Exp 68: The Force Between Current Carrying Conductors

Discussion:

The MKS unit of current, the ampere, is defined as follows:

> One ampere is that unvarying current, which if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2×10^{-7} Newtons per meter of length

The current balance is an instrument which measures the force experienced by two parallel conductors carrying the current. Since the method is based upon the definition of the ampere, this measurement provides an absolute determination of the ampere.

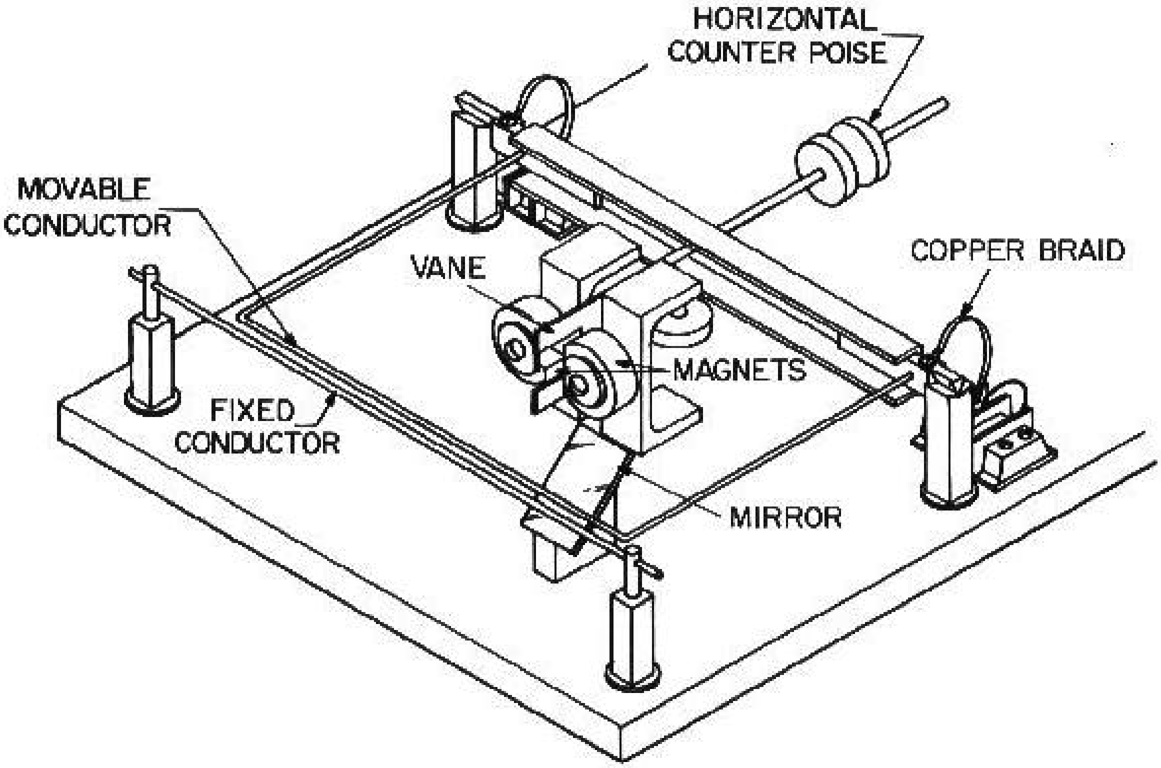


Figure 1: The Current Balance

The instrument consists of two wires mounted so that they are parallel (over some part of their lengths) and are connected electrically in series. The lower wire is rigidly mounted to the instrument base while the upper one is suspended above it by means of knife edges. The operation of this balance is the same in principle as that of a conventional analytical balance. Initially, the wires are in equilibrium separated by a small distance adjusted by moving the counter-weight. Pieces of accurately weighed wire are placed on the upper wire, displacing it downward. If current passes through the system the wires repel each other and the initial equilibrium can be reestablished by proper adjustment of the current. The equilibrium position can be noted using a microscope which allows measurement of the separation of the wires. When equilibrium is established with a given current and a known mass on the wire, the magnitude of the current may be determined in terms of the force using the fundamental force equations and basic geometry.

We need an expression for the force experienced by parallel current-carrying conductors. Since each conductor lies in the magnetic field set up by the other, each will experience a force which depends on the current flowing through both wires. If the lower wire, carrying current I' is very long compared to *the center to center* distance d_0 between the two wires (so that end-effects may be neglected) the magnetic field B which it produces at the position of the upper wire is

$$B = \frac{\mu_0 2I'}{4\pi d_0} \tag{1}$$

where d_0 is the distance between the conductors in meters and $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T}m\mathrm{A}^{-1}$. The field produces a force F on length L of the upper wire carrying a current I

$$F = ILB = \frac{\mu_0}{4\pi} L \frac{2I\ I'}{d_0} \tag{2}$$

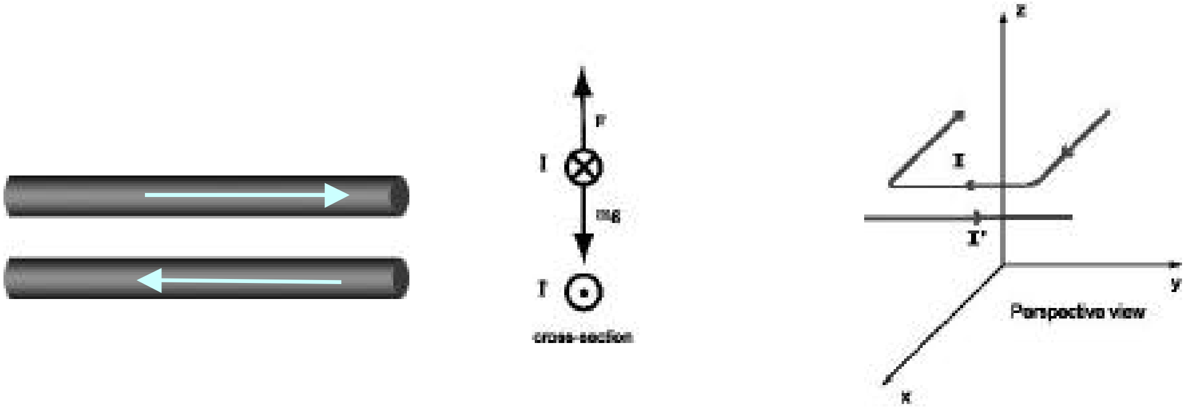
Since the two conductors in this balance are connected in series, I and I' are equal and their product may be replaced by I^2. Since the currents flow in opposite directions the conductors repel one another.

