.

The effects of impurity scattering on an interacting Luttinger liquid has been considered by a number of authors.⁶⁻⁹ The renormalization-group calculation by Giamarchi and Schulz⁶ reveals clearly the phase boundary separating an insulating from a conducting phase. Working in momentum space, they integrate over the field $\theta(k, \omega_n)$, for a shell of modes with $\Lambda/b < k < \Lambda$, and rescale as $k' = bk$ and $\omega'_n = b^z \omega_n$. Here $\Lambda \sim 1/a$ is a cutoff and ω_n is a Matsubara frequency. The dynamical exponent z is chosen to keep the velocity v invariant. To leading order in W the RG recursion relations are ($\ell = \ln b$)

$$\partial W / \partial \ell = (3 - 2K)W, \quad (2.14)$$

$$\partial K / \partial \ell = -\frac{K^2}{2}W, \quad (2.15)$$

with $z = 1 - KW/2$. These equations describe a phase transition between a conducting phase, in which the disorder strength W scales to zero, and an insulating disorder dominated phase, as sketched in Fig. 2. For small W the phase boundary is at $K = 3/2$, and increases to larger K with increasing W .

For the IQHE line junction ($\nu = 1$), the largest value of K is one, so that the system is always in the localized phase. However, for a $\nu = 1/3$ FQHE line junction, the maximum value of K is 3, which occurs when the modes are well separated and W is small. This puts the system well into the conducting phase. With decreasing gate potential, both the tunneling (W) and the interactions (λ) increase, which moves the system along the trajectory sketched in Fig. 2. The system will undergo a phase transition into a localized state.

This localization transition should be observable in FQHE line junctions. In the next section we consider the behavior of the transport along the line junction, first under the as-

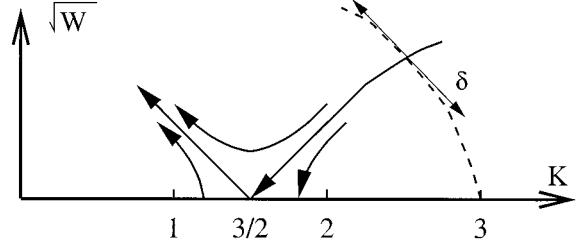


FIG. 2. Renormalization-group flow diagram for a 1D metal-insulator transition, with disorder strength W , and interaction parameter K . The dashed line represents the initial values of W and K for $\nu = 1/3$ as the voltage on the line junction gate is varied. The parameter δ measures the ‘‘distance’’ to the transition.

sumption that the line junction is infinitely long. We then describe the predicted behavior for a finite length line junction fed by QHE edge states, as depicted in Fig. 1. In Sec. IV we argue that a $\nu = 2/3$ line junction should exhibit a similar localization transition.

III. BULK CONDUCTIVITY

Transport along the line junction is characterized by a one-dimensional conductivity σ . Of interest is the temperature dependence in the insulating and conducting phases, as well as near the transition.

In the insulating phase at low temperatures, the transport presumably takes place via variable-range hopping processes between nearby localized states. This gives

$$\sigma(T) \sim e^{-(T_0/T)^{1/2}}. \quad (3.1)$$

The temperature scale T_0 is set by the localization length ξ_{loc} , varying as $T_0 \sim v/\xi_{\text{loc}}$. Deep within the localized phase, $\xi_{\text{loc}} \sim a$ and the temperature scale should be large. Upon approaching the transition from the insulating side, the localization length diverges as

$$\xi_{\text{loc}} \sim a e^{c/\delta^{1/2}}, \quad (3.2)$$

where δ is the distance to the transition and c is a constant. The parameter δ may be tuned by varying the gate voltage V_G , $\delta \propto V_{Gc} - V_G$.

In the conducting phase, the disorder W scales to zero, and the conductivity should be infinite at zero temperature. Finite temperature cuts off the RG flows before W reaches zero, and a large but finite conductivity is expected. In this regime, the system is characterized by two length scales. The scattering mean free path ℓ is the distance an electron travels in the right moving mode, say, before suffering an intermode backscattering event. In addition, the thermal length $L_T = v/T$ describes the loss of phase coherence *within* a single mode due to thermal smearing. On length scales longer than L_T , scattering events are uncorrelated.¹⁰

