Einstein's Miracle Year - Exploring Brownian Motion
Einstein’s Miracle Year (1905)
What did he write about in 1905?

Brownian Motion
- Evidence for existence of Atoms/Molecules
- Estimation of the size of atoms/molecules
- Estimation of Avogadro’s number
- Evidence for microscopic ‘fluctuations’ of the properties of matter
- Random walks in nature
What did he write about in 1905?

- Theory of Special Relativity
  - New notions about the measurement of time and length intervals
  - Energy associated with mass of an object, $E = mc^2$

"Earth magic landscapes"
http://www.flytrap.it/public/faiweb/
What did he write about in 1905?

- **Photoelectric Effect**
  - Birth of quantum mechanics
  - Defined energy carried by photons (the smallest “particles” of light)
Brownian Motion: What is it? Who was Robert Brown?

Why do the particles move?
Brownian Motion all Around Us
Brownian Motion: Size Dependence

2 μm and 10 μm diameter beads in water
Particle Tracking

Profile

Centroid

Trajectory

$\mu$m

$\mu$m
Brownian Motion: Viscosity Dependence

2 μm beads in water

2 μm beads in 50% glycerol
Physics around 1900

What are ‘things’ made of?
If you keep dividing a drop of water into two parts, then what do you end up with?

Atomic Hypothesis: Matter is composed of atoms and molecules.
• Useful for understanding ‘chemical combination’, the behavior of gases, and other phenomena.
• BUT no direct evidence for atoms.
Physics around 1900

Thermodynamics from Statistical Behavior of Many Atoms

- Fluctuations

From: A Review of the Universe http://universe-review.ca
Einstein’s Picture of Water

- Brownian Particles are about 1 micron in diameter (1/1000th of a millimeter)
- Water Molecules are about 10,000 times smaller in diameter!
Einstein’s Picture of Water
Analysis of Trajectories
Analysis of Trajectories: Raw Images

t = 0 sec.  t = 2 sec.  t = 4 sec.

  t = 6 sec.  t = 8 sec.  t = 10 sec.
Analysis of Trajectories: Displacements

\[ \Delta r = \sqrt{\Delta x^2 + \Delta y^2} \]
## Analysis of Trajectories: Table

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<th>t (sec)</th>
<th>$\Delta x$ (µm)</th>
<th>$\Delta y$ (µm)</th>
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Mean Square Displacement vs. time

Particle Trajectory

Analysis of Trajectories: Plots
Analysis of Student’s Results
Computer Analysis of Tracking

\[ \Delta r^2 \propto t \]
\[ \Delta x^2 = \Delta y^2 = 2Dt \quad (1D) \]

\[ \Delta r^2 = 4Dt \quad (2D) \]

\[ \Delta r^2 = 6Dt \quad (3D) \]
What did Einstein Tell Us About Brownian Motion?

• **Microscopic picture of water.**
  – Composed of tiny particles (atoms/molecules)
  – Molecules have an average kinetic energy (KE) related to the water temperature \( \frac{1}{2} m v^2 \sim kT \)
  – Kinetic energy fluctuates! \( \Delta KE \sim kT \)

• **Collisional fluctuations of molecules**
  “hitting” large sphere cause the Brownian particle to move.
What did Einstein Tell Us About Brownian Motion?

- Collisions occur in random directions and at random times. Brownian particle ‘moves’ like a random walker.

\[
\Delta r^2 \sim D \, t ; \quad (\Delta r^2)^{1/2} \sim t^{1/2}
\]

\[
D = \frac{kT}{6\pi \eta a}
\]

- Particle Diffusion Coefficient
- Constant \( (N_{\text{Avogadro}} \, k = R) \)
- Temperature
- Viscosity
- Particle Radius
What did Einstein Tell Us About Brownian Motion?

- Results broadly applicable
  (e.g. small molecules dissolved in water)
**Key Elements of Random Walks**

**Step Size** = \( l^* \)
- (Distance ‘walker’ travels before its direction is changed)

**#Steps** = \( N \)
- (Total number of steps taken by ‘walker’)

**Mean-square-Displacement** = \( \Delta r^2 \)
- (Square of the ‘Net’ Distance walker moves)

\[ \Delta r^2 \sim N(l^*)^2 \; ; \; (\Delta r^2)^{1/2} \sim N^{1/2} (l^*) \]
Other Examples of Random Walks

Polymers

Very, very long chain molecules

Polyethylene: \( \cdots - \text{CH}_2 - \text{CH}_2 - \text{CH}_2 - \text{CH}_2 - \cdots \) or \([\text{CH}_2]_N\)

\(N\) repeats of fundamental chemical unit.
Other Examples of Random Walks (Polymers)

What do polymer chains really look like?

Not straight!
Polymer chains are built from ‘steps’ or ‘chainlinks’ of molecules.

Direction of neighboring ‘chainlinks’ changes randomly.
Other Examples of Random Walks (Polymers)

Conformation (i.e. the shape) of a Polymer Chain traces out a random walk like path!

Stepsize = ‘chainlink’ length = \( l^* \)

\#Steps = Total length/stepsize = \( \frac{L}{l^*} \)

Typical size \( \sim (\frac{L}{l^*})^{1/2} l^* \)

Chester Liu
Universal Molecular Simulator
http://rati.pse.umass.edu/~cht/research/usim/
Other Examples of Random Walks (Light)
Other Examples of Random Walks (Light)

Light diffuses through lot’s of media!

When light is scattered its direction changes.

Stepsizes = \((\text{speed of light}) \times (\text{time between scattering events})\)

\#Steps = \((\text{Total time}) / (\text{time between scattering events})\)

Distance traveled \(\sim (\text{Total time})^{1/2}\)
Other Examples of Random Walks (Light)

Current Applications: Imaging in Human Tissue

20th Century Fox: Minority Report
More on Einstein

Light Absorption and Emission

- Stimulated and Spontaneous Emission – Basis for Modern Day Lasers

From: R. Keenan, Ph.D.
Providence High School, Charlotte, NC

From: University of Aberdeen
http://www.eng.abdn.ac.uk/comms/candi/facilities.html
More on Einstein

Theory of General Relativity
(A fundamentally better description of gravity)

From: ZARM - Center of Applied Space Technology and Microgravity
http://www.zarm.uni-bremen.de

From: NASA http://www.nasa.gov
More on Einstein

Critic of Quantum Mechanics

Taken from: http://chaos.swarthmore.edu
More on Einstein

Influence on World Affairs

- Urged development of Atomic Bomb during WW II
- Later became a pacifist
Photoelectric Effect

ALBERT EINSTEIN

Fundamental ideas and problems of the theory of relativity

Lecture delivered to the Nordic Assembly of Naturalists at Gothenburg*
July 11, 1923

If we consider that part of the theory of relativity which may nowadays in a sense be regarded as bona fide scientific knowledge, we note two aspects which have a major bearing on this theory. The whole development of the theory turns on the question of whether there are physically preferred states of motion in Nature (physical relativity problem). Also, concepts and distinctions are only admissible to the extent that observable facts can be assigned to them without ambiguity (stipulation that concepts and distinctions should have meaning). This postulate, pertaining to epistemology, proves to be of fundamental importance.