In operation the horizontal counterpoise is adjusted so that in the *absence* of currents or *additional mass* on the wire the upper conductor is balanced several millimeters above the lower conductor. A known mass is then placed on the upper wire, which will move downward. The force exerted by this mass (mg) may now be counterbalanced by passing sufficient current through the circuit to restore equilibrium. The required current may be found by equating the magnetic repulsion given by (2) to the downward force of gravity:

$$mg = F = \frac{\mu_0 2L}{4\pi d_0} I^2 \tag{3}$$

Therefore, a plot of the added mass, m, as a function of the square of the current needed to restore equilibrium will be a straight line as long as L and d_0 are kept constant.

The direction of the force between the two copper conductors of the current balance is shown in Figure 2. When the current in the upper conductor (I) is directed into the page,



The conductors as they look in the microscope and cross section diagram of the currents into and out of the page. The repulsive force is directed upward.

Perspective drawing of the currents in the conductors

Figure 2: The current carrying conductors

Magnetic Damping of the Balance:

Procedure

The balance is a delicate apparatus and should be handled with care at all times. Watch out for air currents and vibrations.

1. Trace the flow of current through the balance. Verify that the current is in opposite directions in the parallel conductors.

2. Verify that the moveable conductor is directly above and parallel to the fixed conductor. If it is not ask for assistance.

3. Adjust the horizontal counterpoise so that the visible space between the parallel conductors at initial equilibrium is approximately 1 mm. Be careful that this adjustment does not disturb the alignment in (2).

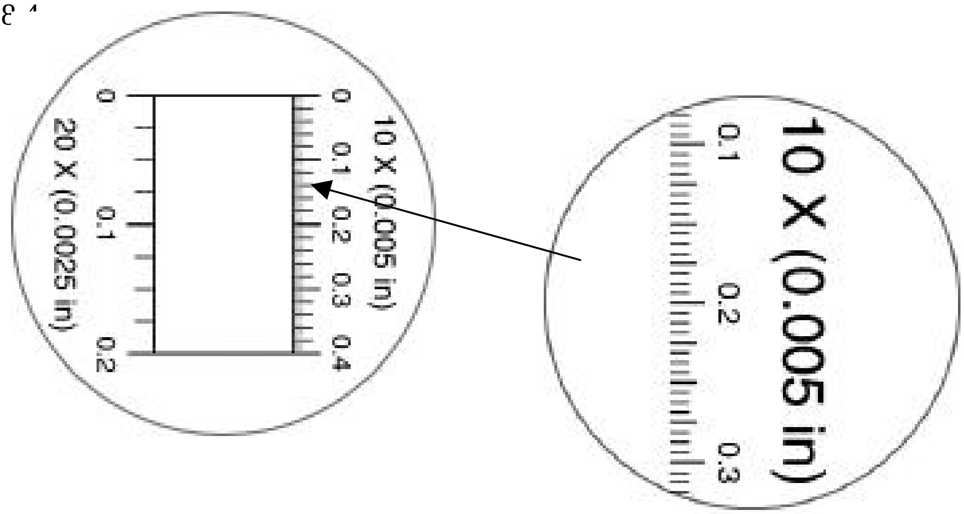


Figure 3: The microscope Reticle The 10X scale shown enlarged

4. Adjust the microscope and mirror so that you can see the two conductors in the mirror. Measure the separation between the conductors at equilibrium using the $10 \times$ scale of the microscope. Calculate the center to center distance d_0 between the conductors. The conductors are made of (nominally) $\frac{1}{8}$ inch diameter wire. Measure this diameter with a micrometer. Be very careful not to disturb the balance. Use this diameter to check the actual dimensions as read with the reticle scale.

5. Measure the appropriate length L. This is the length of the upper wire over which the force exerted by the lower wire acts.

6. Carefully place a 100 mg weight (piece of wire) on the upper wire and increase the current through the balance until it returns to the initial equilibrium position in (3). Record the current.

7. Carefully remove the weight and check that the moveable conductor returns to the same equilibrium position. If it does not, readjust the counterweight and start over. If it does, record the current and repeat this measurement four more times. Calculate the mean current and its uncertainty.

8. Repeat this measurement for other added masses up to a maximum current of 20 amperes. Plot the force as a function of I^2. Find the slope of the resulting line and calculate $\frac{\mu_0}{4\pi}$ and its uncertainty. Compare this to the accepted value. (Be careful of units - all must be MKS.)

9. Readjust the horizontal counterpoise so that the edge separation between the wires at equilibrium is approximately 2 to 3 mm and repeat (7) and (8).

10. From you graph, determine your best value of $\frac{\mu_0}{4\pi}$ and the associated uncertainty. How does your value compare with the defined value of $10^{-7}\,mA^{-1}$? Discuss this in terms of your error analysis.

11. Assuming your graph is a fairly good straight line and the slope is about right when uncertainties are considered, what physical relationships have you confirmed in this experiment?