Following Giamarchi and Schulz,⁶ the temperature dependence of ℓ may be deduced from scaling arguments. Under a rescaling transformation by a factor ‘‘ b ’’ one can write

$$\ell(W, T/\omega_c) = b \ell(b^{3-2K}W, bT/\omega_c). \quad (3.3)$$

Generally temperature scales as b^z , but $z = 1$ to leading order in W . With the choice $b = \omega_c/T$, the effective temperature on

the right side becomes comparable to the cutoff frequency. Quantum interference effects should be absent at such high temperatures, and a perturbative evaluation of the scattering length should be valid. Since the scattering rate should be linear in W one expects $\ell(W,1) = ca/W$, with some constant c . This gives

$$\ell(W, T/\omega_c) = \frac{ca}{W(T/\omega_c)^{2(K-1)}}. \quad (3.4)$$

Since $K > 3/2$ throughout the conducting phase, $\ell \gg L_T$ as $T \rightarrow 0$. This implies that successive backscattering events are incoherent, so that quantum interference effects are absent. A Boltzmann transport description is then appropriate, which relates the conductivity and scattering lengths $\sigma = (e^2/h)\ell$, so that

$$\sigma \propto (e^2/h)(a/W)(T/\omega_c)^{2(1-K)}. \quad (3.5)$$

In the Appendix we show that this result may also be obtained from the Kubo formula, treating the disorder perturbatively within a Born approximation.⁸ For $1 < K < 3/2$ it appears naively that perturbation theory should be valid, since ℓ diverges at low temperature. However, since L_T diverges faster, successive scattering events become coherent. This leads to a breakdown of Boltzmann transport and to localization.

To describe the conductivity near the metal-insulator transition, it is necessary to include the renormalization of K . The precise temperature dependence of the conductivity in the crossover region may be determined by integrating both flow equations (2.14) and (2.15) out to a temperature-dependent length scale, $\ln b = \ln(\omega_c/T)$. Let $\delta \propto K_c - K$ be the ‘‘distance’’ to the transition, as shown in Fig. 2. To leading order in $K - 3/2$, the renormalized value of W in the insulating phase, $\delta > 0$, is

$$W_R = (8/9)c^2|\delta|/\sin^2(c\sqrt{\delta}\ln\omega_c/T). \quad (3.6)$$

The conductivity is then given by Eq. (3.5) with $K = 3/2$ and W replaced by W_R ,

$$\sigma \propto (e^2/h)a(\omega_c/T)\sin^2(c\sqrt{|\delta|}\ln\omega_c/T)/c^2|\delta|. \quad (3.7)$$

For the conducting phase $\delta < 0$ the expressions are similar, except ‘‘sin’’ is replaced by ‘‘sinh.’’ These expressions are accurate provided $\delta \ll 1$ and $W_R \ll 1$. The latter condition is equivalent to $L_T \ll \xi$, where the localization length ξ is given in Eq. (3.2).

At high temperatures, the conductivity in the transition region is dominated by the prefactor in Eq. (3.7), $\sigma = A/T$, where $A \sim (e^2/h)a\omega_c$. In Fig. 3 we plot the temperature dependence of $\sigma T/A$ from Eq. (3.7) for several values of δ above and below the transition. On the metallic side, $T\sigma/A$ diverges as $(T/\omega_c)^{-\alpha}$ at low temperatures, where the exponent $\alpha = c\sqrt{\delta}$. As the transition is approached from above, $\alpha \rightarrow 0$, and logarithmic corrections to the power-law behavior develop. Precisely at the transition, the conductivity varies as

$$\sigma T/A = \ln^2 \omega_c/T. \quad (3.8)$$

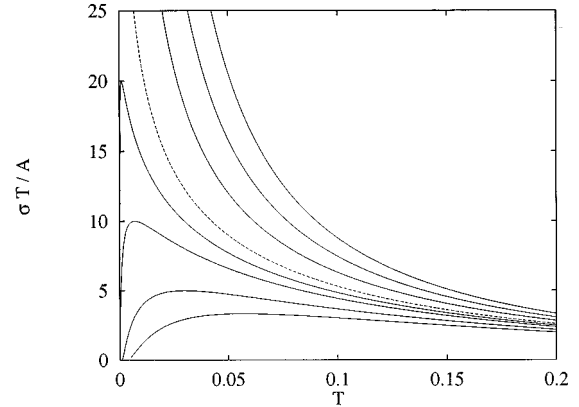


FIG. 3. Temperature dependence of the conductivity obtained from Eq. (3.7) for $c^2\delta = -0.3, -0.2, -0.1, -0.05, 0, 0.1, 0.2, 0.3$ (σ decreasing). Negative (positive) values of δ correspond to the metallic (insulating) phase. The dashed line corresponds to $\delta = 0$ — precisely at the metal-insulator transition.

Slightly below the transition σT increases logarithmically as the temperature is lowered, down to a temperature of order $T^* \approx \omega_c \exp[-c/\sqrt{\delta}]$. Below T^* the conductivity decreases, signaling the crossover to the insulating regime. As the temperature is lowered further W flows out of the perturbative regime when $\sigma T/A < 1$. The conductivity should then follow the Mott variable-range hopping law (3.1) with $T_0 \sim T^*$.

IV. TRANSPORT WITH LEADS

In the previous section we discussed the temperature dependence of the line junction conductivity for an infinitely long wire. In practice, of course, the line junction will have some finite length L . Moreover, it is initially unclear how the bulk line junction conductivity can be extracted from a standard Hall conductance measurement. In this section we discuss how this can be achieved.

Consider the geometry shown in the Fig. 1. A Hall bar of width L is cut into two by a line junction. The easiest quantity to measure experimentally is the Hall conductance, passing a current from source to drain, and measuring the voltage drop between the two Hall voltage probes (denoted 1 and 2) that straddle the line junction. Ignoring contact resistances at the source and drain electrodes, this is equivalent to the two-terminal source-to-drain conductance,

$$G_{sd} = I_{sd}/(V_s - V_d), \quad (4.1)$$

where V_s and V_d are the source and drain voltages.

Imagine starting in equilibrium with $V_s = V_d = 0$, and then raising the source voltage to $V_s = V$. This injects an extra incident current, $I_{in} = (ve^2/h)V$, from the source electrode along the top edge. At the line junction this current splits, with a current I passing along the line junction, and $I_{in} - I$ continuing along the edge into the drain electrode. The transmitted source-to-drain current is thus $I_{sd} = I_{in} - I$. Since the two ends of the line junction are separated by the voltage V , the current flowing in the line junction is $I = GV$, where G is the two-terminal *conductance* of the line junction. The source-to-drain conductance is thereby related to the line junction conductance:

$$G_{sd} = \nu e^2/h - G. \quad (4.2)$$

The two-terminal conductance of the line junction depends on the length L of the junction. Provided L is long compared to the thermal coherence length L_T , a classical description is possible in terms of the bulk *conductivity* of an infinitely long line junction. In particular, solving a Boltzmann equation¹¹ subject to the boundary condition that the channels incident to the left and right of the line junction are separated by a voltage V , one arrives at the two-terminal conductance

$$G = \nu \frac{e^2}{h} \frac{\sigma}{\sigma + \nu(e^2/h)L}. \quad (4.3)$$

In the localized phase, $\sigma \ll (e^2/h)L$, so that Eq. (4.3) reduces to the classical expression $G = \sigma/L$, with a small conductance. In the extended phase this may also be true at high temperatures. However, upon cooling, σ grows, and the backscattering length l will eventually become comparable to or larger than the line junction length L . In this opposite limit, $\sigma \gg (e^2/h)L$, an electron will typically be transported all the way along the line junction length without suffering any backscattering collisions. The line junction conductance will be very close to perfect, $G \approx \nu e^2/h$, whereas the source-to-drain conductance will be much smaller than the quantum unit $G_{sd} \ll \nu e^2/h$. In this low-temperature regime of the metallic phase, one thus has $G_{sd} \propto (e^2/h)(L/a)W(T/\omega_c)^{2(K-1)}$.

Finally, at very low temperatures in the metallic phase, when $L_T \gg L$, the system length L replaces temperature as a cutoff. In this limit the line junction behaves effectively as a point contact, with ideal leads. One expects the source-to-drain conductance to vary as $G_{sd} \propto (e^2/h)(L/a)W(T/\omega_c)^{(2/\nu-2)}$.

To extract the bulk line junction conductivity, and avoid the complications associated with the various different regimes, it is clearly desirable to make the line junction as long as possible. The temperature range should then be restricted so that the source-to-drain conductance is close to its quantized value. The bulk conductivity can be readily extracted:

$$\sigma = L(\nu e^2/h - G_{sd}) \frac{\nu e^2/h}{G_{sd}}. \quad (4.4)$$

V. LOCALIZATION TRANSITION FOR 2/3

Hierarchical FQHE states at filling factors different from $1/\nu$ an odd integer, are believed to have multiple propagating modes on a single edge. This necessarily complicates the analysis of a line junction in such states. For simplicity, we discuss only the experimentally most robust hierarchical state — $\nu = 2/3$.

We first briefly review the theory of a single edge of a $\nu = 2/3$ fluid. MacDonald¹² and Wen¹ originally argued that the edge consists of two modes: a forward propagating mode similar to a $\nu = 1$ edge, and a backward propagating mode similar to a $\nu = 1/3$ edge. The appropriate Hamiltonian is

$$\mathcal{H}_0 = \pi v_1 n_1^2 + 3\pi v_2 n_2^2, \quad (5.1)$$

where $n_1 = \partial_x \phi_1/2\pi$ is the charge density propagating downstream at velocity v_1 , and $n_2 = -\partial_x \phi_2/2\pi$ the density propagating upstream at v_2 . These densities satisfy the commutation relations:

$$[n_1(x), n_1(x')] = (i/2\pi) \partial_x \delta(x-x'), \quad (5.2)$$

$$[n_2(x), n_2(x')] = -(1/3)(i/2\pi) \partial_x \delta(x-x'). \quad (5.3)$$

Generally, these two modes will interact via a term of the form $\mathcal{H}_{\text{int}} \sim v_{12} n_1 n_2$.

Operators that add charge to the edge take the general form $O_{n_1, n_2} = e^{i(n_1 \phi_1 + n_2 \phi_2)}$, for integer n_i . These operators add n_1 electrons to channel one, and n_2 $1/3$ -charged Laughlin quasiparticles to mode two, creating a total charge $Q(n_1, n_2) = e(n_1 + n_2/3)$.

To account for the observed quantized Hall conductance at $\nu = 2/3$, it is essential to incorporate processes that can equilibrate the two edge modes. The dominant process is $O_{1, -3}$, which transfers a unit charge from mode two to mode one. This process must be mediated by impurities, since the two edge modes will generally be at different momenta. In Ref. 9 we have analyzed in detail the effects of such impurity-induced tunneling processes. We find that there are two possible phases, depending on the impurity strength and the interchannel Coulomb interaction v_{12} . For a very clean edge with small v_{12} the impurity scattering scales to zero at low energies, and charge propagates in both directions. But for a dirtier edge, the system has a phase transition into a disorder-dominated phase. In this phase the two modes restructure, forming a charge mode,

$$n_\rho = n_1 - n_2 = \partial_x \phi_\rho / 2\pi, \quad (5.4)$$

propagating downstream, and a neutral mode,

$$n_\sigma = n_1 - 3n_2 = \partial_x \phi_\sigma / 2\pi, \quad (5.5)$$

moving upstream. [More precisely, the actual neutral mode that propagates is related to n_σ by a spatially random $SU(2)$ rotation — see Ref. 9.] The effective Hamiltonian becomes

$$\mathcal{H}_0 = (\pi v_\rho / \nu) n_\rho^2 + (\pi v_\sigma / 2) n_\sigma^2. \quad (5.6)$$

In the following we consider the behavior of a line junction, supposing that the $\nu = 2/3$ edges on either side of the junction are in this disorder-dominated phase. For this analysis, we will need the ‘local’ scaling dimensions δ of the edge operators, defined via $\langle O(x, \tau) O(x, 0) \rangle \sim \tau^{-2\delta}$. Using the above definitions, one finds

$$\delta(n_1, n_2) = \frac{3}{4} \left(n_1 + \frac{n_2}{3} \right)^2 + \frac{1}{4} (n_1 + n_2)^2. \quad (5.7)$$

Consider now a line junction, which will consist of two charge and two neutral modes, one above and the other below the junction. We denote these as $\phi_{\rho, a}$ for the ‘top’ and $\phi_{\rho, b}$ for the ‘bottom’ charge mode — and similarly for the two neutral modes. Tunneling processes that transfer charge from the top to bottom are expressed as products, $O^{a\dagger} O^b$. For example, the operator $O_{1,0}^{a\dagger}(x) O_{1,0}^b(x)$ tunnels an electron from top to bottom at position x along the junction. The appropriate term to add to the Hamiltonian is

$$\mathcal{H}_1 = \xi(x) O_{1,0}^{a\dagger}(x) O_{1,0}^b(x) + \text{H.c.}, \quad (5.8)$$

where again $\xi(x)$ is a random (complex) tunneling amplitude.

Generally, all tunneling processes that transfer charge from top to bottom edges in integer units of the electron charge are allowed. Of interest are the most relevant (or least irrelevant) of such operators, or equivalently those with the smallest scaling dimensions δ . There are three of these, with the same scaling dimension, $\delta(1,0) = \delta(2,-3) = \delta(1,-3) = 1$. The first two transfer a charge e , whereas zero charge is transferred for $(1,-3)$. A perturbative RG calculation for small disorder W , gives

$$\partial W / \partial \ell = (3 - 2\Delta)W, \quad (5.9)$$

where $\Delta = \delta_a + \delta_b$ is the total (local) scaling dimension of the operator $O^a O^b$.

If we ignore any Coulomb interactions between the modes on the top and bottom sides of the line junction, then we can use Eq. (5.7) to evaluate the scaling dimension, giving $\Delta = 2$. This implies that *all* electron tunneling processes are irrelevant. The line junction is in a conducting phase, with the electron backscattering strength scaling to zero at low energies. This should be the case when the gate potential is adjusted so that the top and bottom modes are well separated.

But as the gate potential is reduced, the modes get closer together, and the Coulomb interaction increases. Since the neutral modes do not carry any charge, the Coulomb interaction acts only between the top and bottom charge modes. Since these modes move in opposite directions, a Coulomb interaction will modify the scaling dimensions, just as in Sec. II. To see this consider the total Hamiltonian for the clean line junction, $\mathcal{H}_0 = \mathcal{H}_0^a + \mathcal{H}_0^b$. From Eq. (3.6) this factorizes into a sum of the charge and neutral sectors, $\mathcal{H}_0 = \mathcal{H}_0^\rho + \mathcal{H}_0^\sigma$, with

$$\mathcal{H}_0^\rho = (\pi v_\rho / \nu) [n_{\rho,a}^2 + n_{\rho,b}^2 + \lambda n_{\rho,a} n_{\rho,b}], \quad (5.10)$$

where we have included a Coulomb interaction, with (dimensionless) strength λ . Since the charge densities satisfy

$$[n_{a/b}(x), n_{a/b}(x')] = \pm (i\nu/2\pi) \partial_x \delta(x - x'), \quad (5.11)$$

one sees that the charge sector for $\nu = 2/3$ is formally identical to the full theory for ν^{-1} an odd integer, Eqs. (2.1) and (2.2). As in Sec. II, we can diagonalize the Hamiltonian \mathcal{H}_0^ρ by defining fields, $\phi_{\rho,a/b} = \sqrt{\pi}(\phi \pm \theta)$. In this way, \mathcal{H}_0^ρ takes the form of Eq. (2.10) with a charge stiffness,

$$K_\rho = \frac{1}{\nu} \left[\frac{1-\lambda}{1+\lambda} \right]^{1/2}. \quad (5.12)$$

Once the Hamiltonian is diagonalized, one can readily obtain the scaling dimension for the electron tunneling operator that appears in Eq. (5.9), giving

$$\Delta = \frac{1}{2} + \frac{3K_\rho}{2}, \quad (5.13)$$

where the first term is from the neutral sector. In the absence of Coulomb interactions between the top and bottom charge modes, $K_\rho = 1$, and the tunneling is irrelevant, as before.

However, with increasing Coulomb interaction, K_ρ decreases, and eventually $\Delta < 3/2$. When this happens, even weak backscattering grows under the RG transformation, and the system scales into a disorder-dominated phase. Although the properties of this phase are perturbatively inaccessible, it is natural to presume that both the charge and neutral excitations are localized in this phase.

We thereby conclude that a line junction in a $\nu = 2/3$ fluid should be qualitatively similar to a $\nu = 1/3$ junction. Two phases should be present, a conducting phase when the modes are well separated, and a localized phase. Upon tuning the gate potential, one should be able to pass through the localization phase transition separating the two phases. In the conducting phase, the electrical conductivity should diverge as a power law of temperature, precisely as in the analysis of Sec. III.

VI. CONCLUSION

A line junction in the fractional quantum Hall effect offers a unique opportunity for observing a one-dimensional localization transition. In this geometry, the 1D system is formally equivalent to a 1D electron gas with *attractive* interactions. With sufficiently strong attraction, localization ceases to be operative in 1D, and the system can undergo a metal-insulator transition. The key signature of this 1D metal-insulator transition is the temperature dependence of the conductivity, which diverges as a (variable) power of temperature in the metallic phase. In the insulating phase the conductivity is expected to drop rapidly with cooling, following a variable-range hopping law, whereas right at the transition a $1/T$ behavior with logarithmic corrections is predicted. As discussed in detail, the 1D conductivity can be extracted from a standard Hall transport measurement, in the regime where the Hall conductance is close to its quantized value.

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APPENDIX A: LINEAR RESPONSE THEORY FOR THE CONDUCTIVITY

Here we evaluate the conductivity in linear response theory, treating the disorder perturbatively in the Born approximation. This is valid in the conducting phase, where W scales to zero at low temperatures and scattering events are uncorrelated. The Kubo formula expresses the conductivity as a current-current correlation function:

$$\sigma = \frac{1}{\omega_n} \int_{x,\tau} \mathcal{D}_I(x,\tau) e^{-i\omega_n \tau} \Big|_{\omega_n \rightarrow i\omega + \epsilon} \quad (A1)$$

where

$$\mathcal{D}_I(x-x', \tau - \tau') = [\langle T_\tau I(x,\tau) I(x',\tau') \rangle]_{\text{ens}}. \quad (A2)$$

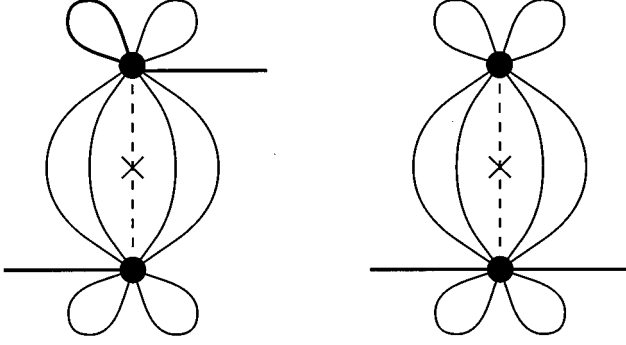


FIG. 4. Diagrams for the self-energy. The solid lines represent the propagators $\mathcal{D}_{\theta,0}$ and the dashed lines represent the impurity scattering vertex. A sum over all possible combinations of these lines is implied.

Here the square brackets denote an ensemble average over realizations of the disorder. After ensemble averaging the current correlation function is translationally invariant. Fourier transforming gives

$$\sigma = \frac{1}{\omega_n} \mathcal{D}_I(k=0, \omega_n) \Big|_{\omega_n \rightarrow i\omega + \epsilon}. \quad (\text{A3})$$

An expression for the current operator can be obtained by noting that the total one-dimensional density is $n = n_R + n_L = (\nu/\sqrt{\pi})\partial_x\theta$. Thus $\theta(x)$ is proportional to the integrated charge less than x , so that current operator is

$$I = \frac{\nu}{\sqrt{\pi}} \partial_t \theta. \quad (\text{A4})$$

The conductivity then becomes

$$\sigma = \frac{\omega_n \nu^2}{\pi} \mathcal{D}_{\theta}(k=0, \omega_n) \Big|_{\omega_n \rightarrow i\omega + \epsilon}, \quad (\text{A5})$$

where we have defined

$$\mathcal{D}_{\theta}(x-x', \tau-\tau') = [\langle T_{\tau} \theta(x, \tau) \theta(x', \tau') \rangle]_{\text{ens}}. \quad (\text{A6})$$

In the absence of any disorder,

$$\mathcal{D}_{\theta,0} = \frac{\nu K}{v^2 k^2 + \omega_n^2}. \quad (\text{A7})$$

With disorder present we write

$$\mathcal{D}_{\theta}^{-1} = \mathcal{D}_{\theta,0}^{-1} + \Sigma, \quad (\text{A8})$$

with a self-energy $\Sigma(k, \omega)$. This self-energy can readily be evaluated to leading order in the disorder strength. The relevant diagrams are shown in Fig. 4 and may be evaluated as

$$\Sigma(k, \omega_n) = \Delta_W \int_0^{\beta} d\tau (1 - e^{i\omega_n \tau}) \left[\frac{\pi T / \omega_c}{\sinh \pi T \tau} \right]^{2K}. \quad (\text{A9})$$

Analytically continuing to real time, this becomes $\Sigma(k, \omega) = c_K i \omega W / a (T / \omega_c)^{2(K-1)}$, where c_K is a numerical factor that depends on K . Then using Eqs. (3.10) and (3.13), we recover the conductance given in Eq. (3.5).

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